

# Worksheet - Vector Spaces

Math 2010

Due Friday, March 2 by 4:00 p.m.

1. (2 points each) Determine whether the given set together with the given operations is a vector space. If it is not a vector space, show that at least one property that fails to hold. If it is a vector space, you must show all 10 properties hold.

- (a) The set of all ordered triples of real numbers  $(x, y, z)$  with the operations

$$(x, y, z) \oplus (x', y', z') = (x', y + y', z')$$

and

$$c \odot (x, y, z) = (cx, cy, cz).$$

- (b) The set of all ordered triples of real numbers of the form  $(0, 0, z)$  with the operations

$$(0, 0, z) \oplus (0, 0, z') = (0, 0, z + z')$$

and

$$c \odot (0, 0, z) = (0, 0, cz).$$

2. (2 points each) Determine whether the subset  $W$  is a subspace of the given vector space  $V$ .

- (a)  $W$  is the set of all ordered pairs of real numbers  $(x, y)$  such that  $x > 0$  and  $y > 0$ ;

$$[x, y] \oplus [x', y'] = [x + x', y + y']$$

and

$$c \odot [x, y] = [cx, cy].$$

Determine if  $W$  is a subspace of  $\mathbb{R}^2$ .

- (b)  $W$  is the set of all polynomials of the form  $at^2 + bt + c$  where  $a, b$ , and  $c$  are real numbers with  $b = a + 1$ ;

$$(a_1t^2 + b_1t + c_1) \oplus (a_2t^2 + b_2t + c_2) = (a_1 + a_2)t^2 + (b_1 + b_2)t + (c_1 + c_2)$$

and

$$r \odot (at^2 + bt + c) = (ra)t^2 + (rb)t + rc.$$

Determine if  $W$  is a subspace of  $P_2$ .

- (c)  $W$  is the set of all diagonal 3x3 matrices with normal addition and scalar multiplication. Determine if  $W$  a subspace of  $M_{3,3}$ .