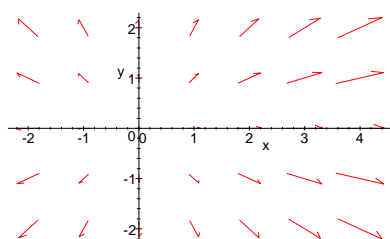


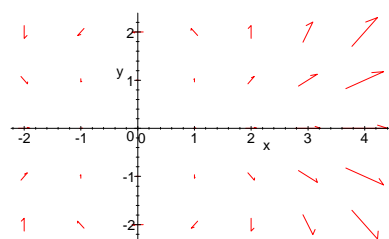
Chapter 5 Solutions

Section 5-1:

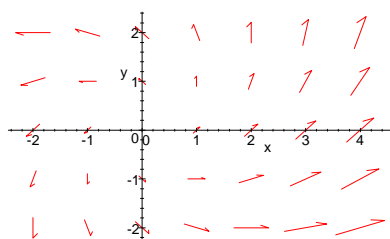
1.



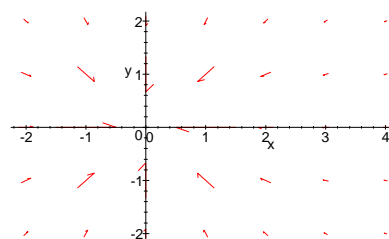
5.



3.



7.



8.

9. $\nabla U = \langle 3, 4 \rangle$
11. $\nabla U = \langle y, x \rangle$
13. $\nabla U = \langle 3x^2 \cos(xy) - x^3y \sin(xy), -x^4 \sin(xy) \rangle$
15. $\nabla U = \langle \cos(x) \sinh(y), \sin(x) \cosh(y) \rangle$
17. $\nabla U = \langle 2xy, x^2 + 2yz, y^2 \rangle$
19. $\nabla U = \langle \sin(yz), xz \cos(yz), xy \cos(yz) \rangle$
21. $\text{curl}(\mathbf{F}) = \mathbf{0}$, $\text{div}(\mathbf{F}) = 3x^2 + 2y + 2z$, conservative
23. $\text{curl}(\mathbf{F}) = \langle 0, 0, 2x - 2y \rangle$, $\text{div}(\mathbf{F}) = 2x - 2y$
25. $\text{curl}(\mathbf{F}) = \langle 0, 0, 4y \rangle$, $\text{div}(\mathbf{F}) = 4x + 2z$
27. $\text{curl}(\mathbf{F}) = \langle 0, 0, 0 \rangle$, $\text{div}(\mathbf{F}) = 1$, conservative
29. $\text{curl}(\mathbf{F}) = \langle 0, -xe^x, ye^x \rangle$, $\text{div}(\mathbf{F}) = ze^x + e^x$

Section 5-2:

- | | | | |
|-----|-------------------|-----|--------------------------|
| 1. | $\frac{2}{3}$ | 17. | $\frac{206}{105}$ |
| 3. | 0 | 19. | $8\pi^2$ |
| 5. | $\frac{\pi}{2}$ | 21. | $\frac{5\sqrt{5}-1}{12}$ |
| 7. | π | 23. | $10\pi^2$ |
| 9. | $\frac{\pi^2}{2}$ | 25. | $-\pi$ |
| 11. | 0 | 27. | 18 |
| 13. | $-\pi$ | 29. | 0 |
| 15. | 0 | | |

Section 5-3:

- | | | | |
|-----|---|-----|-----------------|
| 1. | $U(x, y) = xy + C$ | 17. | 1 |
| 3. | $U(x, y) = xe^x + ye^y + C$ | 19. | 1 |
| 5. | $U(x, y) = \sin(x) \cos(y) + y + C$ | 21. | 2 |
| 7. | not conservative | 23. | $\frac{41}{12}$ |
| 9. | $U(x, y) = e^x \sin(y) + C$ | 25. | 2 |
| 11. | $U(x, y, z) = -32z + C$ | 27. | 12 |
| 13. | $U(x, y, z) = \frac{x^2}{2} + xz + \frac{y^3}{3} + \frac{z^4}{4} + C$ | 29. | 3 |
| 15. | not conservative | | |

