

1. Section 3-1

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| 1. $z = -\frac{1}{3}x + \frac{11}{3} - \frac{1}{3}y$ | 15. $z = 2x + 2y - 2$ |
| 3. $z = -\frac{1}{2}x + 3 - \frac{1}{4}y$ | 17. $z = 3x + 4y + 2$ |
| 5. $3x + 4y + 2z = 13$ | 19. $z = x + y - 1$ |
| 7. $z = x + y - 1$ | 21. $z = x - 4 + 2y$ |
| 9. $z = -x + 2 - y$ | 23. $12t^{11}$ |
| 11. $x = 1$ | 25. 0 |
| 13. $x = 1$ | 27. 0 |

2. Section 3-2

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| 1. $x^2 + y^2 = z^2$ | | |
| 3. $x^2 + y^2 + z^2 = 1$ | | |
| 5. $\frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{4} = 1$ | | |
| 7. $x^2 - y^2 + z^2 = 1$ | | |
| 9. $x^2 + y^2 - z^2 = 1$ | | |
| 11. $(x^2 + y^2)z^2 = 1$ | | |
| 13. $\mathbf{r}_u = \langle v \cos u, -v \sin u, 0 \rangle$ | $\mathbf{r}_v = \langle \sin u, \cos u, 1 \rangle$ | orthogonal |
| 15. $\mathbf{r}_u = \langle \cos u \cos v, -\sin u, \cos u \sin v \rangle$ | $\mathbf{r}_v = \langle -\sin u \sin v, 0, \sin u \cos v \rangle$ | orthogonal |
| 17. $\mathbf{r}_u = \langle e^v \cos u, -e^v \sin u, 0 \rangle$ | $\mathbf{r}_v = \langle e^v \sin u, e^v \cos u, -e^{-v} \rangle$ | orthogonal |
| 19. $\mathbf{L}(du, dv) = \langle \frac{\sqrt{3}}{2}, \frac{1}{2}, \frac{\pi}{3} \rangle + \langle \frac{1}{2}, -\frac{\sqrt{3}}{2}, 1 \rangle du + \langle \frac{\sqrt{3}}{2}, \frac{1}{2}, \frac{\pi}{3} \rangle dv$ | | |
| $\mathbf{L}(du, dv) = \langle \frac{\sqrt{3}}{2} + \frac{du}{2} + \frac{\sqrt{3}}{2} dv, \frac{1}{2} - \frac{\sqrt{3}}{2} du + \frac{1}{2} dv, \frac{\pi}{3} + du + \frac{\pi}{3} dv \rangle$ | | |
| 21. $\mathbf{L}(du, dv) = \langle 1, \sqrt{3}, 2 \rangle + \langle \sqrt{3}, -1, 0 \rangle du + \langle \frac{1}{2}, \frac{1}{2}\sqrt{3}, 1 \rangle dv$ | | |
| $\mathbf{L}(du, dv) = \langle 1 + du\sqrt{3} + \frac{1}{2}dv, \sqrt{3} - du + \frac{\sqrt{3}}{2}dv, 2 + dv \rangle$ | | |
| 23. $\mathbf{L}(du, dv) = \langle 0, -e, e^{-1} \rangle + \langle -e, 0, 0 \rangle du + \langle 0, -e, -e^{-1} \rangle dv$ | | |
| $\mathbf{L}(du, dv) = \langle -e du, -e - e dv, e^{-1} - e^{-1} dv \rangle$ | | |
| 25. $\mathbf{r}(u, v) = \langle u, u \sin(v), u \cos(v) \rangle$ | $\mathbf{r}_u = \langle 1, \sin v, \cos v \rangle$ | orthogonal |
| $\mathbf{r}_v = \langle 0, u \cos v, -u \sin v \rangle$ | | |
| 27. $\mathbf{r}(u, v) = \langle u, (u - u^2) \sin(v), (u - u^2) \cos(v) \rangle$ | $\mathbf{r}_u = \langle 1, (1 - 2u) \sin v, (1 - 2u) \cos v \rangle$ | orthogonal |
| $\mathbf{r}_v = \langle 0, (u - u^2) \cos v, -(u - u^2) \sin v \rangle$ | | |
| 29. $\mathbf{r}(u, v) = \langle u, \cosh(u) \sin(v), \cosh(u) \cos(v) \rangle$ | $\mathbf{r}_u = \langle 1, \sinh u \sin v, \sinh u \cos v \rangle$ | orthogonal |
| $\mathbf{r}_v = \langle 0, \cosh u \cos v, -\cosh u \sin v \rangle$ | | |

