ASTR-1010: Astronomy I
Course Notes
Section III

Dr. Donald G. Luttermoser
Department of Physics and Astronomy
East Tennessee State University

Edition 2.0

Abstract

These class notes are designed for use of the instructor and students of the course ASTR-1010: Astronomy I taught by Dr. Donald G. Luttermoser at East Tennessee State University.

III. Astronomical History: Planetary and Lunar Motion

A. History of Astronomy

1. The ancient Greeks were the first group of people to investigate the Universe in a systematic way (400 BC – 200 AD).

a) Developed fundamental mathematics to help describe the Universe: Euclid & Pythagoras.

b) Some Greek philosophers reasoned that the Earth orbited the Sun: Aristarchus of Samos $\Longrightarrow$ **heliocentric universe**.

c) Eratosthenes measured the Earth's actual size (and realized it was round) by noting the Sun's position in the sky at 2 locations (*i.e.*, latitudes) on the Earth on the summer solstice.

i) He noted a shift in the Sun's position in the sky in two cities: Synene in Egypt on the Earth's equator, and Alexandria, which was north of Synene.

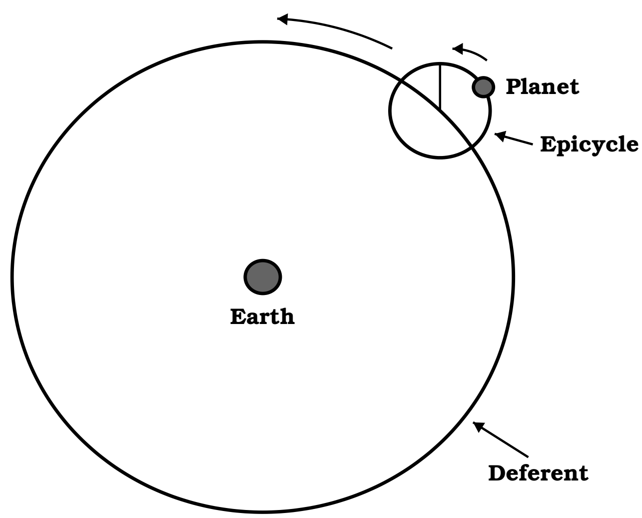
ii) The angle shift of the Sun's position on the meridian and the known distance between these two cities enable Eratosthenes to measure the Earth's circumference, hence diameter, fairly accurately.

d) Hipparchus made star charts and developed the magnitude system to describe a star's brightness $\Longrightarrow$ brightest stars are at 1st magnitude, faintest stars he could see are 6th magnitude.

e) Aristotle rejected observation and experimental processes and reasoned that the Universe could be understood with thought alone. He also believed that the Earth was at the center of the Universe and that the planets, Sun, and stars revolve around the Earth $\Longrightarrow$ **geocentric universe**.

f) The Christian religion adopted Aristotle's viewpoints. These viewpoints, taught by the Church, lead to the dark ages which lasted nearly 1500 years. During this time, science was not practiced and pseudoscience flourish.

g) Based on Aristotle's views, Ptolomy developed a geocentric model of the Solar System.



i) The main circular planetary orbit was called an **deferent** with the Earth at the center of the orbit.

ii) The planet then orbited a point on the deferent with a smaller circular orbit called an **epicycle**.

iii) The epicycles were needed to explain the **retrograde** (*i.e.*, reversed or westward) **motion** of the planets when they are near opposition.

iv) Note that planets normally have a **prograde** (*i.e.*, direct or eastward) **motion** in the sky.

