Astronomy 1010: Astronomy I
Homework 4 Solutions
Solution Set for Universe: Origins & Evolution
by Snow & Brownsburger

This is the solution set for problems assigned in ASTR-1010: *Astronomy I*. This set is from problems assigned from the current textbook in the class, *Universe: Origins and Evolution* by Snow and Brownsburger.

1. **Review Question 9-3:**
   Discuss the general structure of the surfaces of Mercury, Venus, and Mars, explaining the similarities and differences among them. In your explanation, mention the role played by tectonic activity and volcanism.

   Similarities between the surfaces of Mercury, Venus, and Mars:

   - All have impact craters, except Venus and Mars have quite a bit fewer than Mercury. This results primarily from the fact that both Venus and Mars have atmospheres and weather and Mercury does not, and that Venus' and Mars' surfaces have been active since their formation, whereas Mercury has not.
   - All have mountains and valleys.
   - None have liquid water on their surfaces.
   - Currently, none of these planets show signs of plate tectonics.

   Differences between these planetary surfaces include:

   - Mars shows evidence that liquid water once flowed on its surface, Venus and Mercury do not show any evidence for this.
   - Mercury's surface is extremely old (4.6 billion years) based upon the number of craters seen. Also, based on crater numbers, Mars' surface is old (about 3 billion years), but younger than Mercury's. Finally, Venus' surface is not quite as old as Mars' or Mercury's surface (approximately 0.5 billion years).
   - Mars and Venus both have an atmosphere and Mercury does not.

2. **Review Question 9-6:**
   Would you expect Mars, Venus, and Mercury to have charged particle belts surrounding them similar to the Earth's Van Allen belts? Explain.

   For Mercury and Venus, <u>no</u>, since they rotate so slowly (59 and 243 days, respectively) and terrestrial planetary magnetic fields are generated by the dynamo effect which requires a hot molten Ni-Fe core and a rapid rotation. Mars' rotation rate is similar to that of Earth, but Mars being so much smaller probably does not have a hot core any more, as such, Mars should have a weak (or no) magnetic field, which is indeed what is observed.

3. **Review Question 9-8:**
   Why is it thought that life might once have existed on Mars? Would you expect that the same thing might have happened on Venus? Explain.

   The main ingredient required for life to form is a supply of liquid water. Mars in its early history once had flowing liquid water, and as such, might have given rise to primitive life. Venus, if it once had liquid water, such water would have evaporated and dissociated by solar UV radiation very quickly. Hence, Venus would not have had sufficient time for life to get going before the water disappeared.

4. **Review Question 10-3:**
   Explain how astronomers deduce the internal properties of the gaseous giant planets.

   The internal structure of the Jovian planets is determined by mathematical models based on the laws of physics. These models are then compared to the observed surfaces and the magnetic field strengths to support their validity. The current models do well in matching the observations.

5. **Review Question 10-4:**
   Explain why the giant planets all have belts of charged particles that are very intense compared with the Earth's Van Allen belts.

   The dynamo effect in the giant planets is much more efficient than Earth's since (a) they rotate much faster than the Earth, and (b) they all contain metallic liquid hydrogen which is a much better conductor of electricity than liquid Ni-Fe.

6. **Review Question 10-6:**
   Explain the Roche limit and its role in the formation of ring systems.

   The Roche limit is a limiting distance from a planet (or anything massive) where anything of large size (*i.e.*, a moon) cannot form due to the extreme tidal forces on it. Within this limiting distance, the tidal force would simply rip anything of large size apart.

7. **Review Question 10-10:**
   Explain how the discovery of Charon has helped astronomers learn about the properties of Pluto.

   Masses of astronomical objects are deduced from the separation and period as one body orbits another body via Kepler's 3rd law of planetary motion. One cannot determine the mass of a planet accurately as it orbits the Sun due to the fact that the Sun is so much larger than a planet. With the discovery of Pluto's moon Charon, we were able to deduce the mass of both through Kepler's 3rd law and the relative geometry of their orbits about each other. Periodically, Charon eclipses Pluto which tells us the actual sizes of each. From the mass and diameters, we can calculate the average density of each, and from this information we know that both are very small (as compared to the other planets) and composed primarily of ices.

8. **Problem 10-4:**
   Calculate the position (distance from the center of Saturn) of the Cassini division, assuming it is the location where the ring particles would have exactly half of the orbital period of Mimas.

(*Hint:* Use Kepler's third law.)