3. Section 3-3

1. $y = 4x^2$
3. $y = 2x$
5. $\frac{x^2}{16} + \frac{y^2}{9} = 1$
7. $x = \frac{y^2}{4} - 4$
9. $x^2 + y^2 = 1$

11. unit square with lower left vertex at $(1, 5)$
13. unit square
15. region between x -axis, $x = \frac{y^2}{4} - 1$, and $x = 1 - \frac{y^2}{4}$
17. region between x -axis, $y = x$, $y = e^{-2}x$ and $xy = 1$

$$19. \begin{bmatrix} \frac{1}{2}\sqrt{3} & -\frac{1}{2}\sqrt{3} \\ \frac{1}{2}\sqrt{3} & \frac{1}{2} \end{bmatrix} \quad y = -x \frac{\sqrt{3}+2}{-1+2\sqrt{3}}$$

$$21. \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \quad y = 1$$

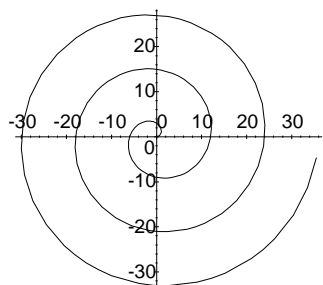
$$23. \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \quad y = -x - 2$$

25. $u^2 + \frac{v^2}{4} = 1$
27. $u^2 - v^2 = 2$
29. $\frac{u^2}{16} + \frac{v^2}{9} = 1$

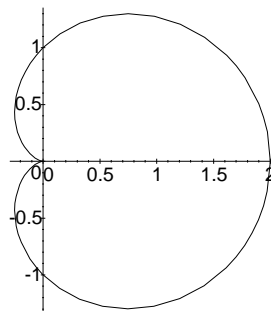
4. Section 3-4

1. a. $(-2, 0)$ b. $(-1, 0)$ c. $\left(\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$ d. $(1, -\sqrt{3})$

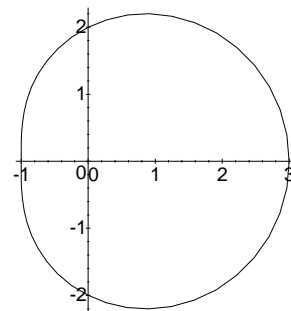
3.



5.



7.



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| 9. $r = 4$ | 21. $r = 2 \cos(\theta)$ |
| 11. $r = \sec(\theta)$ | 23. $r = \sqrt{2 \sec(\theta) \csc(\theta)}$ |
| 13. $r = \frac{2}{\sin(\theta) - 3 \cos(\theta)}$ | 25. $p = 3, \varepsilon = \frac{3}{4}$ |
| 15. $r = \sec(\theta) \tan(\theta)$ | 27. $p = 2, \varepsilon = 0.2$ |
| 17. $r = \frac{\cos \theta \pm \sqrt{1 + 15 \sin^2 \theta}}{8 \sin^2 \theta}$ | 29. $p = 2, \varepsilon = 0.5$ |
| 19. $r = \sqrt{2 \sec(\theta) \csc(\theta)}$ | |

