Astronomy 1010: Astronomy I
Homework 1 Solutions
Solution Set for Universe: Origins & Evolution
by Snow & Brownsburger

This is the solution set for problems assigned in ASTR-1010: *Astronomy I*. This set is from problems assigned from the current textbook in the class, *Universe: Origins and Evolution* by Snow and Brownsburger.

1. **Review Question 1-3:**
   Discuss how you might test the validity of astrology: that is, describe an experiment or observation that would show whether predictions made by astrologers are correct or incorrect.

   The first thing that needs to be done is to have the astrologer give specific events which will occur for each zodiac sign on a given date in the future. Note that *global* claims such as *you will have a nice day* would not be allowed since it is too general and more times than not, one usually has a nice day. Then, send out questionnaires prior to the date in question to a large sample of people. The questions would concern things that happened to them on that date. Once the questionnaires are returned, numerical scores would be given for each one based upon whether or not the astrologer's predictions are matched. These numbers can then be compared to those of random chance (which will vary based upon the number of people in the survey). Random chance percentages are typical in the 10% range, so if the astrologer's predictions were no better than 10%, then the astrologer is a quack. To take all of astrology into account, then one also needs a large sample of astrologers to make predictions about a given date. Then one needs to compare all of the astrologer's predictions to the questionnaires results and to each others predictions. Should these numbers be no better than random chance, then one could say that astrology has no basis in truth. (It should be noted here that such experiments have been done by scientists in the past and astrology never does any better that random chance.)

2. **Review Question 1-5:**
   We have noted that the planets appear to travel along the same strip across the sky. What does this tell us about the orbits of the planets around the Sun?

   That they orbit the Sun in the same plane and formed in a flattened disk when the solar system came into being.

3. **Problem 1-2:**
   The Sun's angular diameter is 30 arcminutes. What is the diameter in degrees and in arcminutes?

   There are 60' (arcminutes) per degree, so the Sun's angular size (α) in degrees is

$$\alpha = 30' \times \frac{1^\circ}{60'} = 0.5^\circ.$$

There are 60" (arcseconds) in one arcminute, so

$$\alpha = 30' \times \frac{60''}{1'} = 1800''.$$

4. **Problem 1-3:**
The Moon takes 29.5 days to orbit the Earth, as seen by an observer on the Earth. How far must the Moon travel each day, in units of degrees per day? How far must it travel each hour? How does this hourly distance compare with the angular diameter of the Moon, which is 0.5°? Would this motion be easily observed?

The Moon's orbital period is 29.5 days which means it takes this long to make a complete circuit around the sky (*i.e.*, 360°). The rate, r, the Moon moves on the sky of degrees per day is just a ratio of these two numbers:

$$r = \frac{360^\circ}{29.5 \text{ days}} = 12.2^\circ/\text{day}.$$

There are 24^h in a day, so

$$r = 12.2^\circ/\text{day} \times \frac{1 \text{ day}}{24^h} = 0.51^\circ/\text{h}.$$

Since the Moon's diameter is 0.5°, the Moon moves one of its diameter per hour! If you note the Moon's exact position with respect to nearby stars, after a few hours it will have moved quite noticeably.

5. **Review Question 2-1:**
Explain the difference between solar time, sidereal time, and the time kept by atomic clocks.

Solar time is based upon the Sun passage across your meridian, sidereal time upon when a given star passes your meridian, and atomic clock time has nothing to do with the Earth's rotation. Instead, it is based upon *physical time*, that is, time based upon oscillations inside an atom which do not change with time.

6. **Review Question 2-4:**
Why do we have a north pole star, but not a south pole star? Will we always a north pole star? Explain.

Currently, there just happens to be a 2nd magnitude star near the north celestial pole. It won't stay that way forever since the Earth's axis precesses over a 26,000 year period. Most of this period, there are no bright stars close to the NCP, just as is the case today with the SCP. It's just a coincidence that today, Polaris is near the NCP.

