Astronomy 1010: Astronomy I
Homework 2 Solutions
Solution Set for Universe: Origins & Evolution
by Snow & Brownsburger

This is the solution set for problems assigned in ASTR-1010: *Astronomy I.* This set is from problems assigned from the current textbook in the class, *Universe: Origins and Evolution* by Snow and Brownsburger.

1. **Review Question 6-1:**
   Explain why we say that light has properties of both waves and particles.

   This is due to the fact that when experiments are performed with light to test a *wave* nature, it shows such a nature, meanwhile, when tests are performed to test a *particle* nature, it also shows that type of nature. **Diffraction** and **interference** are wave-like properties, the **photoelectric effect** represents a particle-like property.

2. **Review Question 6-2:**
   Discuss possible reasons why the human eye is sensitive to the particular set of wavelengths that we know as visible light. (Hint: Consider the wavelengths that might be seen by life-forms living on a planet orbiting a much hotter or much cooler star than the Sun.)

   The Earth's atmosphere allows sunlight from 4000 Å to 7000 Å (*i.e.*, the visible band) to pass through unimpeded to it surface from space (it also allows radio waves to pass). Evolution has allowed those species capable of seeing well at those wavelengths to survive and propagate. The Sun's temperature of 6000 K causes the maximum amount of sunlight to be emitted at 5000 Å (yellow light, this is why the Sun looks yellow to us). The human eye, as well as the eyes of most species on Earth, are most sensitive at 5000 Å, once again, due to evolution — those that see the best at wavelengths where the Sun produces most of it light will be able to hunt better and survive.

3. **Review Question 6-3:**
   Explain how the color of a star is related to its surface temperature.

   By Wien's law, we know that the hotter then star, the bluer it is, the cooler the star, the redder it is.

4. **Review Question 6-6:**
   Explain how the same atom can form emission lines in one situation and absorption lines in another.

   An **absorption line** results when a photon is absorbed by an atom, causing an electron in an atom to jump from a lower energy state to a higher energy state, whereas, an **emission line** results when an excited electron jumps back from a higher to lower energy level producing a photon. If one is looking at gas in the line-of-sight of a continuum source, atoms in that gas will selectively absorb light from the continuum at specific wavelengths corresponding to the energy differences between the electronic states of the atoms in the gas, producing dark lines on the bright continuum of colors. Then the electrons cascade back down producing photons that can be sent out in any direction. As such, if we look at the same gas, but now with the continuum source off to one side, we see emission lines coming from the same atoms that caused the absorption lines in the line-of-sight direction.

5. **Review Question 6-7:**
   What is ionization and how does it occur?

   Ionization is caused by an electron being ripped off of an atom causing the atom to have an excess positive charge. This can happen in one of two ways: (1) energetic collisions between atoms can knock off electrons and (2) high energy photons can be absorbed by the atom and knock off a bound electron from the atom.

6. **Problem 6-1:**
   Calculate the frequency and energy of a photon whose wavelength is $\lambda = 5560$ Å.

   The frequency ν is related to wavelength λ by $\nu = c/\lambda$, where $c = 3.00 \times 10^8$ m/s is the speed of light, so

$$\begin{aligned}\nu = \frac{c}{\lambda} &= \frac{3.00 \times 10^8 \text{ m/s}}{5560 \text{ Å} \times 10^{-10} \text{ Å/m}} \\ &= \frac{3.00 \times 10^8 \text{ m/s}}{5.560 \times 10^{-7} \text{ m}} = 5.40 \times 10^{14} \text{ /s} \\ &= 5.40 \times 10^{14} \text{ Hz}\end{aligned}$$

   The energy of a photon is given by $E = h\,c/\lambda$, where $h = 6.63 \times 10^{-34}$ J·s is Planck's constant, so

$$\begin{aligned}E = \frac{h\,c}{\lambda} &= \frac{(6.63 \times 10^{-34} \text{ J·s})\,(3.00 \times 10^8 \text{ m/s})}{5560 \text{ Å} \times 10^{-10} \text{ Å/m}} \\ &= 3.57 \times 10^{-19} \text{ J}\ .\end{aligned}$$

7. **Problem 6-3:**
   How long does it take to send a radio message from Earth to a spacecraft flying by Saturn? (Assume Saturn is at opposition.) With this time frame in mind, would it make more sense to control the probe directly from Earth as it flies by Saturn or to preprogram the probe's maneuvers before the planetary encounter?