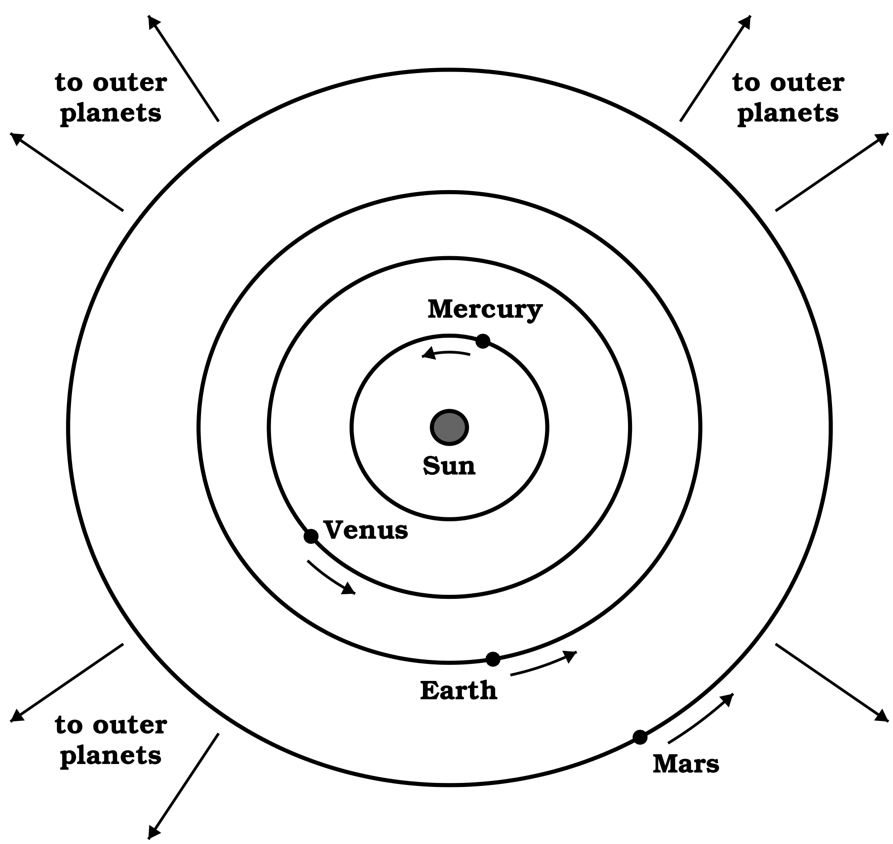
v) This model lasted 1500 years (primarily due to fear of losing one's life if you went against it since the Church adopted it)!

2. Nicolaus Copernicus.

a) Copernicus, a Polish astronomer, lived from 1473 to 1543.

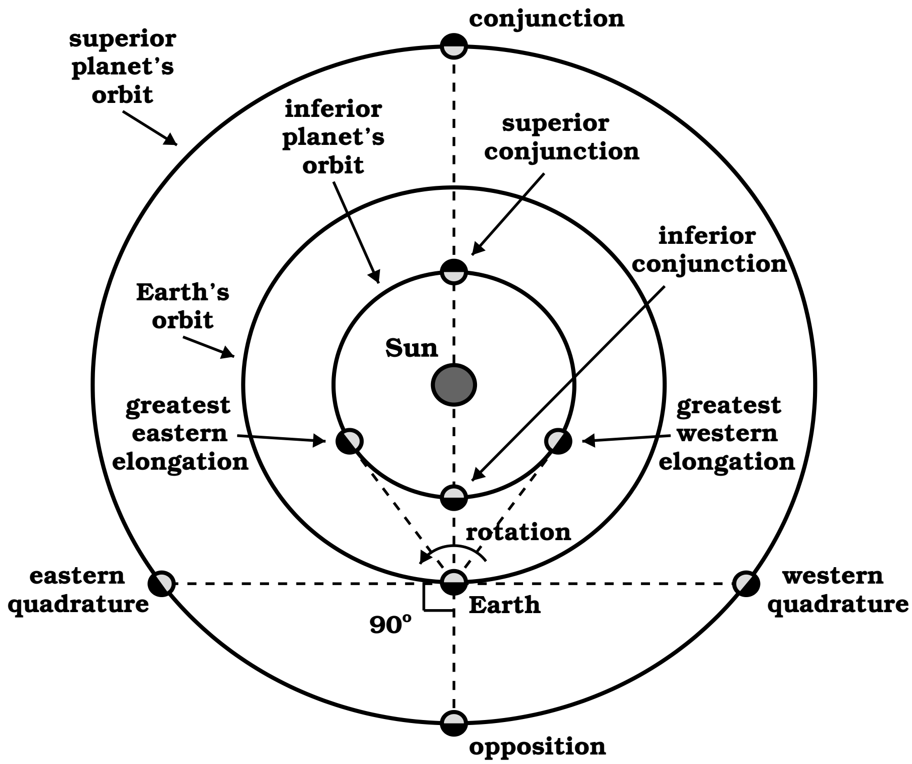
b) He rejected Ptolomy's geocentric model and developed a heliocentric *i.e.*, Sun-centered) model of the Solar System.

c) However, Copernicus assumed the planets orbited the Sun circular paths since circles were considered a 'perfect' shape.



