MATH 2010

Test # 3

April 19, 2012

Name:____

You must **show all work** to receive full credit. Parts of questions will not necessarily be weighted equally.

1. Let $\mathbf{u} = [1, 1, 0]$ and $\mathbf{w} = [0, -\sqrt{2}, 0]$.
   (a) (6 points) Find a unit vector parallel to $\mathbf{u}$, having the same direction.
   (b) (8 points) Find the angle between $\mathbf{u}$ and $\mathbf{w}$.
   (c) (6 points) Find the value of y such that $[-3, y, 10]$ is perpendicular (orthogonal) to $\mathbf{u}$.
   (d) (8 points) Find all vectors in $\Re^3$ that are simultaneously perpendicular (orthogonal) to both $\mathbf{u}$ and $\mathbf{w}$.

2. (2 points) Find the domain and codomain of the transformation $T_A(x) = Ax$ where A is a 3x5 matrix.

3. (10 points) Determine whether the linear transformation

$$T(x, y, z) = (x + 1, -3y)$$

   is linear. Show all necessary steps.

4. Given the linear transformation

$$T([v_1, v_2, v_3]) = [-2v_1 + 4v_2, -v_1 + 2v_2, -3v_3]$$

   (a) (8 points) Find the standard matrix for the operator T.
   (b) (8 points) Find the preimage of $\mathbf{w} = [6, 3, -3]$ for the transformation.
   (c) (8 points) Find a basis for $ker(T)$.
   (d) (8 points) Find a basis for the range of T.

5. Given the linear transformation

$$T([v_1, v_2]) = [3v_1 + 6v_2, -v_2]$$

   (a) (2 points) Determine if the transformation is one-to-one.
   (b) (8 point) Does an inverse operator exist? If so, find the standard matrix for the inverse operator.
   (c) (4 points) Write the inverse linear transformation in the form above, i.e. $T^{-1}([w_1, w_2]) = [\_,\_]$.

6. Assume A is a $6x6$ matrix with characteristic equation given by

$$\lambda^2(\lambda-3)^3(\lambda+4)=0$$

(a) (6 points) List the distinct eigenvalues and give the algebraic multiplicity for each eigenvalue.

(b) (8 points) Given the following cases, determine in each case whether or not A is diagonalizable. If A is diagonalizable, then find a matrix P that diagonalizes A and $P^{-1}AP$ *without* calculating P^{-1}.

- **Case 1:** For one of the eigenvalues, there is one associated eigenvector

$$x_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

For the other eigenvalue, there are three associated eigenvectors given by

$$x_2 = \begin{bmatrix} 2 \\ -3 \\ 9 \\ 0 \\ 0 \\ 0 \end{bmatrix}, x_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 3 \\ 0 \\ 0 \end{bmatrix} x_4 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 3 \\ 0 \end{bmatrix}$$

- **Case 2:** For one of the eigenvalues, there is one associated eigenvector

$$x_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

For another eigenvalue, there are two associated eigenvectors given by

$$x_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, x_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

For a final eigenvalue, there are three associated eigenvectors

$$x_4 = \begin{bmatrix} 2 \\ -3 \\ 9 \\ 0 \\ 0 \\ 0 \end{bmatrix}, x_5 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 3 \\ 0 \\ 0 \end{bmatrix} x_6 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 3 \\ 0 \end{bmatrix}$$