ASTR-1020: Astronomy II
Course Lecture Notes
Section III

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Abstract

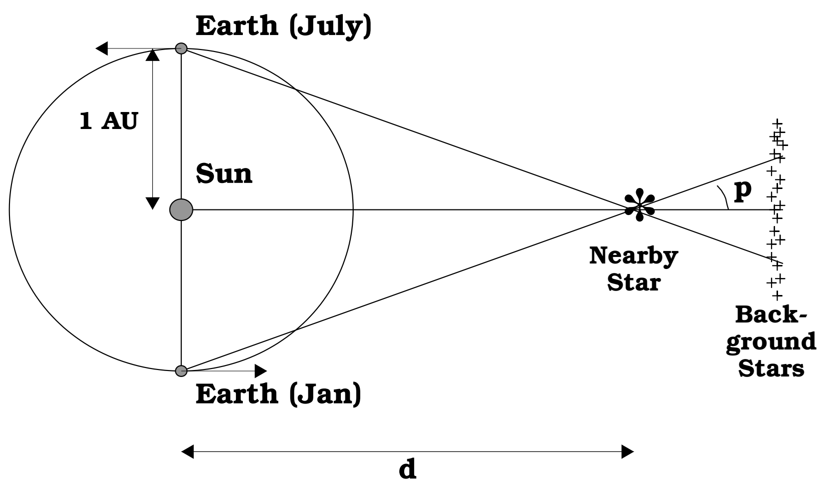
These class notes are designed for use of the instructor and students of the course ASTR-1020: Astronomy II at East Tennessee State University.

III. Stellar Properties and the HR Diagram

A. Foundation of the Distance Scale

1. Stars that are relatively nearby to the Sun (*i.e.*, within approximately 1500 light years) have their distances determined via **trigonometric parallax**.

   a) Use the Earth's orbit as a measuring stick:



   b) The distance to a star, d, (in parsecs) is equal to the reciprocal of the parallax angle, p (in arcseconds):

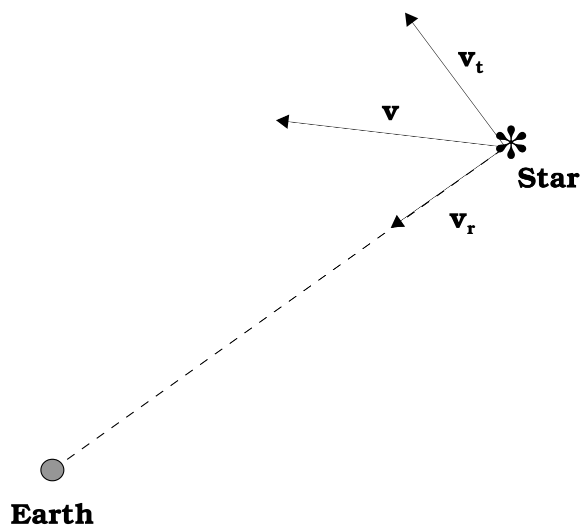
$$d = \frac{1}{p}. \qquad \text{(III-1)}$$

      i) A star that has a measured parallax angle of $p =$ 1 arcsec would be at a distance of 1 parsec (note 1 pc = 3.26 ly).

      ii) The nearest stellar system to the Sun is the "triple-star" α Centauri system (α Cen A, α Cen B, and Proxima Cen) which has a parallax angle of 0.76 arcsec $\Longrightarrow$ 1.32 pc = 4.3 ly.

c) Prior to the *Hipparcos* satellite mission by ESA, we could only measure parallaxes out to 50 parsecs accurately from the ground. Hipparcos has now measured stellar parallaxes out to 500 pc!

