MATH 2010

Test # 3

April 21, 2011

Name:____

You must **show all work** to receive full credit. Parts of questions will not necessarily be weighted equally.

1. Consider the basis $B = \{[-1,-1],[0,1]\}$ and $B' = \{[1,2],[2,5]\}$ and assume

$$[x]_B = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

   (a) (6 points) Find x in the standard basis.

   (b) (6 points) Find the transition matrix from B to B'.

   (c) (5 points) Find $[x]_{B'}$.

   (d) (6 points) If T is a linear transformation from $\Re^2 \to \Re^3$ such that $T([1,2]) = [1,-1,0]$ and $T([2,5]) = [0,1,-2]$, find $T(x)$ where $[x]_B$ is given as

$$[x]_B = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

   Hint: use the information obtained in part (c).

2. (8 points) Find a subset of the vectors $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4, \mathbf{v}_5, \mathbf{v}_6\}$ that forms a basis for the space spanned by the vectors where

$$\mathbf{v}_1 = [0,2,2], \mathbf{v}_2 = [0,4,4], \mathbf{v}_3 = [-2,-10,-5], \mathbf{v}_4 = [0,6,6], \mathbf{v}_5 = [6,12,-5], \mathbf{v}_6 = [12,28,-1]$$

3. Given

$$A = \begin{bmatrix} 1 & 3 & -2 & 0 & 2 & 0 \\ 2 & 6 & -5 & -2 & 4 & -3 \\ 0 & 0 & 5 & 10 & 0 & 15 \\ 2 & 6 & 0 & 8 & 4 & 18 \end{bmatrix}$$

   and it's reduced row echelon form

$$B = \begin{bmatrix} 1 & 3 & -2 & 0 & 2 & 0 \\ 0 & 0 & 1 & 2 & 0 & 3 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

   (a) (5 points) Find a basis for the row space of A.

   (b) (5 points) Find a basis for the column space of A.

   (c) (7 points) Find a basis for the nullspace of A.

   (d) (1 point) What is the nullity of A?

   (e) (1 point) What is the rank of A?

4. (2 points) Find the domain and codomain of the transformation $T_A(x) = Ax$ where A is a 3x5 matrix.

5. (8 points) Determine whether the linear transformation

$$T(x, y, z) = (x + 1, -3y)$$

is linear. Show all necessary steps.

6. Given the linear transformation

$$T([x_1, x_2, x_3]) = [-2x_1 + 4x_2, -x_1 + 2x_2, -3x_3]$$

   (a) (6 points) Find the standard matrix for the operator T.
   (b) (7 points) Find the preimage of $\mathbf{w} = [6, 3, -3]$ for the transformation.
   (c) (4 points) Determine if the transformation is one-to-one.
   (d) (1 point) Does an inverse operator exist, if so, find the standard matrix for the inverse operator.
   (e) (5 points) Find a basis for $ker(T)$.
   (f) (5 points) Find a basis for the range of T.

7. Given

$$A = \begin{bmatrix} 4 & 2 \\ 0 & -1 \end{bmatrix}$$

   (a) (5 points) Find the eigenvalues of A.
   (b) (7 points) Find the corresponding eigenvectors for the matrix.