7. **Review Question 2-6:**
How would our seasons be different if the Earth's axis was not tilted (that is, if the axis were perpendicular to the Earth's orbital plane)? What would the seasons be like if the Earth's axis were tipped 90° instead of 23.5°?

If the Earth's axis was not tilted, there would be no seasons — the Sun would always be the same altitude above the horizon at noon throughout the year. For your given location, the daily temperatures would not change by much for a given time of day over the course of a year. If the Earth's axis was tilted by 90°, the seasons would be extreme since in the winter, the northern hemisphere would be dark for much of the day and the southern hemisphere, light for 24^h, and just the opposite would take place in the summer. Life would have probably never arose under such circumstances.

8. **Problem 2-2:**
   The Moon moves eastward on the sky at an angular distance of about 12° per day. How much later does it set each evening, compared with the evening before?

   Since there are 360° in 24^h and the Moon moves 12° in one day, it would set

   $$12^\circ \times \frac{24^h}{360^\circ} = 0.8^h = 48^m$$

   later each day.

9. **Problem 2-5:**
   Precession causes the Sun's position at any given date to move gradually along the zodiac. If each major constellation occupies about 1/12 th of the zodiac (that is, the constellations are centered about 30° apart), how long does it take for the Sun's position to shift from one zodiac sign (constellation) to the next?

   It takes precession 26,000 years to complete one circuit around the sky. Therefore, the vernal equinox moves by 360° / 26,000 yrs = 0.014°/yr = 0.83'/yr = 50"/yr. Since the center of each zodiac constellation is 30° apart, the vernal equinox moves 1 constellation every 30° / 0.014°/yr = 2100 years.

10. **Problem 2-10:**
    Suppose a new superior planet is discovered and found to have a synodic period of 6.85 years. What is its sidereal period?

    If S is the synodic period of the planet, P_o is the sidereal period of the superior planet and $S = 6.85$ years, then the sidereal period is

    $$\begin{aligned}\frac{1}{S} &= 1 - \frac{1}{P_o}\\ \frac{1}{P_o} &= 1 - \frac{1}{S}\\ \frac{1}{P_o} &= 1 - \frac{1}{6.85}\\ \frac{1}{P_o} &= 1 - 0.14599\\ \frac{1}{P_o} &= 0.85401\\ P_o &= 1.17 \text{ years}\end{aligned}$$

11. **Problem 3-2:**
Aristarchus measured the angular diameters of the Sun and Moon, finding each to be about 2°, whereas the correct values are about 0.5°. What would be the length of 1 AU (in kilometers) if the Sun actually were close enough to have an angular diameter of 2°?

Using the small angle formula we developed in class, let s be the actual size of the Sun ($= 1.39 \times 10^6$ km) and the angular size $\theta = 2^\circ \times 2\pi \text{ rad}/360^\circ = 3.49 \times 10^{-2}$ rad. Then the distance of the Sun r is

$$\begin{aligned} s &= \theta\, r \\ r &= \frac{s}{\theta} = \frac{1.39 \times 10^6 \text{ km}}{3.49 \times 10^{-2}} \\ &= 3.98 \times 10^7 \text{ km}. \end{aligned}$$

The actual Astronomical Unit is 1.50×10^8 km, so the AU would be 4 times shorter than it actually is if the Sun was 2° instead of its actual 0.5°.

12. **Problem 4-3:**
Tycho's great quadrant consisted of an quarter circle with a radius of 9.6 m, covering an angle of 90°. Marks on the quadrant corresponded to degrees and minutes of arc. How far apart, in centimeters, were the degree marks? The minute marks?

From the equation above, the length of the arc of Tycho's quadrant s would have been

$$s = \theta\, r = \frac{\pi}{2}\, 9.6 \text{ m} \;= 15.1 \text{ m},$$

(note that $90^\circ = \pi/2$ radians). Since there are 90° in this length, the distance between each degree marker would have been $15.1 \text{ m}/90^\circ = 0.167$ m per degree or 16.7 cm/deg. Since there are 60'/deg, the distance between each minute mark would have been (16.7 cm/deg)/(60'/deg) = 0.279 cm/arcmin = 2.79 mm/arcmin.