B. Planetary Motion

1. In the course of their respective orbits, planets obtain various configurations with respect to the Earth and Sun.



2. Planetary Periods.

a) The **synodic period** S of a planet is the interval that elapses between 2 successive conjunctions (or oppositions) $\Longrightarrow$ this is an orbit period with respect to the Earth's position.

b) The **sidereal period** P is the true orbital period of a planet $\Longrightarrow$ time it takes the planet to orbit the Sun relative to the stars (*sidereal* means "star").

c) We can compare the sidereal periods of two planets to the synodic period of one planet as seen from the other by the equation

$$\frac{1}{S} = \frac{1}{P_i} - \frac{1}{P_o}, \qquad \text{(III-1)}$$

where P_i represents a sidereal period of the *inner* planet (*i.e.*, closer to the Sun), and P_o, the sidereal period of the *outer* planet (*i.e.*, farther from the Sun).

i) Since we make our observations from Earth, if we are observing an **inferior** planet (*i.e.*, closer to the Sun than Earth), then $P_o = P_\oplus = 1$ yr and

$$\frac{1}{S} = \frac{1}{P_i} - 1, \qquad \text{(III-2)}$$

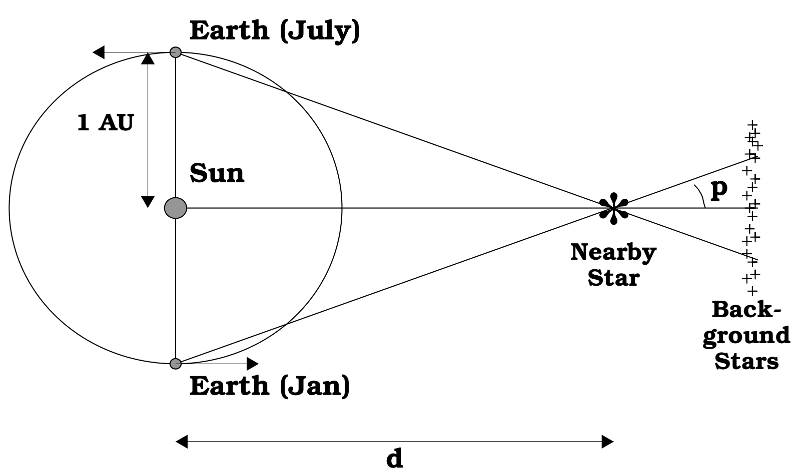
$\Longrightarrow$ Venus: $S = 1.6$ yrs (584 days) $\Rightarrow 1/S = 0.625$ $\Rightarrow 1/P = 1/S + 1 = 1.625 \Rightarrow P = 0.615$ yrs = 225 days.

ii) If we are observing a **superior** planet (*i.e.*, farther from the Sun than Earth), then $P_i = P_\oplus = 1$ yr and

$$\frac{1}{S} = 1 - \frac{1}{P_o}, \qquad \text{(III-3)}$$

$\Longrightarrow$ Mars: $S = 2.14$ yrs (780 days) $\Rightarrow 1/S = 0.467$ $\Rightarrow 1/P = 1 - 1/S = 0.533 \Rightarrow P = 1.88$ yrs = 687 days.

3. Tycho Brahe (1546 – 1601), the last of the great naked-eye astronomers, rejected the Copernican model because he could not detect *stellar parallaxes*:



a) Tycho made detailed observations of Mars.

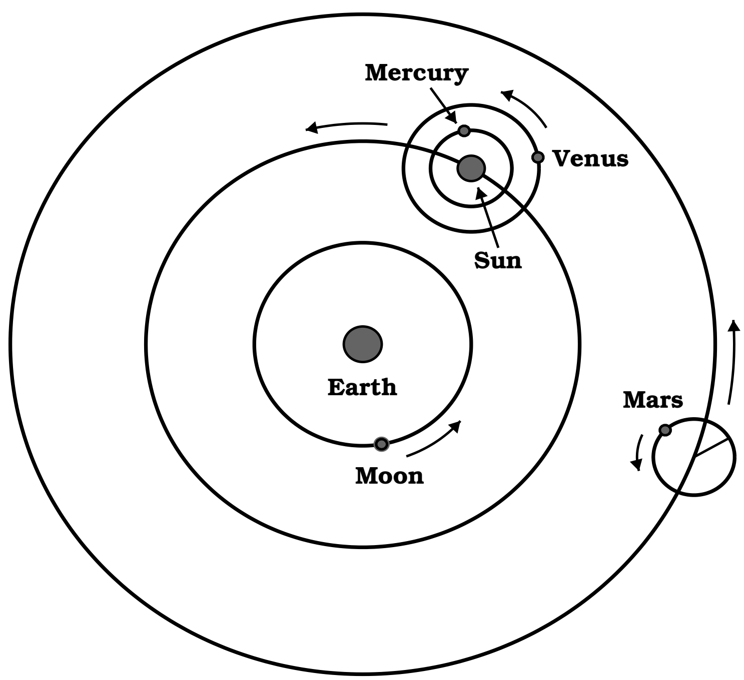
b) He hired Kepler as an assistant to help him reduce the data.

