Homework 6
Part II – Stochastic Models and Gillespie Algorithm

There are three classes of models we have talked about in the past two weeks:

- Models consisting of a system of ordinary differential equations
- Models consisting of a system of stochastic differential equations
- Stochastic models using Markov chains

Random behavior can be introduced into a model using a system of stochastic differential equations (SDEs) which can be derived using information about the system. SDEs are differential equations with a Weiner process included to account for the stochastic nature of the environment. Stochastic models can be derived using discrete-time or continuous-time Markov chains. Stochastic models are not only good in adding random behavior to a model and more accurately describing a system, but they are necessary models when the population size you are considering is *small*. Ordinary differential equations can *only* be used to accurately describe large population systems. To analyze the difference in stochastic models and ordinary differential equations models, you will be looking at the two codes, *run_VRE.m and VRE_SSA.m*. To solve stochastic models, the Gillespie algorithm is used:

1. Initialize the state of the system
2. For a given state of the system, calculate the transition rates (λ_i)
3. Calculate the sum of all transition rates, $\lambda = \sum_{i=1}^{n} \lambda_i$
4. Simulate the time until the next transition, τ
5. Simulate the transition type
6. Update the new time $t = t + \tau$ and the new system state
7. Iterate steps 2-6 until $t \geq t_{stop}$

Look at the code in VRE_SSA to answer the following questions:

1. What is the line used to calculate Step 3 above?
2. What is the line used to generate Step 4 above? What type of probability distribution is the random number generated from?
3. Is the random number used to determine the transition type (Step 5) the same as that used to time to the next transition (Step 4)? If not, what type of probability distribution is that number chosen from? Give the line of code used.
4. What time of loop is used to complete Step 7?
5. Now run the code run_VRE.m with various values of N (total population size) :
   - N=10;
   - N=25;
   - N=75;
   - N=100;
   - N=250;
   - N=500;
   - N=750;
   - N=1000;
   - N=1500;
   - N=2000;

   a. At what population value N does it appear as if the variation in the stochastic realizations are relatively small and all three realizations are approaching the mean (the solution to the ordinary differential equation)?

   b. What happens to the run time as N increases? Why do you think this happens?