Astronomy 1010: Astronomy I
Homework 3 Solutions
Solution Set for Universe: Origins & Evolution
by Snow & Brownsburger

This is the solution set for problems assigned in ASTR–1010: *Astronomy I*. This set is from problems assigned from the current textbook in the class, *Universe: Origins and Evolution* by Snow and Brownsburger.

1. **Review Question 7–2:**
   Summarize the distinction between terrestrial and gas giant (*i.e.*, Jovian) planets.

   The differences are itemized here:

   - Terrestrials are closer to the Sun (*i.e.*, inside the asteroid belt) than the Jovians, which are outside the asteroid belt.
   - Terrestrials are smaller than the Jovians.
   - Terrestrials are rocky with thin or no atmospheres whereas the Jovians are gaseous throughout (with possibly small rocky/metallic cores), though both Uranus and Neptune may have substantial icy mantles as well.
   - The Jovians have large moon systems whereas the terrestrials have 1, 2, or no moons.

2. **Review Question 7–5:**
   Explain why hydrogen is not a dominate component of the atmospheres of the terrestrial planets, even though it is the most common element in the Universe and in the solar system.

   Two things dictate whether or not a planet holds onto a gas: the temperature of the planet, which dictates the speed of the gas particles, and the mass of the planet, which dictates the escape velocity of the planet. The square of the gas velocity is inversely proportional to the mass of the gas particle, so the lighter the gas, the faster it will move. The terrestrial planets are close to the Sun, hence have higher temperatures than the Jovians. They are also of smaller mass, such that the velocity of the hydrogen molecules is much greater than the escape velocity. As such, the terrestrials have lost all of their free hydrogen.

3. **Review Question 7–9:**
   Summarize the factors that determine the composition of a planet's atmosphere. Include all of the atmospheric gain and loss mechanisms you can think of.

   First, the cosmic abundance of the elements. Second, the distance that the planet is from the Sun coupled with the mass of the planet as described in the answer above. Finally, if liquid water can form, then CO_2 is filtered out, and if life forms, some of this CO_2 is converted to O_2.

4. **Problem 7–5:**
A new planet is discovered in our solar system. Its orbital period is 3.6 years, its mass is 4.0×10^{25} kg, and its radius is 12,500 km. Calculate the orbital semimajor axis and the density of the body, and use your results to decide whether it is a terrestrial or gas giant planet. Explain how you reached your conclusion.

$P = 3.6$ yrs, $M = 4.0 \times 10^{25}$ kg, and $R = 12,500$ km. The semimajor axis is determined from Kepler's 3rd law:
$$a = P^{2/3} = (3.6)^{2/3} = 2.3 \text{ AU}.$$
The density is determined from Eq. (VII–1) in the class notes:
$$\rho = \frac{M}{4\pi R^3} = \frac{4.0 \times 10^{25} \text{ kg}}{4\pi \times (12,500 \text{ km } \times 1000 \text{ m/km})^3} = \frac{4.0 \times 10^{25} \text{ kg}}{2.45 \times 10^{22} \text{ m}^3} = 1630 \text{ kg/m}^3,$$
which is 1.63 as dense as liquid water. From the semimajor axis, this planet would be in the asteroid belt between the orbits of Mars and Jupiter and have the same characteristics as Jupiter's moon Callisto since their densities are similar — *i.e.*, a lot of ice mixed in with rock.

5. **Problem 7–6:**
Calculate the escape speed from the surface of the new planet described in the problem above. If the average speed of hydrogen molecules in this planet's atmosphere is 4.5 km/s, would you expect to find hydrogen in the atmosphere of the planet? Explain.

The escape velocity would be
$$\begin{aligned} v_{esc} &= \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2 \times 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2 \times 4.0 \times 10^{25} \text{ kg}}{1.25 \times 10^7 \text{ m}}} \\ &= \sqrt{\frac{5.34 \times 10^{15} \text{ Nm}^2/\text{kg}}{1.25 \times 10^7 \text{ m}}} \\ &= \sqrt{4.27 \times 10^8 \text{ m}^2/\text{s}^2} \\ &= 2.07 \times 10^4 \text{ m/s } = 20.7 \text{ km/s}. \end{aligned}$$
Since $v_H = 4.5$ km/s $< v_{esc}$, this planet should be able to hold onto its hydrogen.