5. Section 3-5

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| 1. $\mathbf{w} = \langle 1, 2 \rangle, J = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, \mathbf{z} = \langle 3, -1 \rangle$ | 15. $\mathbf{v} = \langle -2, -1 \rangle$ |
| 3. $\mathbf{w} = \langle 1, 3 \rangle, J = \begin{bmatrix} 2uv & u^2 \\ v^2 & 2uv \end{bmatrix}, \mathbf{z} = \langle 36, 108 \rangle$ | 17. $dA = 2dudv$ |
| 5. $\mathbf{w} = \langle 1, 0 \rangle, J = \begin{bmatrix} \sec(v) & u \sec(v) \tan(v) \\ \tan(v) & u \sec^2(v) \end{bmatrix}, \mathbf{z} = \langle 1, 0 \rangle$ | 19. $dA = (6u^2 + 2v^2) dudv$ |
| 7. $\mathbf{v} = \left\langle \frac{-\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right\rangle$ | 21. $dA = 4 u dudv$ |
| 9. $\mathbf{v} = \langle 2, 4 \rangle$ | 23. $dA = 12 u dudv$ |
| 11. $\mathbf{v} = \langle -\sqrt{3}, 0 \rangle$ | 25. $dA = 2e^{2u} dudv$ |
| 13. $\mathbf{v} = \left\langle -\frac{\pi}{8}\sqrt{2}, \sqrt{2} + \frac{\pi}{8}\sqrt{2} \right\rangle$ | 27. $dA = \cos(2u) dudv$ |

6. Section 3-6

1. a. $(\frac{3}{2}, \frac{3}{2}\sqrt{3}, 3)$ b. $(0, 7, 0)$ c. $(5, 0, 0)$ d. $(-4, 0, -2)$
3. a. $(-\frac{3}{2}\sqrt{3}, 0, \frac{3}{2})$ b. $(\frac{7}{2}\sqrt{2}, \frac{7}{2}\sqrt{2}, 0)$ c. $(-1, 0, 0)$ d. $(0, 0, 5)$

In 5-11, substitute the value for r into the parameterization

$$\mathbf{r}(\theta, z) = \langle r \cos(\theta), r \sin(\theta), z \rangle$$

In 13-19 and 23, substitute the value for ρ into the parameterization

$$\mathbf{r}(\phi, \theta) = \langle \rho \sin(\phi) \cos(\theta), \rho \sin(\phi) \sin(\theta), \rho \cos(\phi) \rangle$$

In 25-19, substitute the value for r into the parameterization

$$\mathbf{r}(t) = \langle r \cos(t), r \sin(t), r \rangle$$

5. $r = 5,$
7. $r = \sqrt{z^2 + 1}$
9. $r = \frac{2}{3 \cos(\theta) + 4 \sin(\theta)}$
11. $r = z$
13. $\rho = 5$
15. $\rho = \csc(\phi) \sec(\theta)$
17. $\rho = \sqrt{-\sec(2\phi)}$
19. $\rho = \frac{1}{\cos(\phi) + 2 \sin(\phi) \sin(\theta)}$
21. $\phi = \pi/4, \mathbf{r}(\rho, \theta) = \frac{\rho}{\sqrt{2}} \langle \cos(\theta), \sin(\theta), 1 \rangle$
23. $\rho = \sec(\phi) \sin(2\theta)$
25. $r = \frac{1}{1 - \frac{1}{2} \cos(\theta)}$
27. $r = \frac{2}{1 - \cos(\theta)}$
29. $r = \frac{1}{1 - 2 \cos(\theta)}$

7. Section 3-7

1. $\mathbf{n} = \left\langle \frac{-1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} \right\rangle$
3. $\mathbf{n} = \langle 0, \sin(u), \cos(u) \rangle$
5. $\mathbf{n} = \frac{1}{\sqrt{2}} \langle \sin(u), \cos(u), -1 \rangle$
7. $\mathbf{n} = \frac{\langle -e^{-u}, -e^u, 2 \rangle}{\sqrt{e^{-2u} + e^{2u} + 4}}$
9. $\mathbf{n} = \frac{1}{\sqrt{2z^2 + 1}} \langle x, y, -z \rangle$
11. $\mathbf{n} = \left\langle \frac{2}{3}, \frac{2}{3}, \frac{1}{3} \right\rangle$
13. $\mathbf{n} = \frac{1}{\sqrt{\theta^2 + r^2 z^2}} \langle -\theta \sin \theta, \theta \cos \theta, z \rangle$
15. $\mathbf{n} = \frac{\langle \theta \cos \theta - \sin \theta, \theta \sin \theta + \cos \theta, 2z \rangle}{\sqrt{1 + \theta^2 + 4z^2}}$
17. $\boldsymbol{\rho} \cdot \mathbf{r}_u = 4, \boldsymbol{\rho} \cdot \mathbf{r}_v = 2,$ not a geodesic
19. $\boldsymbol{\rho} \cdot \mathbf{r}_u = 32t + 24, \boldsymbol{\rho} \cdot \mathbf{r}_v = -(4t + 3),$ not a geodesic
21. $\boldsymbol{\rho} \cdot \mathbf{r}_u = 0, \boldsymbol{\rho} \cdot \mathbf{r}_v = 0,$ is a geodesic
23. $\boldsymbol{\rho} \cdot \mathbf{r}_u = 0, \boldsymbol{\rho} \cdot \mathbf{r}_v = 0,$ is a geodesic
25. $\boldsymbol{\rho} \cdot \mathbf{r}_u = \sec^2(t) + \tan^2(t), \boldsymbol{\rho} \cdot \mathbf{r}_v = 1,$ not a geodesic
27. $\mathbf{r}(t) = [\sin(t), \sin(t), \cos(t) \sqrt{2}],$ distance = $\frac{\pi\sqrt{2}}{2}$
29. $\mathbf{r}(t) = \langle 2, 2, 1 \rangle \cos(t) + \left\langle \frac{2}{\sqrt{17}}, -\frac{7}{\sqrt{17}}, \frac{10}{\sqrt{17}} \right\rangle \sin(t)$
distance = $\cos^{-1}\left(\frac{8}{9}\right) \approx 0.47588$

