MATH 2010

Test # 2

March 29, 2011

Name:____

You must **show all work** to receive full credit. Parts of questions will not necessarily be weighted equally.

1. (10 points) Use *adjoints* to find the inverse of the following matrix

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 5 & 2 \\ 0 & 0 & 3 \end{bmatrix}$$

2. (10 points) Given $\mathbf{u} = (1, -1, 2)$, $\mathbf{v} = (0, 2, 3)$ and $\mathbf{w} = (0, 1, 1)$, find $\mathbf{x}$ provided

$$5\mathbf{u} - 2\mathbf{x} = 3\mathbf{v} + \mathbf{w} - (\mathbf{u} \cdot \mathbf{w})\mathbf{x}.$$

3. (10 points) Given $\mathbf{u} = (10, -5, 15)$ and $\mathbf{v} = (-2, 1, -3)$, find

   (a) the angle between $\mathbf{u}$ and $\mathbf{v}$

   (b) a unit vector in the direction of $\mathbf{v}$

4. (10 points) Find the representation for all vectors in R^4 that are simultaneously orthogonal to all both vector: $\mathbf{u} = (1, 1, 1, 1)$ and $\mathbf{v} = (-2, -2, -2, 3)$. (Hint: you may need to solve a linear system.)

5. (10 points) Find $(3\mathbf{u} - \mathbf{v}) \cdot (\mathbf{u} - 3\mathbf{v})$ given $\mathbf{u} \cdot \mathbf{u} = 8$, $\mathbf{u} \cdot \mathbf{v} = 7$, and $\mathbf{v} \cdot \mathbf{v} = 6$.

6. (10 points) Determine whether the set of all ordered triplets of real numbers with the operations

$$[x, y, z] \oplus [x', y', z'] = [0, 0, 0]$$

and

$$c \odot [x, y, z] = [cx, cy, cz]$$

is a vector space. Show all supporting work for credit. If V is a vector space, you must show all properties are satisfied. If V is not a vector space, you need to only correctly show one property that fails.

Recall, a vector space V must satisfy the following properties:

(A) If $\mathbf{u}$ and $\mathbf{v}$ are any elements of V, then $\mathbf{u} \oplus \mathbf{v}$ is in V.

- (A1) $\mathbf{u} \oplus \mathbf{v} = \mathbf{v} \oplus \mathbf{u}$, for $\mathbf{u}$ and $\mathbf{v}$ in V.
- (A2) $\mathbf{u} \oplus (\mathbf{v} \oplus \mathbf{w}) = (\mathbf{u} \oplus \mathbf{v}) \oplus \mathbf{w}$, for $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ in V.
- (A3) There is an element $\mathbf{0}$ in V such that $\mathbf{u} \oplus \mathbf{0} = \mathbf{0} \oplus \mathbf{u} = \mathbf{u}$ for all $\mathbf{u}$ in V.
- (A4) For each $\mathbf{u}$ in V, there is an element $-\mathbf{u}$ in V such that $\mathbf{u} \oplus -\mathbf{u} = \mathbf{0}$.

(S) If $\mathbf{u}$ is any element in V and c is any real number, then $c \odot \mathbf{u}$ is in V.

- (S1) $c \odot (\mathbf{u} \oplus \mathbf{v}) = c \odot \mathbf{u} \oplus c \odot \mathbf{v}$ for all real numbers c and all $\mathbf{u}$ and $\mathbf{v}$ in V.
- (S2) $(c + d) \odot \mathbf{u} = c \odot \mathbf{u} \oplus d \odot \mathbf{u}$ for all real numbers c and d and all $\mathbf{u}$ in V.
- (S3) $c \odot (d \odot \mathbf{u}) = (cd) \odot \mathbf{u}$ for all real numbers c and d and all $\mathbf{u}$ in V.
- (S4) $1 \odot \mathbf{u} = \mathbf{u}$ for all $\mathbf{u}$ in V.

7. (10 points) Consider the set W of all vectors in 2x2 matrices of the form

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

such that

$$b = 0 \quad \text{and} \quad a + c + d = 0$$

with standard operations in $M_{2,2}$:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \oplus \begin{bmatrix} a' & b' \\ c' & d' \end{bmatrix} = \begin{bmatrix} a+a' & b+b' \\ c+c' & d+d' \end{bmatrix}$$

and

$$k \odot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} ka & kb \\ kc & kd \end{bmatrix}.$$

Is W a subspace of $M_{2,2}$? Show **all** supporting work for credit.

8. (10 points) Determine whether the polynomials $p_1 = 2t^2 + t + 1$, $p_2 = 3t^2 + 1$, $p_3 = 5t^2 + t + 2$ and $p_4 = 6t^2 + 2$ span P_2.

9. (10 points) Determine if the set $S = \{[1, 1, 2, 3], [4, 1, 5, 6], [7, 1, 8, 3]\}$ is linearly independent or dependent.

10. (a) (2 points) State the requirements for a set S to be a basis for a vector space V.

    (b) (8 points) Determine if the set $S = \left\{ \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}, \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} -3 & 0 \\ -1 & 1 \end{bmatrix}, \begin{bmatrix} 4 & 1 \\ 0 & 0 \end{bmatrix} \right\}$ is a basis for $M_{2,2}$. Explain how what you did shows that S is or is not a basis for $M_{2,2}$, i.e., explain how what you did shows that S satisfies or does not satisfy all the requirements stated in part (a).