13. **Problem 4-5:**
The Earth's distance from the Sun ranges from 147 million to 152 million km. Calculate the eccentricity of the Earth's orbit.

The perihelion distance is $r_p = 1.47 \times 10^8$ km and the aphelion distance is $r_a = 1.52 \times 10^8$ km. From the book, the perihelion and aphelion distances are related to the semimajor axis a and eccentricity e of an orbit by

$$\begin{aligned} r_p &= a(1-e) \\ r_a &= a(1+e) \end{aligned}$$

If we divide both equations by each other we get

$$\begin{aligned} \frac{r_p}{r_a} &= \frac{(1-e)}{(1+e)} \\ r_p(1+e) &= r_a(1-e) \\ r_p + e\, r_p &= r_a - e\, r_a \end{aligned}$$

$$\begin{aligned} e\,r_a + e\,r_p &= r_a - r_p \\ e(r_a + r_p) &= r_a - r_p \\ e &= \frac{r_a - r_p}{r_a + r_p} \\ &= \frac{(1.52\times10^8 - 1.47\times10^8)\ \mathrm{km}}{(1.52\times10^8 + 1.47\times10^8)\ \mathrm{km}} \\ &= \frac{0.05\times10^8}{2.99\times10^8} \\ &= 0.0167 \end{aligned}$$

14. **Problem 4-7:**
If asteroid Grumpy has an orbital eccentricity of 0.47 and a perihelion distance of 3.54 AU, how far is it from the Sun at aphelion?

Grumpy has $e = 0.47$ and $r_p = 3.54$ AU. First, calculate its semimajor axis (see problem above):

$$a = \frac{r_p}{1-e} = \frac{3.54\ \mathrm{AU}}{1-0.47} = \frac{3.54\ \mathrm{AU}}{0.53} = 6.68\ \mathrm{AU}.$$

Use the semimajor axis along with the eccentricity to get its aphelion distance:

$$r_a = a\,(1+e) = 6.68\ \mathrm{AU}\ (1+0.47) = 6.68\ \mathrm{AU}\ \times 1.47 = 9.82\ \mathrm{AU}.$$

15. **Problem 4-8:**
What is the period of a planet whose semimajor axis is 3 AU? What is the period of a planet whose semimajor axis is 25 AU?

If $a = 3$ AU, then its period (from Kepler's 3rd law, $P^2(\mathrm{yr}) = a^3(\mathrm{yr})$) is

$$P = a^{3/2} = (3)^{3/2} = (27)^{1/2} = \sqrt{27} = 5.2\ \mathrm{yr}.$$

If $a = 25$ AU (and note that 25 is a perfect square of 5), then its period is

$$P = a^{3/2} = (25)^{3/2} = (\sqrt{25})^3 = (5)^3 = 125\ \mathrm{yr}.$$

16. **Review Question 5-2:**
Explain, in your own words, the difference between weight and mass.

Weight is a force (mass times acceleration), where the acceleration is the Earth's surface gravitational acceleration, whereas mass is a measure of how much material is there. Weight can change from planet to planet, but mass stays the same.

17. **Review Question 5-6:**
Explain why an astronaut in an orbiting spacecraft is said to be "weightless." Is there really no force of gravity acting on the astronaut?

Actually the Earth's gravitational field is still very strong in Earth orbit. However, weight is a measure of how much force is being exerted against an "immovable" surface (like the surface of a planet). Something in orbit is actually falling towards the Earth, however its forward motion causes it to always *miss* the Earth as it is falling. This is what is meant by being in orbit. Since you're falling at the same rate as the spacecraft, you appear to be weightless with respect to the spacecraft.