2. We can determine a star's **space velocity** by measuring its **radial velocity** (line-of-sight velocity) and its **tangential velocity** (velocity in the plane of the sky).



a) The radial velocity, v_r, is determined from the **Doppler Effect**:

$$\frac{\lambda - \lambda_\circ}{\lambda_\circ} = \frac{\Delta\lambda}{\lambda_\circ} = \frac{v_r}{c}. \qquad \text{(III-2)}$$

i) λ = observed wavelength of a spectral line.

ii) $\lambda_\circ$ = rest (or laboratory) wavelength of a spectral line.

iii) $\Delta\lambda$ = change in wavelength of the line (redward [+] shift = object is moving away, blueward [–] shift = object is moving towards us).

iv) c = speed of light = 3.00×10^5 km/s.

Example III–1. What would the radial velocity of a star be if the Na I line at 5895 Å is shifted to 5892 Å?

$$\begin{aligned} v_r &= \frac{\Delta\lambda}{\lambda_\circ}\, c \\ &= \frac{5892 \text{ Å} - 5895 \text{ Å}}{5895 \text{ Å}}\, (3.00 \times 10^5 \text{ km/s}) \\ &= \frac{-3 \text{ Å}}{5895 \text{ Å}}\, (3.00 \times 10^5 \text{ km/s}) \\ &= (-5.089 \times 10^{-4})(3.00 \times 10^5 \text{ km/s}) \\ &= -153 \text{ km/s} \qquad \text{(hence moving towards us)} \end{aligned}$$

b) The tangential velocity, v_t, is determined from the distance, d, to the star and the **proper motion**, μ, (small change of position on sky over time) of the star:

$$v_t = 4.74\, \mu\, d, \qquad \text{(III-3)}$$

where d is in parsecs, μ is in arcsec/yr, and v_t is in km/s.

Example III–2. Assume the star in the example above has a parallax of 0.025 arcsec and a proper motion of 0.124 arcsec/yr. What is the tangential velocity of the star?

$$d = \frac{1}{p} = \frac{1}{0.025} = 40 \text{ pc}$$

$$\begin{aligned} v_t &= 4.74\, \mu\, d \\ &= 4.74\, (0.124)\, (40) \text{ km/s} \\ &= 23.5 \text{ km/s} \end{aligned}$$

c) The space velocity is the *vector sum* of these 2 velocities:

$$v = \sqrt{v_r^2 + v_t^2}. \qquad \text{(III-4)}$$

Example III–3. What is the space velocity of the star in the examples above?

$$\begin{aligned} v &= \sqrt{v_r^2 + v_t^2} \\ &= \sqrt{(-153 \text{ km/s})^2 + (23.5 \text{ km/s})^2} \\ &= \sqrt{(23,409 \ (\text{km/s})^2) + (552 \ (\text{km/s})^2)} \\ &= \sqrt{23,961 \ (\text{km/s})^2} \\ &= 155 \text{ km/s} \end{aligned}$$

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3. We can use the motion of nearby star clusters to deduce the distance to these clusters $\Longrightarrow$ **moving cluster method**.

a) The tangential velocities for the group can be deduced from the radial velocities of each star and the *movement angles* each proper motion makes on the sky.

b) The distance is then determined from the v_t equation above.

c) The **Hyades** star cluster's distance has been determined in this manner. The distance to the Hyades is used as a base for calibration of more distance indicators (more to come).

B. Stellar Magnitudes

1. Hipparchus, an ancient Greek astronomer, classified stars by their brightness.

a) The brightest = *1st magnitude*.

b) The faintest he could see = *6th magnitude*.

2. In the 19*th* century, astronomers realized that 1st magnitude stars were about 100 times brighter than the 6th magnitude stars $\Longrightarrow$ each magnitude is 2.512 times fainter than the next lowest magnitude.

a) The observed *energy flux* f (energy per sec per unit surface area per wavelength band: W/m^2/Å) of a star is related to the observed *magnitude* m by the relation

$$m_2 - m_1 = -2.5 \log \left(\frac{f_2}{f_1} \right) , \qquad \text{(III-5)}$$

where "1" represents star #1 and "2" represents star #2.

b) Note that the magnitude system is logarithmic since the human eye is a logarithmic detector.

c) **Apparent Magnitude (m):** the brightness of a star *as seen from Earth* ($m_\odot = -26.78$) $\Longrightarrow$ note that $\odot$ is the astronomical symbol for the Sun.

d) **Absolute Magnitude (M):** the brightness as seen at a distance of 10 parsecs ($M_\odot = +4.82$).