6. **Review Question 12–5:**
Summarize the modern view of star formation, with special emphasis on the formation of the protosolar disk.

An interstellar gas cloud will collapse when its weight (*i.e.*, gravitational force) exceeds its internal pressure as determined from the density and temperature of the gas. As a large cloud contracts, it becomes unstable and fragments into many dense cloudlets. Each little cloudlet continues to collapse. As the large cloud was collapsing, there were many internal eddies and turbulent motions. Each little cloudlet has a rotation associated with it that was induced from one of these eddies. As the cloudlet contracts, it spins faster due to the conservation of angular momentum. This increased spin causes the equatorial region to bulge outward which flattens the cloudlet. This continues until a central bulge with an equatorial disk forms. As this contraction continues, the temperature and the pressure at the center of the cloud rises. There comes a time when this center gets so hot, it starts emitting visible photons. The surrounding gas and dust in the cloudlet (*i.e.*, the cocoon) absorbs the visible light and re–emits it as infrared light. The contracting cloudlet now is called a **protostar**. In the surrounding disk, dust grains begin to stick together from condensation and accretion, building in size to form planetesimals. These planetesimals conglomerate further into protoplanets. The visible light now reaches the surface of the protostar (still being power by gravitational contraction) and the pressure from this light starts to push out the unused material in the planetary disk. This spring cleaning phase is called the **T–Tauri stage** of the star. At the center, the temperature and pressure build so high that nuclear reactions start $\Longrightarrow$ **A STAR IS BORN**.

7. **Review Question 12–8:**
Describe the process whereby planets form within the protosolar disk.

Dust and ice begin to conglomerate together in the disk in a process known as **advection** (similar to building a snowman), building bigger and bigger particles. This process continues until boulder to mountain sized objects exist $\Longrightarrow$ the **planetesimals**. Planetesimals smash into each other from time to time, sometimes destroying each other, sometimes sticking together to form even bigger planetesimals $\Longrightarrow$ **accretion**. The outer planets formed first since the temperature were low enough for condensation to occur there first. Four giant planets formed with enough mass to gravitationally attract much of the H and He gas in the vicinity. The inner planets than began to form after the formation of the Jovian planets. However, they were too close to the Sun, hence too hot, and not massive enough to hang onto the H and He gas, which was later lost when the Sun went through its T–Tauri stage.

8. **Problem 12–1:**
Calculate the orbital period for a planet orbiting at a distance of 1 AU from a star having a mass equal to one-half the mass of the Sun; a two solar-mass star; a 10 solar-mass star; and a 50 solar-mass star.

$a = 1$ AU, $M_1 = 0.5\ M_\odot$, $M_2 = 2\ M_\odot$, $M_3 = 10\ M_\odot$, and $M_4 = 50\ M_\odot$. We use Kepler's 3rd law modified by Newton (and noting that the planet's mass is much less than the star's mass):

$$P = \sqrt{\frac{4\pi^2}{GM}}\ a^{3/2}.$$

Note that we can set up a ratio with respect to the Earth/Sun values:

$$P_{yrs} = \sqrt{\frac{M_\odot}{M}}\ a_{AU}^{3/2}.$$

Note that for each of our stellar systems above $a_{AU}^{3/2} = 1^{3/2} = 1$, so

$$\begin{aligned} P_1 &= \sqrt{\frac{M_\odot}{0.5 M_\odot}}\ \text{yrs}\ = 1.4\ \text{yrs}\ = 516\ \text{days} \\ P_2 &= \sqrt{\frac{M_\odot}{2 M_\odot}}\ \text{yrs}\ = 0.71\ \text{yrs}\ = 258\ \text{days} \end{aligned}$$

$$P_3 = \sqrt{\frac{M_\odot}{10M_\odot}} \text{ yrs } = 0.32 \text{ yrs } = 115 \text{ days}$$

$$P_4 = \sqrt{\frac{M_\odot}{50M_\odot}} \text{ yrs } = 0.14 \text{ yrs } = 52 \text{ days}$$

9. **Problem 12–4:**
Suppose a protostar, buried inside a dense interstellar cloud, has a temperature of 1,000 K. Calculate the wavelength at which it radiates most intensely. What kind of telescope is needed to observe the protostar most efficiently.