18. **Problem 5-1:**
Suppose you are a construction worker, laboring in Earth orbit to build a space station. You are free-floating (space-walking) outside of the station, and you have to move a massive beam, also free-floating. The beam has a mass of 1,000 kg (10 times your mass), and you push it with a force of 200 N. What is the acceleration of the beam?

From Newton's 2nd law of motion, $F = m\,a$, the acceleration is

$$a = \frac{F}{m} = \frac{200\ \mathrm{N}}{1000\ \mathrm{kg}} = 0.2\ \mathrm{m/s}^2,$$

note that $1\ \mathrm{N} = 1\ \mathrm{kg\ m/s}^2$.

19. **Problem 5-2:**
Jupiter is about 5 times farther from the Sun than the Earth is, and its mass is about 300 times the Earth's mass. Compare the gravitational force between Jupiter and the Sun with that between the Earth and the Sun. Perform the same calculation for Saturn, which has 100 times the mass of the Earth and orbits about 10 times farther from the Sun.

Let $F_\oplus$ be the force between the Earth and Sun, F_J the force between Jupiter and the Sun, and F_S the force between Saturn and the Sun. Newton's law of gravity is $F = G\,M\,m/r^2$, where here, M is the mass of the Sun, m the mass of the individual planets, and r the distance between each planet and the Sun. If we set up a ratio of forces with respect to Earth, we can get rid of a couple of constants:

$$\frac{F_J}{F_\oplus} = \frac{G\,M\,m_J/r_J^2}{G\,M\,m_\oplus/r_\oplus^2} = \frac{m_J/r_J^2}{m_\oplus/r_\oplus^2} = \frac{m_J}{m_\oplus}\left(\frac{r_\oplus}{r_J}\right)^2.$$

Since $r_J = 5\,r_\oplus$ and $m_J = 300\,m_\oplus$, we get

$$\frac{F_J}{F_\oplus} = \frac{300\,m_\oplus}{m_\oplus}\left(\frac{r_\oplus}{5\,r_\oplus}\right)^2 = 300 \times \left(\frac{1}{5}\right)^2 = \frac{300}{25} = 12,$$

or $F_J = 12\,F_\oplus$. For Saturn, $r_S = 10\,r_\oplus$ and $m_S = 100\,m_\oplus$, we get

$$\frac{F_S}{F_\oplus} = \frac{100\,m_\oplus}{m_\oplus}\left(\frac{r_\oplus}{10\,r_\oplus}\right)^2 = 100 \times \left(\frac{1}{10}\right)^2 = \frac{100}{100} = 1,$$

or $F_S = F_\oplus$!

20. **Problem 5-5:**
What would you weigh on the surface of Titan, the giant satellite of Saturn? Use data from Appendix 8.

Weight is determined from Newton's law of gravity: $F = G\,M\,m/r^2$. In this problem, $G = 6.67 \times 10^{-11}$ N m^2/kg^2, $M = 1.35 \times 10^{23}$ kg is the mass of Titan, $m = 70$ kg is your mass (a weight of approximately 160 lbs, note that 1 N = 0.225 lb and the surface gravity of Earth is 9.8 m/s^2), and $r = 2.58 \times 10^6$ m is your distance from the center of this moon, which since you are on the surface, is just the radius of Titan. Then your weight is

$$\begin{aligned} F &= \frac{(6.67 \times 10^{-11}\ \mathrm{N\ m^2 kg^{-2}})(1.35 \times 10^{23}\ \mathrm{kg})(70\ \mathrm{kg})}{(2.58 \times 10^6\ \mathrm{m})^2} \\ &= \frac{6.30 \times 10^{14}\ \mathrm{N\ m^2}}{6.66 \times 10^{12}\ \mathrm{m^2}} \\ &= 95\ \mathrm{N}. \end{aligned}$$

Using the conversion factor above, this means your weight on Titan (assuming its 160 lb on Earth) is 21 lbs!