Example III–4. How much brighter is a 3.51 magnitude star with respect to a 5.67 magnitude star?

$$m_2 - m_1 = -2.5 \log \left(\frac{f_2}{f_1} \right)$$

$$\begin{aligned}
\log\left(\frac{f_2}{f_1}\right) &= -0.4 \times (m_2 - m_1) \\
\frac{f_2}{f_1} &= 10^{-0.4\times(m_2-m_1)} \\
&= 10^{-0.4\times(3.51-5.67)} \\
&= 10^{0.864} \\
&= 7.31 \\
f_2 &= 7.31\, f_1
\end{aligned}$$

The 3.51 magnitude star (*i.e.*, star #2) is 7.31 times brighter than the 5.67 magnitude star (*i.e.*, star #1).

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3. The **absolute bolometric magnitude** $M_{\rm bol}$ of a star is the magnitude as measure across the entire spectrum (*e.g.*, all wavelengths) of the star. It is related to the star's **luminosity** L — energy output per second over the entire stellar surface ($M_{\rm bol}(\odot) = 4.75$ and $L_\odot = 3.90 \times 10^{26}$ watts):

$$M_{\rm bol} - M_{\rm bol}(\odot) = -2.5 \log\left(\frac{L}{L_\odot}\right). \qquad \text{(III-6)}$$

Example III–5. Rigel is a very luminous star with $L = 72,000\, L_\odot$. What is the absolute bolometric magnitude of Rigel?

$$\begin{aligned}
M_{\rm bol} - M_{\rm bol}(\odot) &= -2.5 \log\left(\frac{L}{L_\odot}\right) \\
&= -2.5 \log\left(\frac{7.2 \times 10^4 L_\odot}{L_\odot}\right) \\
&= -2.5 \log\left(7.2 \times 10^4\right) \\
&= -2.5(4.86) = -12.15 \\
M_{\rm bol} &= M_{\rm bol}(\odot) - 12.15 = 4.75 - 12.15 \\
&= -7.40
\end{aligned}$$

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4. By convention, when magnitudes are listed without the *bolometric* extension, it is assumed that the magnitude refers to the visual band (see below).

5. The distance of a star can be determined from m and M via the **distance modulus** formula:

$$m - M = 5 \log d - 5 \tag{III-7}$$

or

$$d = 10^{(m-M+5)/5}, \tag{III-8}$$

with d measured in parsecs.

Example III–6. For Sirius, $m = -1.46$ and $M = 1.42$, how far away is it?

$$\begin{aligned} d &= 10^{(m-M+5)/5} \\ &= 10^{(-1.46-1.42+5)/5} = 10^{2.12/5} \\ &= 10^{0.424} = 2.65 \end{aligned}$$

or 2.65 pc (= 8.65 ly).

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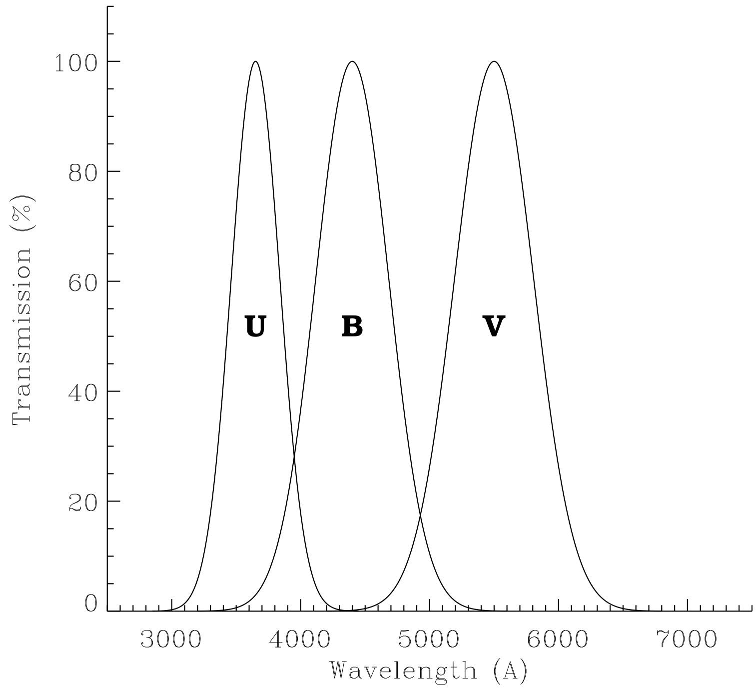
6. Typically stellar magnitudes are measured through various filters: U = ultraviolet, B = blue, V = visual (V matches response of human eye).