c) He developed his own model of the Solar System (see below).

d) Tycho was unaware of the immense distances between the stars. It turns out that stars do show a parallax angle shift (twice the p angle in the diagram above) due to the Earth's motion around the Sun. However even for the closest star, these angles are less than 1 arcsecond (most times much less), and without a telescope, Tycho had no chance of detecting this shift.

e) The distance to a star d (in parsecs) is equal to the reciprocal of the angle p (in arcseconds) in the diagram on the previous page:

$$d = \frac{1}{p} . \qquad \text{(III-4)}$$



4. Kepler's Laws of Planetary Motion.

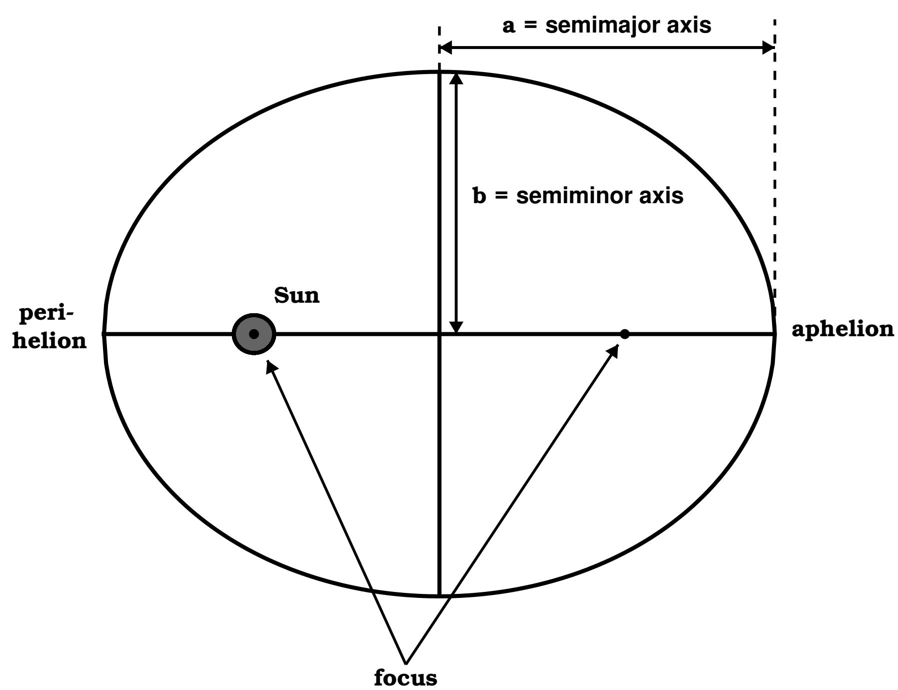
a) Johannes Kepler (1571 – 1630) was a German mathematician and astronomer who used Tycho's observations of the planets (especially Mars) to describe the 3 laws of planetary motion. The data showed that the Copernican model was correct, except that the planets move in elliptical and not circular paths around the Sun.

b) **Law 1:** The orbit of a planet about the Sun is an ellipse with the Sun at one focus.

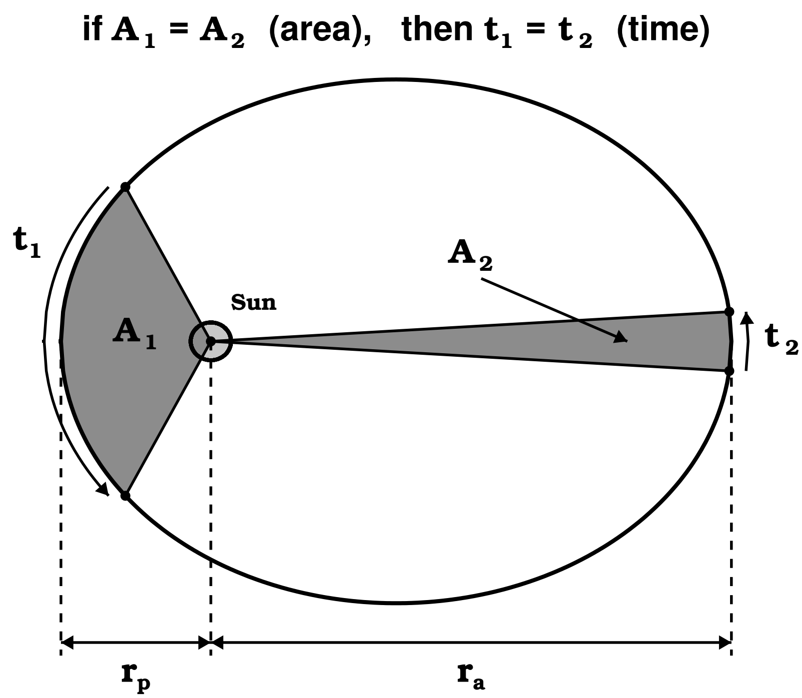
i) **Semimajor axis:** Half of the longest axis of an ellipse (a = semimajor axis length).

