## Maple Questions

Here are some sample Maple assessment questions for this chapter.

1. Create a worksheet in which a user supplies numbers $a$, and $b$, and then *Maple* determines the domain of the function

$$f(x,y) = \sqrt{\frac{x^2+a}{y+b}}$$

The worksheet should also produce sketches of the domain and determine if it is open, closed or neither; bounded or unbounded; and connected or not connected. Be sure to consider each of the cases $a$ positive, negative, or zero.

2. Create a worksheet which solves the vibrating string problem

$$\frac{\partial^2 u}{\partial t^2} = a\frac{\partial^2 u}{\partial x^2}$$

for some constant $a$ subject to the free end boundary conditions

$$\frac{\partial u}{\partial x}(0,t) = 0 \quad \textit{and} \qquad \frac{\partial u}{\partial x}(L,0) = 0$$

where $L$ is the length of the string. Assume the string is plucked, so that

$$u(x,0) = f(x) \qquad \textit{and} \qquad \frac{\partial u}{\partial t}(x,0) = 0$$

where $f(x)$ is the shape of the initial string. Set up the worksheet so that if a user supplies $a$, $L$, and the initial shape $f(x)$, then Maple produces the solution and animates the sum of the fundamental and the first 3 harmonics (i.e., the 1st four terms in the Fourier series solution).

3. It is often difficult or even impossible to solve the system of equations $f_x = 0$, $f_y = 0$. However, we can use the fact that the negative gradient of $f(x,y)$ at a point $(x_n, y_n)$ points in the direction of steepest descent of $f(x,y)$, which implies that if $\delta > 0$ is a small number and

$$(x_{n+1}, y_{n+1}) = (x_n, y_n) - \delta \nabla f(x_n, y_n)$$

then $(x_{n+1}, y_{n+1})$ is closer to a minimum of $f(x,y)$ than $(x_n, y_n)$ is. Use this fact to create a worksheet which numerically approximates a minimum of $f(x,y)$ using an initial "guess" of $(x_0, y_0)$.

4. Create a worksheet which allows a user to supply a data set $(t_n, y_n)$, $n = 1, \ldots, N$, where $y_n > 0$ for all $n$, and then minizes the total squared error function
$$E(C,k) = \sum_{n=1}^{N} \left(\ln(y_n) - kt_n - C\right)^2$$
Then graph the data set $(t_n, y_n)$ along with the *exponential fit*
$$y = e^{kt+C}$$
where $C$ and $k$ minimize $E(C,k)$.

5. Create a worksheet in which first a user supplies the coefficients $a$, $b$, $c$, $d$, and $e$ of the curve
$$ax^2 + bxy + cy^2 = dy + e$$
and then Maple uses Lagrange multipliers to determine the point(s) on the curve closest to the origin, graphs the curve, and then labels the points on the curve that are closest to the origin.