Section 5-4:

- | | | | |
|-----|-------|-----|--------------------------------------|
| 1. | π | 17. | $\frac{-2}{3}$ |
| 3. | 0 | 19. | $\frac{8\pi}{3}$ |
| 5. | 0 | 21. | $\frac{1}{12}$ |
| 7. | 1 | 23. | $\frac{60}{12\pi^3} - \frac{1}{\pi}$ |
| 9. | 0 | 25. | $\frac{12}{5\pi}$ |
| 11. | 0 | 27. | $\frac{3\pi}{8}$ |
| 13. | 0 | 29. | 64. 13981 |
| 15. | 0 | | |

Section 5-5:

- | | |
|--|-----------------------------|
| 1. Surface Area = $\sqrt{26}$ | 17. Flux = $\frac{-5}{6}$ |
| 3. Surface Area = $\frac{1}{32}(2 + \pi)\pi$ | 19. Flux = 4π |
| 5. Surface Area = $3\pi\sqrt{2}$ | 21. Flux = $\frac{4\pi}{3}$ |
| 7. Surface Area = $\frac{8}{3}\sqrt{2}$ | 23. Flux = 0 |
| 9. Flux = $\frac{1}{2}$ | 25. Flux = 0 |
| 11. Flux = 0 | |
| 13. Flux = 0 | |
| 15. Flux = 0 | |

Section 5-6: Compute the flux through the boundary ∂S of the solid S . Then use the divergence theorem to calculate the flux.

- | | |
|--|------------------------------|
| 1. Flux = 0 | 13. Flux = $4k\pi$ |
| 3. Flux = $\frac{\pi}{2}$ | 15. Flux = $4k\pi$ |
| 5. Flux = 4π | 17. Flux = $4k_1\pi$ |
| 7. $\int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{x^2+y^2}^{2-x^2-y^2} (2z+2) dz dy dx = 4\pi$ | 19. Flux = $4(k_1 + k_2)\pi$ |
| 9. $\int_0^1 \int_0^{1-x} \int_0^{1-x-y} 3dz dy dx = \frac{1}{2}$ | |
| 11. $\text{div}(\langle x^3, y^3, z^3 \rangle) = 3(x^2 + y^2 + z^2) = 3\rho^2$
$\int_0^1 \int_0^\pi \int_0^{2\pi} (3\rho^2) \rho^2 \sin(\phi) d\rho d\phi d\theta = \frac{192}{5}\pi^5$ | |

Section 5-7:

- | | |
|--------------------|-------------------|
| 1. 2π | 13. 2π |
| 3. 0 | 15. 2π |
| 5. 2π | 17. π |
| 7. 0 | 19. 4π |
| 9. 0 | 21. 100π |
| 11. $\frac{-1}{3}$ | 23. $\pi\sqrt{2}$ |

Section 5-8:

- | | |
|--|-----------------------|
| 1. $d\omega = ydx + xdy + 2zdz$ | 15. 2π |
| 3. $d\omega = dx + dy + dz$ | 17. 0 |
| 5. $d\omega = e^x \cos(yz) dx - ze^x \cos(yz) dy - ye^x \cos(yz) dz$ | 19. $-\frac{1}{2}$ |
| 7. $d\omega = 0$ | 21. 2π |
| 9. $d\omega = (x^2 - 1) dy \wedge dx + z^2 dy \wedge dz$ | 23. 2π |
| 11. $d\omega = 0$ | 25. $\frac{1}{2}$ |
| 13. $d\omega = 3dx \wedge dy \wedge dz$ | 27. $\frac{12\pi}{5}$ |