Homework #2

Math 4257/5257
Due Midnight, October 12

- Create a script file that performs calls the appropriate .m files for the following problems. Publish the report as an html file. Zip all the .m files, figures and html file together and upload zip file to the dropbox in D2L. Your work will be graded on the following criteria:
  - Correct use of appropriate algorithms.
  - Correct answers.
  - Appropriate use of function and script files.
  - Sufficient comments in code.
  - Correct script file to TEST all algorithms for ALL parts of the problems and to publish work as a .html file.

1. The saturation concentration of dissolved oxygen in freshwater can be calculated with the equation

$$\ln o_{sf} = -139.34411 + \frac{1.575701 \times 10^5}{T_a} - \frac{6.642308 \times 10^7}{T_a^2} + \frac{1.243800 \times 10^{10}}{T_a^3} - \frac{8.621949 \times 10^{11}}{T_a^4}$$

where o_{sf} = the saturation concentration of dissolved oxygen in freshwater at 1 atm (mg L^{-1}); and T_a = absolute temperature (K). Remember that $T_a = T + 273.15$, where T = temperature (°C). According to this equation, saturation decreases with increasing temperature. For typical natural waters in temperate climates, the equation can be used to determine the oxygen concentraton ranges from 14.621 mg/L at 0°C to 6.413 mg/L at 40°C. Use the bisection method to determine T as a function of a given oxygen for $o_{sf} = 8, 10$ and 12 mg/L. Use inital guess as 0°C and 40°C.

2. Water flowing in a trapezoidal channel at a rate of $Q = 20$ m^3/s. The critical depth y for such a channel must satisfy the equation

$$0 = 1 - \frac{Q^2}{gA_c^3}B$$

where $g = 9.81\mathrm{m/s}^2$, A_c = the cross-section area (m^2), and B = the width of the channel at the surface (m). For this case, the width and the cross-sectional area can be related to depth y by

$$B = 3 + y$$

and

$$A_c = 3y + \frac{y^2}{2}$$

(a) Use the graphical method to approximate the critical depth y.

(b) Solve for the critical depth using the bisection method with initial guesses of $x_l = 0.5$ and $x_u = 2.5$ and iterate until the approximate error falls below 1% or the number of iterations exceeds 10.

(c) Use the incremental search to refine a possible bracket on the root, use an initial interval of $[0.5, 2.5]$.

(d) Solve for the critical depth using the bisection method with bounds found in the previous part and iterate until the approximate error falls below 1% or the number of iterations exceeds 10.

(e) Explain the any difference in possible roots and number of iterations needed in parts b and d. Give reasons for the differences or similarities.

3. The upward velocity of a rocket can be computed by the following formula:

$$v = u \ln\left(\frac{m_0}{m_0 - qt}\right) - gt$$

where v = upward velocity, u = the velocity at which fuel is expelled relative to the rocket, m_0 = the initial ass of the rocket at time $t = 0$, q = the fuel consumption rate, and g = the downward acceleration of gravity (assumed constant = 9.81 m/s^2). If $u = 2000$ m/s, $m_0 = 150000$ kg, and $q = 2700$ kg/s, compute the time at which $v = 750$ m/s.

(a) Use a graph to determine initial bounds. (Plot for t in the interval $[0, 60]$)

(b) Refine the initial bounds using the incremental search algorithm.

(c) Use the bisection method with the results from part b.

(d) Use the secant method to determine an approximate root.

(e) Use the program fzerotx from class to determine an approximate root.

(f) Use the built-in Matlab function fzero to find an approximate root.

4. In a chemical engineering process, water vapor (H_2O) is heated to sufficiently high temperatures that a significant portion of the water dissociates, or splits apart, to form oxygen (O_2) and hydrogen(H_2):

$$H_2O \rightleftharpoons H_2 + \frac{1}{2}O_2$$

If it is assumed that this is the only reaction involved, the mole fraction x of H_2O that dissociates can be represented by

$$K = \frac{x}{1-x}\sqrt{\frac{2p_t}{2+x}}$$

where K is the reaction's equilibrium constant and p_t is the total pressure of the mixture. If $p_t = 3.5$ atm and $K = 0.04$, determine the value of x that satisfies this equation.

(a) Determine the function and derivative of the function needed to implement using Newton Raphson's method, then use newtraph.m to determine x.

(b) Determine x using the Secant method.

(c) Compare answers from parts a and b.

5. Use the Newton-Raphson method to find the root of

$$f(x) = e^{-0.5x}(4 - x) - 2$$

Employ initial guesses 2, 6, and 8. Explain your results.