$T = 1000$ K, using Wien's Law gives

$$\lambda_{max} = \frac{0.0029 \text{ m K}}{T} = \frac{2.9 \times 10^{-3} \text{ m K}}{10^3 \text{ K}} = 2.9 \times 10^{-6} \text{ m } = 29,000 \text{ Å}.$$

This wavelength is in the infrared, hence you would need an infrared telescope, preferably in space.

10. **Review Question 8–2:**
How do we know anything about the Earth's interior? Do you think the same methods that are used to study the Earth can be applied to other planets?

Studying earthquake waves as they propagate through the planet is used to analyze the Earth's interior $\Longrightarrow$ **seismology**. Sound waves propagate at different speeds through different materials. Besides the Earth, the Moon and Mars have had seismographs left on their surfaces. Unfortunately, these bodies are not active, hence do not produce quakes. The only way to produce a quake is to wait for the occasional meteoroid impact on the planet surface. This would be true for the other terrestrial planets as well if we would plant seismographs there. Since the Jovian planets are gas giants with no solid surface, we couldn't use this technique for these planets.

11. **Review Question 8–3:**
Explain the difference between transverse and compressional waves. Can you think of everyday examples of each kind? Explain how we know that the Earth's core has a liquid component from studying how these waves travel through the Earth's interior.

A **transverse wave** is like a water wave with crests and troughs in a displacement of some material. A **longitudinal** or **compressional wave** is like a sound wave with compressions and rarefactions in a material. Changes in the frequency and velocity of the seismic longitudinal wave show that the Earth's outer core is liquid.

12. **Review Question 8–4:**
If the density of typical surface rocks is 3.5 g/cm^3, and the Earth's average density is 5.5 g/cm^3, what does this tell us about the density in the deep interior? How did this situation arise?

The deep interior must have high density material such as metals. Since Fe (iron) and Ni (nickel) are the most abundant metals in the Universe, the core is composed of these metals (which has been confirmed with seismic waves). When the Earth was completely molten in its early stages of formation, heavier elements (like Fe and Ni) sunk towards the center of the planet whereas lighter elements floated towards the top. This process is known as **differentiation**.

13. **Review Question 8–6:**
Summarize the effects of life-forms on the evolution of the Earth's atmosphere.

Plant life converted much of the remaining CO_2 (*i.e.*, that CO_2 not filtered by the liquid water oceans) in the early Earth's atmosphere into O_2. This has continued until the present day, where O_2 is the second most abundant gas in the Earth's atmosphere.

14. **Review Question 8–9:**
Why does the Earth not have as many craters as the Moon?

The Earth's surface is active, meaning that plate tectonics, erosion from wind and water, and weathering from rainfall have erased the early scars of the Earth's formation.

15. **Problem 8–5:**
If North America is approaching Japan at a rate of 3 cm/yr, and the present distance between North America and Japan is 5,000 km, how long will it take for the two to collide?

Velocity, v, is related to distance traveled, d, and the time it takes to travel that distance, t, by $v = d/t$. Since we want to solve for the time it will take:

$$t = \frac{d}{v} = \frac{5000 \text{ km } \times 10^5 \text{ cm/km}}{3 \text{ cm/yr}} = \frac{5.0 \times 10^8 \text{ cm}}{3 \text{ cm/yr}} = 1.67 \times 10^8 \text{ yrs},$$

or 167 million years!

16. **Problem 8–6:**
Calculate the escape velocity from the surface of the Moon. Comment on the relevance of your answer to the absence of any significant atmosphere on the Moon.

The escape velocity would be

$$\begin{aligned} v_{esc} &= \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2 \times 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2 \times 7.35 \times 10^{22} \text{ kg}}{3.476 \times 10^6 \text{ m}}} \\ &= \sqrt{\frac{9.80 \times 10^{12} \text{ Nm}^2/\text{kg}}{3.476 \times 10^6 \text{ m}}} \\ &= \sqrt{2.82 \times 10^6 \text{ m}^2/\text{s}^2} \\ &= 1.68 \times 10^3 \text{ m/s } = 1.68 \text{ km/s}. \end{aligned}$$

This is a very low velocity and for the Moon's proximity to the Sun, even the heaviest gas particles would move too fast as compared to this escape velocity. As such, the Moon can not hang on to an atmosphere for any length of time.