ii) **Semiminor axis:** Half of the shortest axis of an ellipse (b = semiminor axis length).

iii) 1 Astronomical Unit is the length of the Earth's semimajor axis (1 A.U. = 1.4960×10^{11} m).



c) **Law 2:** A line joining a planet and the Sun sweeps out equal areas in equal amounts of time (*law of equal areas*) $\Longrightarrow$ this means that a planet moves faster when it is near perihelion than at aphelion.



i) **Perihelion:** Point on a orbit when a planet is closest to the Sun (r_p = perihelion distance).

ii) **Aphelion:** Point on a orbit when a planet is farthest from the Sun (r_a = aphelion distance).

d) **Law 3:** The square of the sidereal period of a planet is proportional to the cube of the semimajor axis of a planet's orbit about the Sun (*harmonic law*).

$$\left(\frac{P}{P_\oplus}\right)^2 = \left(\frac{a}{a_\oplus}\right)^3$$

$$\left(\frac{P}{1\ \mathrm{yr}}\right)^2 = \left(\frac{a}{1\ \mathrm{AU}}\right)^3 \qquad \text{(III-5)}$$

OR

$$P_{\rm yr}^2 = a_{\rm AU}^3$$

Example III-1. If a comet has a semimajor axis of 100 AU, what would be its orbital period?

$$\begin{aligned}
\left(\frac{P}{1\ \rm yr}\right)^2 &= \left(\frac{100\ \rm AU}{1\ \rm AU}\right)^3 \\
&= (10^2)^3 = 10^6 \\
\frac{P}{1\ \rm yr} &= (10^6)^{1/2} = 10^3 \\
P &= 1000\ \rm yrs
\end{aligned}$$

---

5. If we define the distance between the 2 foci of an ellipse as h, the the **eccentricity** of the ellipse is defined by

$$e = \frac{h}{2a} = \frac{\sqrt{a^2 - b^2}}{a} \ . \qquad \text{(III-6)}$$

a) A **circular** orbit has $e = 0$ since $h = 0$ (note that the radius of the circle is $r = a$).

b) **Elliptical** orbits have $0 < e < 1$ since $0 < h < 2a$.

c) Note that as $h \rightarrow 2a$, the other side of the ellipse goes out to ∞ and the orbit is no longer **closed** $\Longrightarrow e = 1$ defines a **parabolic** path around the Sun, such an object would leave the Solar System and never return to the vicinity of the Sun.

d) The final type of orbit is a **hyperbolic** orbit where $e > 1$. Both parabolic and hyperbolic orbits are called **open** orbits.

e) The perihelion point r_p is related to the eccentricity of the orbit (*i.e.*, ellipse) and the aphelion point r_a by the equations

$$r_p + r_a = 2a \tag{III-7}$$
$$r_p = a(1 - e) \tag{III-8}$$
$$r_a = a(1 + e) \tag{III-9}$$

Example III-2. If a comet has an eccentricity of 0.8 and a perihelion distance of 0.5 AU, what is its aphelion distance, semimajor axis, and orbital period?

