1. Evaluate the line integral

$$\int_C xdy - ydx$$

where C is the curve $\mathbf{r}(t) = \langle 2t, 3t \rangle$, t in $[0, 1]$.

2. Test for exactness. If exact, find its potential: $\mathbf{F}(x, y) = \langle x^2 + y^2, xy \rangle$

3. Test for exactness. If exact, find its potential: $\mathbf{F}(x, y) = \langle \sin(x + y), \sin(x + y) \rangle$

4. Test for exactness. If exact, find its potential: $\mathbf{F}(x, y, z) = \langle ye^x, e^x + 1, e^z \rangle$

5. Evaluate the integral below using the fundamental theorem for line integrals

$$\int_{(0,0,0)}^{(1,1,1)} (x + y + z)(dx + dy + dz)$$

6. Explain why the integral $\int_{(0,0,0)}^{(1,1,1)} xdy + ydx + zdz$ is independent of path. Then calculate the integral along two different paths from $(0, 0, 0)$ to $(1, 1, 1)$.

7. Let R be the unit square. Use Green's theorem to evaluate the line integral

$$\oint_{\partial R} y^2 dx + x^2 dy$$

8. Let R denote the upper half of the unit disk. Evaluate using Green's theorem:

$$\oint_{\partial R} (xy)(dx + dy)$$

9. Evaluate by using Green's theorem to convert to a line integral over the boundary ($\mathbf{D}$ is the unit disk):

$$\int\int_{\mathbf{D}} \frac{-x}{(x^2 + y^2 + 1)^{3/2}} dA$$

10. Find the area enclosed by the curve $\mathbf{r}(t) = \langle \cos^2(t), \cos(t)\sin(t) \rangle$, t in $[0, \pi]$, using Green's theorem.

11. Calculate the surface area of the surface Σ parameterized by $\mathbf{r}(u, v) = \langle u\cos(v), u\sin(v), u^2 \rangle$ for u in $[0, 1]$ and v in $[0, 2\pi]$.

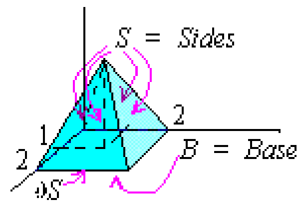
12. Compute the flux of the vector field $\mathbf{F}(x, y, z) = \langle y, x, z \rangle$ through the surface Σ parameterized by

$$\mathbf{r}(u, v) = \left\langle u\cos(v), u\sin(v), u^2 \right\rangle, \qquad u \; in \; [0, 1], \quad v \; in \; [0, 2\pi]$$

13. Show that if $\mathbf{F}(x, y, z) = \langle xy + 2z, yz + 2x, xz + 2y \rangle$, then $curl(\mathbf{F}) = \langle 2 - y, 2 - z, 2 - x \rangle$. Then evaluate

$$\iint_S curl(\mathbf{F}) \cdot d\mathbf{S}$$

when S is the surface of the pyramid with vertices $(2,0,0)$, $(2,2,0)$, $(0,2,0)$, $(0,0,0)$, and $(1,1,2)$ that is not contained in the xy-plane.



14. Use Stoke's theorem for differential forms to calculate

$$\iint_{\partial S} xy\ dy\hat{}dz - z^2\ dy\hat{}dx$$

when S is the solid cube $[0,1] \times [0,1] \times [0,1]$.

15. Compute the flux of the vector field $\mathbf{F}(x,y,z) = \langle x,y,z \rangle$ through the surface of a sphere Σ with radius R centered at the origin. Then show that the divergence theorem produces the same result.