MATH 2010

Test # 1

February 14, 2012

Name:____

You must **show all work** to receive full credit. **No work = no credit**!! Parts of questions will not necessarily be weighted equally.

1. (12 points) Solve the following system by hand using either Gaussian elimination and back substitution *or* Gauss-Jordan elimination. Make sure to clearly mark your solutions and show your work.

$$
\begin{array}{rcrcrcrcr}
-3x_1 & - & 6x_2 & + & x_3 & + & 13x_4 & = & -12 \\
x_1 & + & 2x_2 & & & - & 3x_4 & = & 5 \\
-x_1 & - & 2x_2 & + & 2x_3 & + & 11x_4 & = & 1 \\
2x_1 & + & 4x_2 & + & x_3 & - & 2x_4 & = & 13
\end{array}
$$

2. (4 points each) Determine the solution x for each system $Ax = b$ where the following matrices represent the row-echelon form of the augmented matrix $[A|b]$.

(a) $\begin{bmatrix} 0 & 1 & -1 & 3 & 2 \\ 0 & 0 & 1 & 4 & -5 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & -2 & 3 & 0 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

3. (12 points) Find the values of k for which the following system does *not* have a unique solution.

$$\begin{array}{rcrcr} (k-1)x_1 & + & 3x_2 & = & -2 \\ 2x_1 & + & (k-2)x_2 & = & 0 \end{array}$$

4. (12 points) Find the numbers x and y such that $A^T B = C$ where

$$A = \begin{bmatrix} 2 & y \\ x & 0 \\ y & y \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ x & 1 \\ 1 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & -8 \\ -12 & -6 \end{bmatrix}$$

5. Given the matrix

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 3 & 3 & 4 \\ 2 & 2 & 3 \end{bmatrix}$$

(a) (12 points) Find A^{-1} if it exists.

(b) (4 points) Let

$$b = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}.$$

Use A^{-1} from part (a) to find the solution to $Ax = b$.

6. (12 points) Find $|A|$ for the matrix using **cofactor expansion**

$$A = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -4 & 3 \\ 2 & 1 & 0 \end{bmatrix}$$

7. (6 points) Prove that if A is symmetric and nonsingular, then A^{-1} is symmetric. (Hint: Recall that a matrix B is symmetric if $B^T = B$.)

8. (12 points) Find the eigenvalues and corresponding eigenvectors for the matrix

$$A = \begin{bmatrix} 1 & 0 \\ 2 & -3 \end{bmatrix}$$

9. (1 point each) Suppose that A, B, C, D, and E are matrices with the following sizes:

$$A : (4x1) \quad B : (4x8) \quad C : (8x2) \quad D : (1x2) \quad E : (1x4)$$

Determine whether the given matrix expression is defined. For those that are defined, give the size of the resulting matrix.

(a) EA ____________________

(b) $2A + C$ ____________________

(c) $C^T(EB)^T + D^T$ ____________________

10. (1 point each) For the each of the following, determine if the statement is true or false for general matrices A and B.

(a) A homogeneous system (i.e., a system in which the right hand side, b, equals 0) always has a solution. ____________________

(b) If $AB = AC$, then $B = C$. ____________________

(c) $(A^T)^T = A$ only if A is symmetric. ____________________

(d) $(AB)^{-1} = B^{-1}A^{-1}$. ____________________

(e) There is always a solution to the system $Ax = b$. ____________________

(f) $|4A| = 4|A|$ for all 4x4 matrices A. ____________________

(g) If A is an invertible nxn matrix, then $|A^T| = |A^{-1}|$. ____________________

(h) If A is an invertible nxn matrix, then $|A^T| = \frac{1}{|A^{-1}|}$. ____________________

(i) If A is a 6x4 matrix and B is a mxn matrix such that B^TA^T is a 2x6 matrix, then m must equal 2 and n must equal 4. ____________________