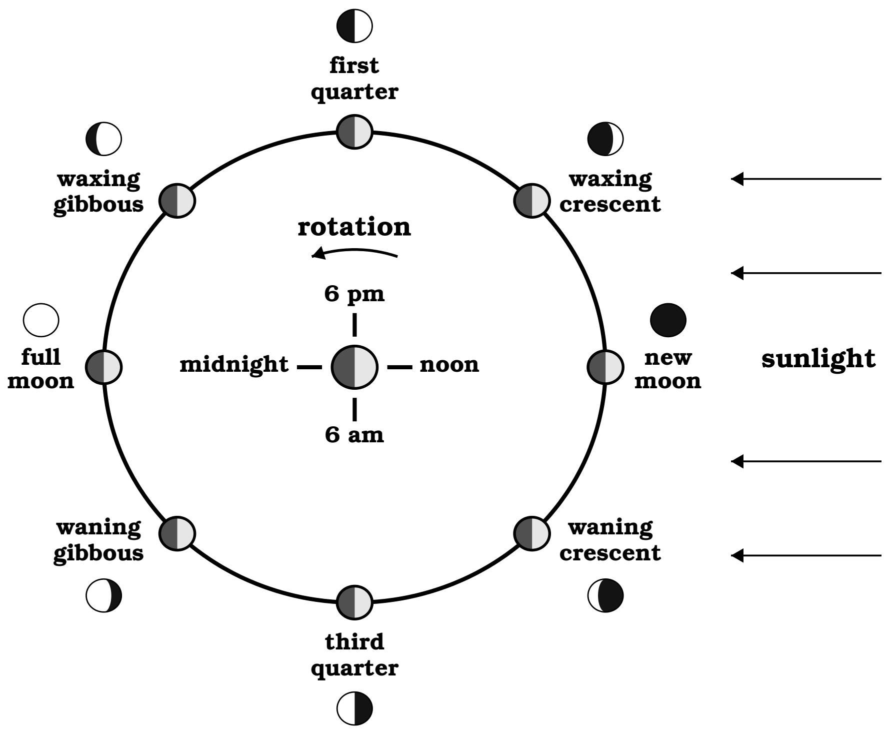
$$\begin{aligned} r_p &= a(1-e) \\ a &= r_p/(1-e) = 0.5 \text{ AU } /(1-0.8) \\ &= 0.5 \text{ AU } /0.2 = 2.5 \text{ AU} \\ r_a &= 2a - r_p = 2(2.5 \text{ AU}) - 0.5 \text{ AU} \\ &= 4.5 \text{ AU} \\ P_{\text{yrs}} &= a^{3/2} = \sqrt{(2.5)^3} = \sqrt{15.625} = 3.95 \text{ yrs} \end{aligned}$$

C. The Moon's Orbit

1. Lunar phases are caused by the Moon's orbital motion (see diagram on next page).

a) The Moon moves from west to east with respect to the stars as it orbits the Earth.

b) The **geocentric** (Earth-centered) phases of the Moon lasts one **synodic month**; these phases occur in the sequence: new (inferior conjunction), waxing crescent, first quarter (quadrature), waxing gibbous, full (opposition), waning gibbous, third quarter (quadrature, also called 'last' quarter), waning crescent, and back to new Moon.



2. If 2 full moons occur in the same calendar month, the second full moon is called a **blue moon**.

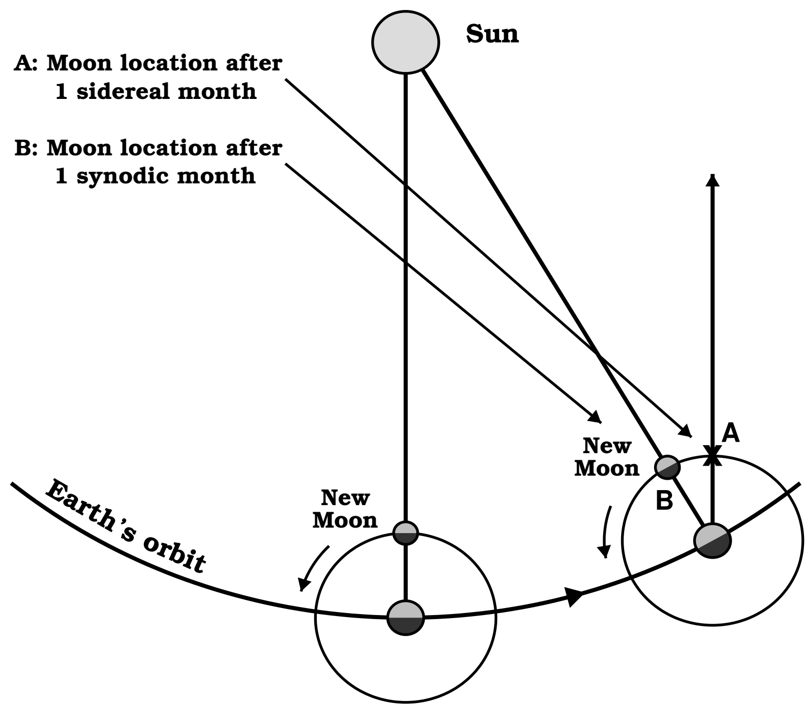
3. The first full moon that falls on or after the autumnal equinox is called the **harvest moon**.

4. *Easter* is always on the first Sunday that falls on or after the first full moon that occurs on or after the vernal equinox.

5. Likewise, *Passover* also is based on the lunar phase.

6. The Moon's sidereal period is the time it takes the Moon to complete one full orbit around the Earth with respect to the background stars (point A on the next diagram): 27.32 days.