8. Section 3-8

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| 1. | $ds^2 = 2du^2 + dv^2$ | 17. | 1 |
| 3. | $ds^2 = v^2 du^2 + 2dv^2$ | 19. | π |
| 5. | $ds^2 = du^2 + \sin^2(u) dv^2$ | 21. | $\frac{4-2\sqrt{3}}{3\sqrt{3}-5}$ |
| 7. | $ds^2 = (9 - 5 \cos^2(u)) du^2 + 4 \sin^2(u) dv^2$ | 23. | 2π |
| 9. | $ds^2 = \cosh^2(v) du^2 + (2 \cosh^2 v - 1) dv^2$ | 25. | π |
| 11. | $ds^2 = (2 \sec^4(u) - \sec^2(u)) du^2 + \sec^2(u) dv^2$ | 27. | 2π |
| 13. | $ds^2 = e^{2v} du^2 + (e^{2v} + e^{-2v}) dv^2$ | 29. | 1.45214 |
| 15. | 2π | | |

9. Section 3-9

1. $\kappa_n(\theta) = \cos^2(\theta)$, $\kappa_1 = 0$, $\kappa_2 = 1$, *Gaussian flat*
3. $\kappa_n(\theta) = 0$, *flat*, minimal, (it's a plane!)
5. $\kappa_n(\theta) = \frac{2 \sin^2(\theta)}{(1 + 4v^2)^{3/2}}$, $\kappa_1 = 0$, $\kappa_2 = \frac{2}{(1 + 4v^2)^{3/2}}$, *Gaussian flat*
7. $\kappa_n(\theta) = \frac{\cos^2(\theta)}{\cosh^2(u)}$, $\kappa_1 = \frac{1}{\cosh^2(u)}$, $\kappa_2 = 0$, *Gaussian flat*
9. $\kappa_n(\theta) = \frac{-\sin(2\theta)}{1 + u^2}$, $\kappa_1 = \frac{-1}{1 + u^2}$, $\kappa_2 = \frac{1}{1 + u^2}$, *Minimal*
11. $\kappa_n(\theta) = \frac{\sin^2(\theta)}{u\sqrt{2}}$, $\kappa_1 = \frac{1}{u\sqrt{2}}$, $\kappa_2 = 0$, *Gaussian flat*
13. $\kappa_n(\theta) = (2 \cos^2(\theta) - 1) \operatorname{sech}^2(v)$, $\kappa_1 = \operatorname{sech}^2(v)$, $\kappa_2 = -\operatorname{sech}^2(v)$, *Minimal*
15. $\kappa_n(\theta) = -\sin(2\theta) \operatorname{sech}^2(v)$, $\kappa_1 = \operatorname{sech}^2(v)$, $\kappa_2 = -\operatorname{sech}^2(v)$, *Minimal*
17. $K = -4$
19. $K = 0$
21. $K = \frac{-2}{v^2(4v^2 + 1)^2}$
23. $K = \frac{\cos u}{2 + \cos u}$