a) **Color index**: subtracting one filter magnitude from another: $(U - B), (B - V)$.

b) The color index or *color* of a star gives us info on the star's temperature.

C. The Hertzsprung-Russell (HR) Diagram

1. Besides stellar colors, the strength of certain spectral lines can give us information about a star's temperature.

   a) Star were first classified based on hydrogen-line strength $\Longrightarrow$ strongest in A stars, then B, etc. $\Longrightarrow$ **spectral class**.

   b) Along the way, various spectral class letters were dropped.

   c) In the 1920s, the temperatures of the various classes were discovered:

	O	**B**	**A**	**F**	**G**	**K**	**M**
hottest							coolest
bluest							reddest

d) Each class is further subdivided into 10 subclasses (..., B8, B9, A0, A1, A2, ...).

e) The Sun's spectral type is G2 ($T_{\rm eff} \approx 5800$ K).

2. The width of spectral lines can tell us the size, hence luminosity, of a star.

a) Thick lines: smaller stars, higher surface gravity.

b) Thin lines: larger stars, lower surface gravity.

3. Besides spectral (temperature) classes, astronomers also have invented **luminosity classes**.

Luminosity Class	**Type of Star**
Ia	Bright Supergiant
Ib	Supergiant
II	Bright Giant
III	Giant
IV	Subgiant
V	Main Sequence (dwarfs)
VI	Subdwarfs
(VII)	White dwarfs

The Sun's luminosity class is V (= main sequence star).

4. Once we know a star's temperature and luminosity, we can determine its radius as described in §I.G.3.e of the notes.

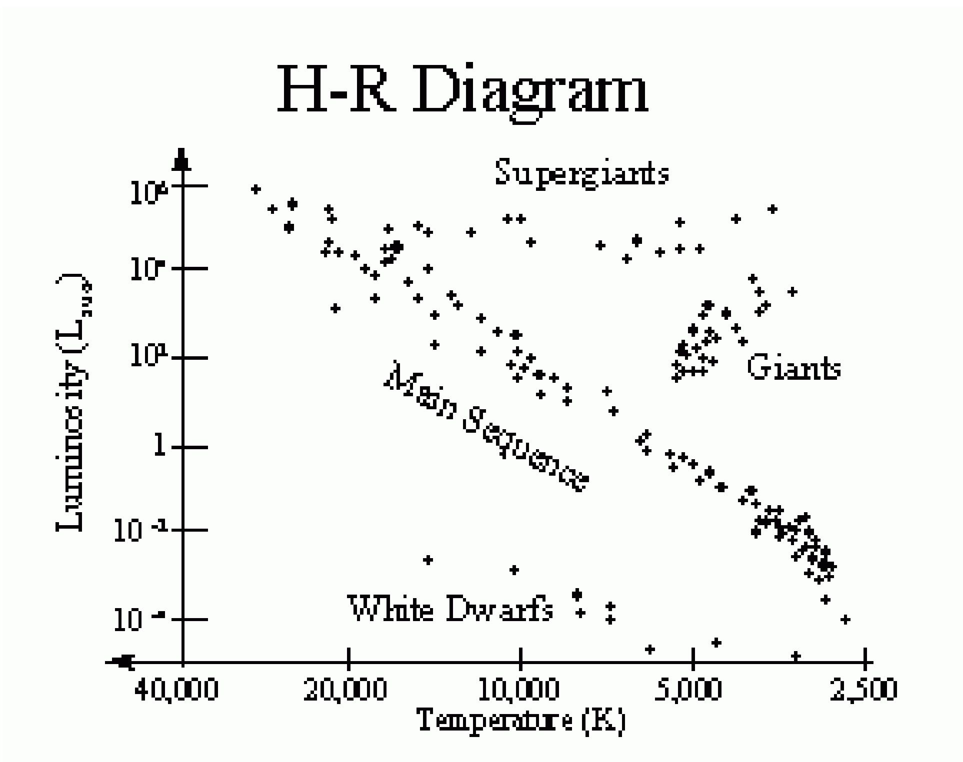
5. Earlier in the 20*th* century, 2 astronomers, Hertzsprung and Russell, discovered a curious pattern when stars are plotted by their brightness versus their colors or spectral types — 4 main patterns appear:

a) **Main sequence** stars (over 90% of the stars seen fall into this class).

b) **Giant** stars (mostly red in color).

c) **Supergiant** stars (very bright).

d) **White Dwarf** stars (very faint).



6. The HR Diagram is one of the most powerful tools ever developed by astronomers $\Longrightarrow$ used for distances, abundances, and stellar evolution studies.

a) *Observational* HR Diagrams plot absolute magnitudes (M), or apparent magnitudes (m or V) for star clusters, versus spectral types or colors ($B - V$).

b) *Theoretical* HR Diagrams plot luminosity ($L/L_\odot$) versus temperature (T) of the star.

7. Once stars are recorded on an *observational* (*e.g.*, V vs. $(B-V)$) HR Diagram, we can use the known *absolute* magnitude for the given spectral-luminosity class of the star to deduce its distance using the distance modulus formula $\Longrightarrow$ this is known as the **spectroscopic parallax** technique.

8. Later, we shall see that the HR Diagram has its peculiar shape due to the evolutionary processes of the stars.

D. Stellar Spectra

1. Three properties of stars are responsible for the appearance of a stellar spectrum:

   a) **Temperature**: Shifts $\lambda_{\rm max} \Longrightarrow$ hot stars are bluish, cool stars are reddish. Affects the ionization and excitation of atoms in the stellar atmosphere $\Longrightarrow$ hot stars have high-ionization lines, cool stars have neutral metal and molecular lines.

   b) **Luminosity**: For a given temperature, the more luminous a star the bigger the radius of the star. The bigger the radius, the lower the pressure in the stellar atmosphere $\Longrightarrow$ For a given temperature, big stars have narrow lines and small stars have thicker lines — **pressure broadening**.

   c) **Chemical Composition**: The higher the chemical abundance of an element, the thicker the spectral line. Chemical abundances (of all elements other than hydrogen and helium) are referred to as the **metalicity** of the star.

      i) Low metalicity stars = low metal abundances (*i.e.*, Population II stars — see §VIII.B of the notes).

ii) High metalicity stars = high metal abundances (*i.e.*, Population I stars, like the Sun).

2. A0 stars ($T_{\rm eff} = 10,000$ K) have the strongest hydrogen Balmer lines. The Balmer lines arise out of the second energy level of hydrogen.

a) Hotter stars (O & B stars) are so hot that hydrogen becomes ionized $\Longrightarrow$ the Balmer lines decrease in strength as you get hotter.

b) Cooler stars (F-M stars) are not hot enough to excite a large number of electrons from the ground state (*i.e.*, first energy level) to the 2nd level $\Longrightarrow$ the Balmer lines decrease in strength as you get cooler.