7. The Moon's synodic period is the time it takes the Moon to make one complete cycle of phases (point B on the diagram below): 29.53 days.



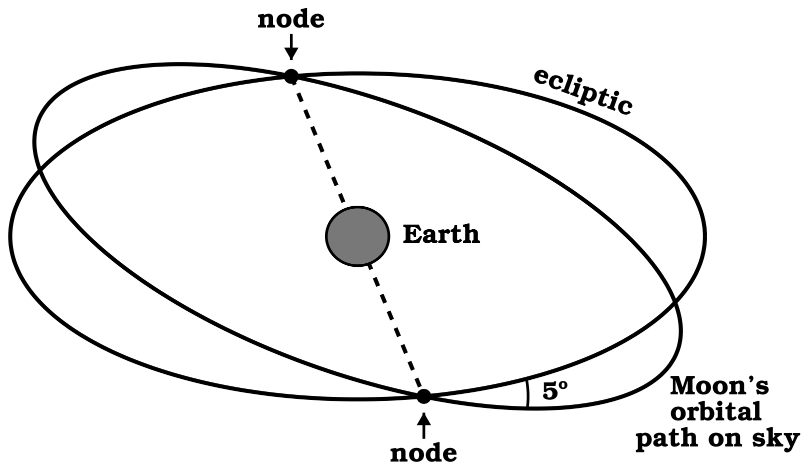
8. The Moon moves eastward in the sky 13.2° per day or slightly more than 0.5° per hour. Since the Moon's diameter is about 0.5°, the Moon move about one lunar angular diameter per hour.

9. The Moon's orbit is tilted 5° 9′ with respect to the Earth's orbit (and ecliptic). The points on the sky where the lunar orbital path intersects the ecliptic are called **node** points (see diagram on next page).

a) If the Moon is at one of its node points when at full phase, a total **lunar eclipse** will occur $\Longrightarrow$ the Moon passes into the Earth's shadow.

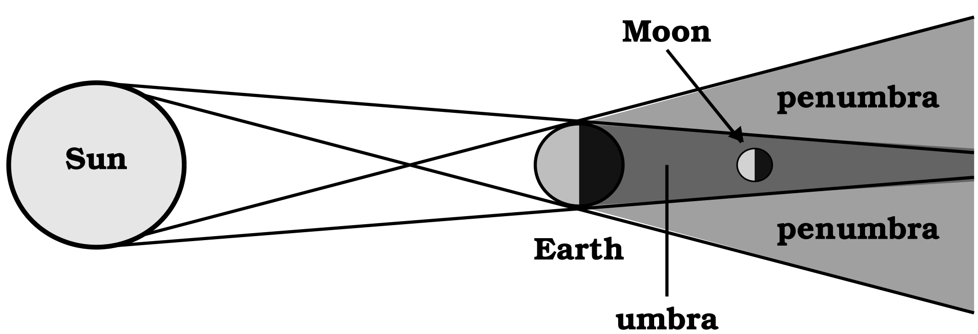
b) If the Moon is at one of its node points when at new phase, a total **solar eclipse** will occur somewhere on the Earth

$\Longrightarrow$ the Earth passes into the Moon's shadow.



c) The *dark* part of the shadow of an eclipse is called the **umbra** and occurs where a *total* eclipse takes place.

d) The *light* part of the shadow of an eclipse is called the **penumbra** and occurs where a *partial* eclipse takes place.



e) On average, 2 lunar eclipses and 2 solar eclipses occur each year.

D. The Physics of Planetary Motion: The Birth of Classical Physics.

1. Galileo Galilei (1564 – 1642), an Italian astronomer and physicist, is the father of experimental physics $\Longrightarrow$ he was the first to make accurate measurements in physical experiments.

a) He determined that objects of different masses fall at the same rate on the Earth's surface (which contradicted the teachings of Aristotle).

b) He came up with the concept of the pendulum clock.

c) He developed the various concepts of motion:

i) **Displacement:** Change of position of an object: d.

ii) **Speed:** The distance an object travels divided by the time it takes to travel that distance: $v = d/t$.

iii) Motion at a constant speed is called **uniform motion**.

iv) **Velocity:** Same as speed except a direction is given along with the magnitude $\Longrightarrow$ velocity is a <u>vector</u> whereas <u>speed</u> is a scalar.

v) **Acceleration:** The change of velocity over a given time interval: $a = (v_{\rm f} - v_{\rm i})/t$.

vi) The acceleration due to the Earth's gravitational field is called the surface gravity: $g = 9.80 \text{ m/s}^2$.

d) First to use the telescope to study the cosmos $\Longrightarrow$ discovered the 4 large moons of Jupiter (*i.e.*, the Galilean moons), that Venus goes through phases (like our Moon), that the Moon's surface wasn't smooth, and that dark spots appear on the Sun (*i.e.*, sunspots) from time to time.

e) Galileo got into a lot of trouble with the Church for these observations and supporting the Copernican model of the Solar System.