   A radio signal travels at the velocity of light c and velocity is related to distance d and the amount of time it takes to travel that distance t by $c = d/t$. So the amount of time it takes a radio signal to travel a distance d is $t = d/c$. Saturn's distance from the Sun (d_{Saturn}) is 9.555 AU and the Earth's ($d_\oplus$) is 1 AU. When Saturn is at opposition it distance from Earth is $d = d_{\text{Saturn}} - d_\oplus = 9.555 - 1.0$ AU $= 8.555$ AU. So the time needed to send a signal to Saturn from Earth is

$$t = \frac{8.555 \text{ AU} \times 1.496 \times 10^{11} \text{ m/AU}}{3.00 \times 10^8 \text{ m/s}} = \frac{1.28 \times 10^{12} \text{ m}}{3.00 \times 10^8 \text{ m/s}}$$

$$
\begin{aligned}
&= 4.27 \times 10^3 \text{ s} \times \frac{1 \text{ hr}}{3600 \text{ s}} \\
&= 1.19 \text{ hours}
\end{aligned}
$$

This means that a round trip communication between spacecraft and Earth would be 2.4 hours which is too long of a time to tell the spacecraft what to do should any unforeseen circumstance happen. One would have to preprogram the spacecraft to react to certain situations.

8. **Problem 6-4:**
An average body temperature for an adult human is 98.6°F (∼310 K). When the person is sick with a fever, however, the body temperature can rise to around 102°F (∼312 K). Assuming a human radiates like a blackbody, how much more energy would a person emit when feverish than when healthy?

Set up the ratio

$$\frac{F_f}{F_n} = \frac{\sigma T_f^4}{\sigma T_n^4} = \left(\frac{T_f}{T_n}\right)^4,$$

where the subscript f represents fever and n represents normal body temperature. Then $F_f/F_n = (312 \text{ K}/310 \text{ K})^4 = (1.00645)^4 = 1.026$. The fractional energy increase is then

$$\frac{F_f - F_n}{F_n} = \frac{F_f}{F_n} - \frac{F_n}{F_n} = \frac{F_f}{F_n} - 1 = 1.026 - 1 = 0.026.$$

Multiplying this by 100% give an increase of energy output of 2.6%.

9. **Problem 6-5:**
Calculate the wavelengths of maximum emission for a star of surface temperature 25,000 K; a star of surface temperature 2,500 K; the Sun's corona at a temperature of 2,000,000 K; and a human body of temperature 310 K. What type of telescope would be best suited to observe each of these bodies?

With $T_1 = 25,000$ K, $T_2 = 2,500$ K, $T_3 = 2,000,000$ K, and $T_4 = 310$ K, use Wien's law to calculate the maximum flux wavelength:

$$\lambda_{max} = \frac{0.0029 \text{ m K}}{T}$$

then

$$
\begin{aligned}
\lambda_{max}(1) &= \frac{0.0029 \text{ m K}}{25,000 \text{ K}} = 1.16 \times 10^{-7} \text{ m} \times \frac{10^{10} \text{ Å}}{1 \text{ m}} = 1160 \text{ Å} \\
\lambda_{max}(2) &= \frac{0.0029 \text{ m K}}{2,500 \text{ K}} = 1.16 \times 10^{-6} \text{ m} \times \frac{10^{10} \text{ Å}}{1 \text{ m}} = 11,600 \text{ Å} \\
\lambda_{max}(3) &= \frac{0.0029 \text{ m K}}{2,000,000 \text{ K}} = 1.45 \times 10^{-9} \text{ m} \times \frac{10^{10} \text{ Å}}{1 \text{ m}} = 14.5 \text{ Å} \\
\lambda_{max}(4) &= \frac{0.0029 \text{ m K}}{310 \text{ K}} = 9.35 \times 10^{-6} \text{ m} \times \frac{10^{10} \text{ Å}}{1 \text{ m}} = 93,500 \text{ Å}.
\end{aligned}
$$