First, the mass of Mimas is much less than the mass of Saturn so we can ignore it in our calculations using Kepler's 3rd law:

$$P^2 = \frac{4\pi^2}{GM}\, a^3,$$

where P is the orbital period, M is the mass of Saturn, G is Newton's Universal Gravitational Constant, and a is the distance that the object is from the planet it is orbiting (*i.e.*, the semi-major axis). We can set up a ratio of the periods of a particle in Cassini's Division, P_C, to that of Mimas, P_M, and solve for the ratio of the semi-major axes:

$$\frac{GM/4\pi}{GM/4\pi} \left(\frac{a_C}{a_M}\right)^3 = \left(\frac{P_C}{P_M}\right)^2.$$

The $GM/4\pi$ terms cancel and set $P_C = 0.5 P_M$, then

$$\begin{aligned}
\frac{a_C}{a_M} &= \left(\frac{P_C}{P_M}\right)^{2/3} \\
\frac{a_C}{a_M} &= \left(\frac{0.5P_M}{P_M}\right)^{2/3} \\
\frac{a_C}{a_M} &= (0.5)^{2/3} = (0.25)^{1/3} = \sqrt[3]{0.25} = 0.63,
\end{aligned}$$

So $a_C = 0.63\, a_M$, since $a_M = 1.855 \times 10^5$ km, $a_C = 0.63 \times 1.855 \times 10^5$ km $= 1.17 \times 10^5$ km. Cassini's Division is 117 thousand kilometers from Saturn's center.

9. **Review Question 11-1:**
How is the asteroid belt similar to the rings of Saturn?

Saturn's rings are composed of small particles as compared to the size of Saturn's moons and the asteroids are composed of small particles as compared to the size of the planets. Gaps exist in Saturn's rings due to various resonances with Saturn's moons and the **Kirkwood gaps** exist in the asteroid belt due to resonances with Jupiter.

10. **Review Question 11-4:**
Summarize the role of Jupiter in influencing asteroids, meteorites, and comets.

Jupiter prevented a small planet from forming in the asteroid belt due to its intense gravitational field and also produces the Kirkwood gaps. The interaction between Jupiter and long-period comets have kept the inner solar system relatively *clean* of comets reducing the amount of cometary impacts with the terrestrial planets. It has also caused a small number of these long-period comets to become short-period comets. The influence of Jupiter's gravitational field on the asteroid belt has caused many interactions of asteroids among themselves. One asteroid colliding with another asteroid causes small meteoroids to form which over time may intersect the Earth's orbit. Should such a meteoroid survive passage through the Earth's atmosphere, it becomes a meteorite.

11. **Review Question 11-6:**
What forces control the direction of a comet's tail? Why do comets tend to have two tails that are not aligned with each other?

A comet's tail is composed of both dust and gas. Sunlight (*i.e.*, photons) impart their momentum on the dust particles which pushes them away from the nucleus in the direction opposite the Sun. The solar wind (*i.e.*, a stream of charged particles emitted from the Sun's corona) ionizes the gas in the tail and these charged particles are pushed away from the nucleus by the same wind that ionizes them. Since the momentum exchange between the dust tail and solar photons is not 100% efficient, the dust tail does not point straight out away from the Sun, but instead curves away. The ion tail does point straight away from the Sun causing two tails to form.

12. **Problem 11-4:**
A comet has an orbital eccentricity of $e = 0.998$. Its perihelion distance (closest approach to the Sun) is 0.240 AU. Find the semimajor axis of its orbit, its aphelion distance, and its orbital period.

We have $e = 0.998$ and $r_p = 0.24$ AU. It's semi-major axis, a, can be found from

$$\begin{aligned} r_p &= a(1-e) \\ a &= r_p/(1-e) = 0.240 \text{ AU}/(1-0.998) \\ &= 0.240 \text{ AU}/0.002 = 120 \text{ AU}. \end{aligned}$$

Its aphelion distance, r_a, can be found from

$$\begin{aligned} r_p + r_a &= 2a \\ r_a &= 2a - r_p = 2(120 \text{ AU}) - 0.240 \text{ AU} = 23\underline{9}.76 \text{ AU} = 240 \text{ AU}. \end{aligned}$$

Its period can be found from Kepler's 3rd Law:

$$\begin{aligned} \frac{P}{P_\oplus} &= \left(\frac{a}{a_\oplus}\right)^{3/2} \\ &= \left(\frac{120 \text{ AU}}{1 \text{ AU}}\right)^{3/2} = (120)^{3/2} \\ &= \sqrt{1.728 \times 10^6} = 1310 \\ P &= 1310\ P_\oplus = 1310 \text{ yrs}. \end{aligned}$$