Practice Test

Chapter 2 Name ______________________________

Instructions. Show your work and/or explain your answers.

1. Find the domain of the function

$$f\left(x,y\right)=\sqrt{y}+\sqrt{x^2-1}$$

   Is the domain open, closed, or neither? Bounded or unbounded? Connnected or not connected?

2. Show the following limit does not exist by showing that different paths through the origin lead to different limits:

$$\lim_{(x,y)\to(0,0)}\frac{\left(x+y\right)^2}{x^2-y^2}$$

3. Does the following limit exist?

$$\lim_{(x,y)\to(0,0)}\frac{\left(x+y\right)^2}{x^2+y^2}$$

4. Find the linearization of $f\left(x,y\right)=x+e^{xy}$ at $\left(1,0\right)$

5. Find the second order derivatives of

$$f\left(x,y\right)=x^2+e^{xy}$$

6. Find the separated solution of

$$\frac{\partial u}{\partial t}+\frac{\partial u}{\partial x}=u$$

7. Find $\partial_u z$ when $z=x^2+y^3$ and $x=u^2+uv$, $y=u^3v$

8. Prove that the derivative of a sum is the sum of the derivatives by applying the chain rule for 2 variables to

$$w=x+y$$

   where $x=f(t)$ and $y=g(t)$.

9. Find the gradient of the function $g\left(x,y\right)=x^2+y^2$, and then show that it is normal to the curve

$$x^2+y^2=25$$

   at the point $\left(3,4\right)$.

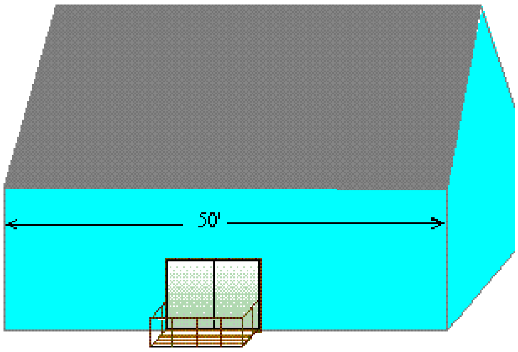
10. In what direction is the function $f\left(x,y\right)=x^2+y^3$ **decreasing** the fastest at the point $\left(1,3\right)$?

11. Find the extrema and saddle points of $f(x, y) = x^2 + 3xy + 2y^2 - 4x - 5y$.

12. Find the extrema and saddle points of $f(x, y) = 4x^3 - 6x^2y + 3y^2$

13. Find the point(s) on the curve $xy = 1$ that are closest to the origin.

14. Use Lagrange Multipliers to solve the following: John wants to build a 500 ft^2 deck behind his house.



His house is 50 feet long, and correspondingly, he wants the deck to be between 5 and 50 feet long. What dimensions of the deck will minimize the lengths of the rail around the 3 exposed sides of the deck?

15. ** Heating of a 2 dimensional surface (such as in a sheet of metal) is modeled by the *2 dimensional heat equation*

$$\frac{\partial u}{\partial t} = k^2 \frac{\partial^2 u}{\partial x^2} + k^2 \frac{\partial^2 u}{\partial y^2}$$

where $u(x, y, t)$ is a function of 3 variables and k is a constant. What is the separable solution of the 2 dimensional heat equation (hint: involves 2 separation constants)?