Homework 6
Numerical Analysis
Math 4257/5257
Due December 6, 2010

1. (15 points) Examine the problem

$$
\begin{gathered}
\frac{d y}{d t}=y t^{2}+1.1 y \\
y(0)=1
\end{gathered}
$$

on the interval $[0,2]$.
a. Solve the problem analytically.
b. Use euler's method by hand to find $y_{1}$ and $y_{2}$ with a step size of 0.5 on the interval $\mathrm{x}=0$ to 2 .
c. Use the Matlab code eulode to approximate y, again with a step size of 0.5 on the interval [0,2]. Compare iterates $y_{1}$ and $y_{2}$ which you did by hand in $\# 2$ with the output from the Matlab routine. They should match.
d. Plot the exact solution and approximate solution on the interval [0,2]. In plotting the exact solution, use 100 points along the interval $[0,2]$ to get smooth curve. Describe the difference.
e. Try using h=0.25, what happens, if anything, when comparing the graph of the approximate solution to the graph of the exact solution?
f. Write a script or function file which compares the iterates of the euler function with the true value at each time step by calculating the error at each time step. Try running your code with a couple of different values of h: $0.5,0.25,0.2,0.1$. Display your results in a chart similar to:

| $\mathbf{t}$ | Exact y | Approx. y | Error (\%) |
| :---: | :---: | :---: | :---: |
|  |  |  |  |
|  |  |  |  |

g. Why does the error keep increasing with each iterate?
2. (10 points) Number 7.1 in your textbook Numerical Computing with MATLAB by Cleve Moler.
3. (25 points) Number $7.15 a-c$ in your textbook Numerical Computing with MATLAB by Cleve Moler. Use an event handler for all of these ( $a-c$ ). Use the values given in $a$ and $b$ to test your stopping criteria. (Hint: use a stopping criteria similar to the twobody problem.)
4. (20 points) Number 7.16 in your textbook Numerical Computing with MATLAB by Cleve Moler.
5. ( 30 points) Number 7.19 in your textbook Numerical Computing with MATLAB by Cleve Moler. (Hint: this problem will involve a stopping criteria similar to the falling body problem and a zero root finder, such as fzero.)