2. Issac Newton (1642 – 1727), an English astronomer and physicist, was perhaps the greatest scientist whoever lived!

a) Invented calculus to describe his physics.

b) Developed the laws of motion.

c) Developed the law of gravity.

d) Invented the reflecting telescope.

e) Developed many theories in optics and showed that white light is composed of the rainbow of colors.

3. Newton's laws of motion:

a) **The First Law:** A body remains at rest, or moves in a straight line at a constant speed, unless acted upon by an external force (**law of inertia**).

i) **Force (F):** Something that produces a change in the state of motion of an object.

ii) **Inertia:** The tendency of an object to remain in uniform motion.

b) **The Second Law:** $F = ma$, where m is the mass of an object and a is the acceleration of an object. Force is measured in *newtons* (N = kg m/s^2). $F = ma$ is the most important equation in all of science!

c) **The Third Law:** Whenever one body exerts a force on second body, the second body exerts an equal and opposite force on the first body. This law is the reason why rockets work. This law is nothing more than the conservation of linear momentum.

4. Newton's Universal Law of Gravity.

a) **Gravity** is a force that every object with mass possesses.

b) The universal law of gravity states: Two bodies attract each other with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them:

$$F = G\left(\frac{m_1 m_2}{r^2}\right). \qquad \text{(III-10)}$$

i) **Mass** measures how much material a body possesses, while **weight** measures the amount of gravitational force on an object.

ii) Weight changes depending on the gravitational field the object is in.

iii) Mass however never changes (unless you are traveling close to the speed of light $\Rightarrow$ special relativity).

iv) Objects in free fall in a gravitational field have mass but are *effectively* weightless $\Longrightarrow$ the Earth has a mass of 5.98×10^{24} kg but is effectly weightless since it is free fall around the Sun.

c) Newton was able to prove Kepler's 3rd law mathematically through his laws of physics. For two masses orbiting each other (m_1 and m_2), they obey the equation:

$$P^2 = \left[\frac{4\pi^2}{G(m_1 + m_2)}\right] a^3. \qquad \text{(III-11)}$$

i) For the case of the Solar System, let $m_1 = M_\odot$ (the mass of the Sun) and $m_2 = m_p$ (the mass of a planet in orbit about the Sun). Let us first take Eq. (III-11) and set up a ratio equation where we compare one planet with respect to the Earth:

$$\begin{aligned}
\left(\frac{P}{P_\oplus}\right)^2 &= \left\{\frac{4\pi^2/[G(M_\odot + m_p)]}{4\pi^2/[G(M_\odot + m_\oplus)]}\right\} \left(\frac{a}{a_\oplus}\right)^3 \\
\left(\frac{P}{1 \text{ yr}}\right)^2 &= \left[\frac{4\pi^2}{G(M_\odot + m_p)} \, \frac{G(M_\odot + m_\oplus)}{4\pi^2}\right] \left(\frac{a}{1 \text{ AU}}\right)^3 \\
P_{\text{yr}}^2 &= \left(\frac{M_\odot + m_\oplus}{M_\odot + m_p}\right) a_{\text{AU}}^3.
\end{aligned}$$

ii) Since the mass of any of the planets in the Solar System is far less than the mass of the Sun, $M_\odot + m_p \approx M_\odot$ and $M_\odot + m_\oplus \approx M_\odot$. Then Newton's form of Kepler's 3rd law becomes

$$P_{\text{yr}}^2 = \left(\frac{M_\odot}{M_\odot}\right) a_{\text{AU}}^3 = a_{\text{AU}}^3, \qquad \text{(III-12)}$$

which is Kepler's 3rd law as written in Eq. (III-5).

d) Newton also was able to mathematically prove Galileo's free fall experiment. $F = ma$, yet gravity says that $F_g = GM_\oplus m/r^2$, where m is the mass of the object accelerating through the Earth's gravitational field and $M_\oplus$ is the mass of the Earth. If we set these 2 forces equal to each other, we get $ma = GM_\oplus m/r^2$ or $a = g = GM_\oplus/r^2 = 9.80 \text{ m/s}^2$, independent of the objects mass!