The 25,000 K star maximum flux is at UV wavelengths (1160 Å), hence one would need an UV telescope above the Earth's atmosphere. The 2,500 K star maximum flux is in the near IR (11,600 Å), hence one would need an IR telescope. The solar corona's maximum flux is at X-ray wavelengths (14.5 Å), hence one would need an X-ray telescope above the Earth's atmosphere. The human body's maximum flux is at IR wavelengths (93,000 Å = 9.3 microns), hence one would need an IR telescope/detector.

10. **Problem 6-8:**
The rest, or laboratory, wavelength of a particular transition of hydrogen called Hα (H–alpha) is 6563 Å. Suppose the Hα transition is observed in a star at a wavelength of 6565 Å. What can you say about the relative motion of this star along your line of sight?

Hα at rest: $\lambda_\circ = 6563$ Å at rest is observed at $\lambda = 6565$ Å. Since 6565 Å is longward of 6563 Å, the observed line is redshifted, hence the object is traveling away from us at a speed:

$$\begin{aligned} v &= \frac{\Delta\lambda}{\lambda_\circ}\, c = \left(\frac{6565 \text{ Å} - 6563 \text{ Å}}{6563 \text{ Å}}\right) 3.00 \times 10^5 \text{ km/s} \\ &= \left(\frac{2 \text{ Å}}{6563 \text{ Å}}\right) 3.00 \times 10^5 \text{ km/s} = (3.05 \times 10^{-4}) \cdot 3.00 \times 10^5 \text{ km/s} \\ &= 91.4 \text{ km/s} \end{aligned}$$

11. **Problem 6-9:**
How much greater is the light-gathering power of a 10 m telescope than that of a 4 cm telescope?

The light gathering power of one telescope with respect to another is

$$\frac{\mathrm{LGP}_A}{\mathrm{LGP}_B} = \frac{D_A^2}{D_B^2} = \left(\frac{D_A}{D_B}\right)^2,$$

where D is the diameter of the objective of the telescope. Let the 10 m telescope be A in the equation above and the 4 cm telescope be B, then

$$\frac{\mathrm{LGP}_A}{\mathrm{LGP}_B} = \left(\frac{10 \text{ m} \times 100 \text{ cm/m}}{4 \text{ cm}}\right)^2 = \left(\frac{1000 \text{ cm}}{4 \text{ cm}}\right)^2 = 250^2 = 62{,}500.$$

The 10 m telescope collects 62,500 times as much light as the 4 cm telescope.

12. **Problem 6-10:**
Suppose you purchase a reflecting telescope with a 20 cm diameter mirror. Ignoring the effects of atmospheric "seeing" and assuming you have perfect optics, what is the smallest angle you can resolve with your new telescope?

$D = 20$ cm and assume $\lambda = 5000$ Å, then

$$\begin{aligned} \alpha = 2.1 \times 10^5 \left(\frac{\lambda}{D}\right) \text{ arcsec} &= 2.1 \times 10^5 \left(\frac{5000 \text{ Å} \times 10^{-8} \text{ cm/Å}}{20 \text{ cm}}\right) \text{ arcsec} \\ &= 2.1 \times 10^5 \left(\frac{5.000 \times 10^{-5} \text{ cm}}{20 \text{ cm}}\right) \text{ arcsec} \\ &= 2.1 \times 10^5 \left(2.50 \times 10^{-6}\right) \text{ arcsec} \\ &= 5.25 \times 10^{-1} \text{ arcsec} = 0.525 \text{ arcsec}. \end{aligned}$$

13. **Review Question 13-4:**
Why are the granules in the solar photosphere brighter than the descending gas that surrounds them?

