MATH 2010

Test # 2

March 27, 2012

Name:____

You must **show all work** to receive full credit. **No work = no credit**!! Parts of questions will not necessarily be weighted equally.

1. (10 points) Determine whether the set of all ordered triplets of real numbers with the operations

$$[x, y, z] \oplus [x', y', z'] = [x + x', y + y', z + z']$$

and

$$c \odot [x, y, z] = [cx, y, cz]$$

is a vector space. Show all supporting work for credit. If V is a vector space, you must show all properties are satisfied. If V is not a vector space, you need to only correctly show one property that fails.

Recall, a vector space V must satisfy the following properties:

(A) If $\mathbf{u}$ and $\mathbf{v}$ are any elements of V, then $\mathbf{u} \oplus \mathbf{v}$ is in V.

(A1) $\mathbf{u} \oplus \mathbf{v} = \mathbf{v} \oplus \mathbf{u}$, for $\mathbf{u}$ and $\mathbf{v}$ in V.

(A2) $\mathbf{u} \oplus (\mathbf{v} \oplus \mathbf{w}) = (\mathbf{u} \oplus \mathbf{v}) \oplus \mathbf{w}$, for $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ in V.

(A3) There is an element $\mathbf{0}$ in V such that $\mathbf{u} \oplus \mathbf{0} = \mathbf{0} \oplus \mathbf{u} = \mathbf{u}$ for all $\mathbf{u}$ in V.

(A4) For each $\mathbf{u}$ in V, there is an element $-\mathbf{u}$ in V such that $\mathbf{u} \oplus -\mathbf{u} = \mathbf{0}$.

(S) If $\mathbf{u}$ is any element in V and c is any real number, then $c \odot \mathbf{u}$ is in V.

(S1) $c \odot (\mathbf{u} \oplus \mathbf{v}) = c \odot \mathbf{u} \oplus c \odot \mathbf{v}$ for all real numbers c and all $\mathbf{u}$ and $\mathbf{v}$ in V.

(S2) $(c + d) \odot \mathbf{u} = c \odot \mathbf{u} \oplus d \odot \mathbf{u}$ for all real numbers c and d and all $\mathbf{u}$ in V.

(S3) $c \odot (d \odot \mathbf{u}) = (cd) \odot \mathbf{u}$ for all real numbers c and d and all $\mathbf{u}$ in V.

(S4) $1 \odot \mathbf{u} = \mathbf{u}$ for all $\mathbf{u}$ in V.

2. (10 points) Consider the subset W in R^2 where $W = \{[x, y] : y = ax^2\}$ with standard operations:

$$[x, y] \oplus [x', y'] = [x + x', y + y']$$

and

$$c \odot [x, y] = [cx, cy]$$

Is W a subspace of R^2? Show **all** supporting work for credit.

3. (10 points) Determine whether the set $S = \{[0, 1, -2, 0], [1, 0, 0, 8], [1, -1, 0, 0], [0, 1, 0, -4], [1, -1, 0, 0]\}$ spans R^4.

4. (10 points) Determine if the set

$$S = \left\{ \begin{bmatrix} 2 & -4 \\ 3 & 1 \end{bmatrix}, \begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & -2 \\ 3 & -1 \end{bmatrix} \right\}$$

is linearly independent or dependent.

5. (10 points) Find *all* values of t for which the set $S = \{[t, t, t], [t, 1, 0], [t, 0, 1]\}$ is linearly independent.

6. (a) (2 points) State the requirements for a set S to be a basis for a vector space V.
   (b) (10 points) Determine if the set $S = \{1-t, 2t+3t^2, t^2-2t^3, 2+t^3\}$ is a basis for P_3. Explain how what you did shows that S is or is not a basis for P_3, i.e., explain how what you did shows that S satisfies or does not satisfy all the requirements stated in part (a).

7. Given

$$A = \begin{bmatrix} 1 & 0 & 2 & 0 & 0 & 2 \\ 2 & -1 & 1 & -2 & 1 & 2 \\ 1 & 0 & 2 & 0 & 1 & -2 \\ -2 & 3 & 5 & 6 & -2 & 0 \\ 1 & -1 & -1 & -2 & 1 & 0 \end{bmatrix}$$

and its row echelon form

$$B = \begin{bmatrix} 1 & -1 & -1 & -2 & 1 & 0 \\ 0 & 1 & 3 & 2 & -1 & 2 \\ 0 & 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

   (a) (1 point) What is the rank of A?
   (b) (4 points) Find a basis for the column space of A.
   (c) (4 points) Find a basis for the row space of A.
   (d) (5 point) Find a basis for the nullspace of A.
   (e) (1 point) What is the nullity of A?

8. (5 points) Find a subset of the vectors $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4, \mathbf{v}_5, \mathbf{v}_6\}$ that forms a basis for the space spanned by S where

$$\mathbf{v}_1 = [1,2,1,-2,1], \mathbf{v}_2 = [0,-1,0,3,-1], \mathbf{v}_3 = [2,1,2,5,-1],$$
$$\mathbf{v}_4 = [0,-2,0,6,-2], \mathbf{v}_5 = [0,1,1,-2,1], \mathbf{v}_6 = [2,2,-2,0,0].$$

Below are two matrices A and a corresponding row echelon form B. Some of this information may be useful in solving this problem.

- $A = \begin{bmatrix} 1 & 2 & 1 & -2 & 1 \\ 0 & -1 & 0 & 3 & -1 \\ 2 & 1 & 2 & 5 & -1 \\ 0 & -2 & 0 & 6 & -2 \\ 0 & 1 & 1 & -2 & 1 \\ 2 & 2 & -2 & 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 & 1 & -2 & 1 \\ 0 & 1 & 0 & -3 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$
- $A = \begin{bmatrix} 1 & 0 & 2 & 0 & 0 & 2 \\ 2 & -1 & 1 & -2 & 1 & 2 \\ 1 & 0 & 2 & 0 & 1 & -2 \\ -2 & 3 & 5 & 6 & -2 & 0 \\ 1 & -1 & -1 & -2 & 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -1 & -1 & -2 & 1 & 0 \\ 0 & 1 & 3 & 2 & -1 & 2 \\ 0 & 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

9. (a) (3 points) Determine the tail of the vector $\mathbf{u} = [-1,-3]$ whose head is at $(0,2)$.
   (b) (4 points) Sketch the vector from part (a) in both the original position and the standard position.

10. (2 points each) Which of the following vectors are scalar multiples of $\mathbf{u} = \left[\frac{1}{2}, -\frac{2}{3}, \frac{3}{4}\right]$?

   (a) $\mathbf{v} = [6, -4, 9]$

   (b) $\mathbf{w} = \left[-1, \frac{4}{3}, -\frac{3}{2}\right]$

11. (10 points) If possible, write $\mathbf{v} = [2, 5, -4]$ as a linear combination of $\mathbf{u}_1$, $\mathbf{u}_2$, and $\mathbf{u}_3$ where

$$\mathbf{u}_1 = [1, 3, 2], \qquad \mathbf{u}_2 = [2, -2, -5], \qquad \mathbf{u}_3 = [2, -1, 3]$$