3. M stars have a large amount of molecular absorption features called **molecular bands**. TiO is very strong in the visible portion of the spectrum.

a) C stars (a special class of cool stars) have a higher abundance of carbon (C/O > 1) with respect to the M stars (which have C/O < 1) $\Longrightarrow$ strong C_2 and CN bands are seen at visual wavelengths.

b) S stars (another special class of cool stars) have C/O $\approx$ 1 $\Longrightarrow$ strong LaO, ZrO, and YO bands are seen.

4. Close examination of the Sun's spectrum reveals millions of lines, a majority belonging to singly-ionized atoms (*e.g.*, Fe II, Ca II, Mg II) and neutral metal lines (*e.g.*, Fe I, Ca I, Mg I).

a) Ionization stages of an atom are labeled with roman numerals in astrophysics: "I" means a neutral species (*i.e.*, # of electrons = # of protons in nucleus).

b) "II" means singly ionized (*i.e.*, one electron taken away).

c) And so on, for instance, if Fe had all but one electron taken away, it would be labeled as Fe XXVI.

E. Binary Stars and Stellar Masses

1. Over half of the stars we see belong to **binary star** systems $\Longrightarrow$ 2 stars in orbit about each other.

   a) Some systems have more than 2 components $\Longrightarrow$ **multiple stellar systems**.

   b) If a binary system is close enough and their separation wide enough, we can see both components in a telescope $\Longrightarrow$ **visual binary**.

   c) Many times we cannot separate the 2 components, but we can deduce their binary nature due to 2 sets of spectral lines that shift back-and-forth with respect to each other due to the **Doppler Effect** as the 2 stars orbit each other $\Longrightarrow$ **spectroscopic binaries**.

   d) If the orbits of the 2 binary stars are edge-on as viewed from Earth, the stars will eclipse each other $\Longrightarrow$ the star periodically appears to change in brightness $\Longrightarrow$ **eclipsing binary** (also called **eclipsing variable star**).

2. For visual binaries, we typically can get accurate readings on the semimajor axis of the star's orbit (a in AU) and its orbital period (P in years). From this we can determine the sum of their masses (in $M_\odot$) of the 2 stars with Kepler's 3rd Law:

$$M_1 + M_2 = \frac{a^3}{P^2}. \qquad \text{(III-9)}$$

Example III–7. We measure the trigonometric parallax of a visual binary star as 0.20 arcsec and measure an angular separation between the pair of stars in this binary as 5 arcsec. Over a few years of observations, we determine the orbital period of this pair to be 30 years. What is the combined mass of these two stars? If one star has a spectrum identical to the Sun and the other an M0 star, what is the luminosity class of the M0 star based upon the masses measured?

First determine the actual semimajor axis. The star's distance is

$$d = \frac{1}{0.20}\ \mathrm{pc}\ = 5.0\ \mathrm{pc}.$$

Now, use the small angle formula to find a:

$$\begin{aligned} a = D &= \frac{\alpha d}{206,265} = \frac{5 \times 5.0\ \mathrm{pc}}{206,265} \\ a &= 1.212 \times 10^{-4}\ \mathrm{pc}\ \times 206,265\ \mathrm{AU/pc} \\ &= 25\ \mathrm{AU} \end{aligned}$$

The sum of the masses is

$$M_1 + M_2 = \frac{25^3}{30^2}\ \mathrm{M}_\odot = \frac{1.56 \times 10^4}{900}\ \mathrm{M}_\odot = 17.4\ \mathrm{M}_\odot.$$

Since one star has a spectrum identical to the Sun, it must be a 1.0 $\mathrm{M}_\odot$ star. Hence star #2 must have a mass of

$$M_2 = 17.4\ \mathrm{M}_\odot - M_1 = 17.4\ \mathrm{M}_\odot - 1.0\ \mathrm{M}_\odot = 16.4\ \mathrm{M}_\odot.$$

Since this was an M0 star, it cannot be a main sequence star since its mass would have to be less than one. Since it is so massive, chances are that it is a supergiant star like Betelgeuse. Actually, Betelgeuse has a mass of 16 $\mathrm{M}_\odot$!