The granule center is brighter than the edge because the gas is hotter at the center than it is at the edges. The gas in the center is rising up from the hotter interior, cools at the surface and sinks back down at the granule edges.

14. **Review Question 13-8:**
Why do we say that the solar activity cycle is 22 years long, even though the sunspot maxima occur every 11 years?

The number of sunspots in the Sun's photosphere goes from a maximum number down to a minimum, then back up to a maximum approximately every 11 years. During this cycle, the magnetic field of the sunspot pairs might have *north* pointing in the direction of rotation. Then during the next 11 year cycle, *south* will be pointing in the direction of rotation. The following cycle will once again have *north* pointing in the direction of rotation. As such, even though maximum to maximum is every 11 years, the complete magnetic cycle is twice this or 22 years.

15. **Problem 13-6:**
Calculate the wavelength of maximum emission for the solar photosphere (temperature of 5,800 K), the chromosphere (20,000 K), and the corona (2×10^6 K). How can each of these layers best be observed?

For $T_{ph} = 5{,}800$ K, $T_{chr} = 20{,}000$ K, and $T_{cor} = 2.00 \times 10^6$ K, we get

$$\begin{aligned}
\lambda_{max}(ph) &= \frac{0.0029 \text{ m K}}{5,800 \text{ K}} = 5.00 \times 10^{-7} \text{ m } \times \frac{10^{10} \text{ Å}}{1 \text{ m}} = 5000 \text{ Å} \\
\lambda_{max}(chr) &= \frac{0.0029 \text{ m K}}{20,000 \text{ K}} = 1.45 \times 10^{-7} \text{ m } \times \frac{10^{10} \text{ Å}}{1 \text{ m}} = 1450 \text{ Å} \\
\lambda_{max}(cor) &= \frac{0.0029 \text{ m K}}{2.0 \times 10^6 \text{ K}} = 1.45 \times 10^{-9} \text{ m } \times \frac{10^{10} \text{ Å}}{1 \text{ m}} = 14.5 \text{ Å}
\end{aligned}$$

For the photosphere, a normal optical telescope can be used since λ_{max} is at visual wavelengths; for the chromosphere, an UV telescope would have to be used; and for the corona, an X-ray telescope would have to be used.

16. **Problem 13-7:**
If the average speed of the solar wind between the Sun and the Earth is 250 km/s, how long does it take for a particle of solar wind to reach the Earth once it is emitted from the Sun? How is your answer related to the time it takes for a solar flare to begin to affect radio communications on the Earth?

Once again, velocity v is defined by $v = d/t$, where t is the time interval over which the object has traveled a distance d. Here, $v = 250$ km/s and $d = 1$ AU $= 1.496 \times 10^8$ km. Hence, the time it will take to for a solar wind particle to get to Earth is

$$t = \frac{d}{v} = \frac{1.496 \times 10^8 \text{ km}}{250 \text{ km/s}} = 5.984 \times 10^5 \text{ s } = 6.93 \text{ days.}$$

Radio communications are typically disrupted 3 or 4 days after a **solar flare** has erupted on the Sun. Since this travel time of the charged particles shot out of the Sun is about half of what the typical solar wind particle travel time is, the particles emitted from a solar flare must be traveling twice as fast (*i.e.*, 500 km/s) as a solar wind particle.

17. **Problem 13-8:**
Suppose the temperature in a sunspot were one half the temperature in the surrounding photosphere. How much less intense would the radiation be from the sunspot than from the photosphere?

This is just a Stefan-Boltzmann Law question. Here $T_{sp} = \frac{1}{2} T_{ph}$, hence the flux would be reduced by

$$\frac{F_{sp}}{F_{ph}} = \left(\frac{T_{sp}}{T_{ph}}\right)^4 = \left(\frac{1/2T_{ph}}{T_{ph}}\right)^4 = \frac{1^4}{2^4} = \frac{1}{16} = 0.0625,$$

or the spot is only 6.25% as bright as the surrounding photosphere.