ASTR-1020: Astronomy II
Course Lecture Notes
Section VIII

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Abstract

These class notes are designed for use of the instructor and students of the course ASTR-1020: Astronomy II at East Tennessee State University.

VIII. Stellar Corpses

A. White Dwarf Stars.

1. Stars will end up in one of 3 states: white dwarf, neutron star, or black hole. Stars that are initially 4 $M_\odot$ or less on the main sequence will wind up as a white dwarf.

2. As explained in the last section of these notes, as the carbon-oxygen core continues to collapse after He-core burning, the outer envelope of the star continues to expand away, helped along by strong stellar winds.

a) The outer envelope detaches itself from the collapsing core and becomes a shell of material surrounding the core.

b) When the shell be comes thin enough so that the hot core can be seen through the shell, UV photons emitted from the core fluoresce the expanding shell $\Longrightarrow$ a **planetary nebula** forms.

c) This shell will continue to expand, getting thinner and thinner as it gets larger until it begins to intermingle with the surrounding ISM.

d) Since the outer envelope of the star was enhanced with C, N, and O from the nuclear reactions due the shell burning phases, the ISM gets enhanced with these elements.

e) Future stars that will be born out of the ISM gas will thus have higher abundances of C, N, and O due to this previous epoch of stellar evolution.

3. The laws of **quantum mechanics** (QM) dictate the characteristics of electrons.

a) Electrons have a variety of **quantum states** associated with them:

i) *Principal* quantum number (n) $\Longrightarrow$ this identifies which electronic (main) *shell* an electron is in. The bigger the shell, the more energy an electron will have.

ii) *Orbital* angular momentum quantum number (ℓ) $\Longrightarrow$ this identifies the *subshell* of the main shell where the electron is located.

iii) *Spin* angular momentum quantum number (s) $\Longrightarrow s = +\frac{1}{2}$ for counterclockwise spin, $s = -\frac{1}{2}$ for clockwise spin.

iv) *Total* angular momentum quantum number ($j = \ell + s$).

b) One fundamental law in QM is the **Pauli Exclusion Principle** (PEP): no two electrons can exist in the same quantum state (*i.e.*, have the same quantum numbers) at the time in the same place.

4. As the stellar core continues to collapse, the free electrons in the core are forced so close together that they all try to sit in the same quantum state at the same location.

a) This cannot happen since it violates the PEP — the electrons resist further compression and become degenerate.

b) The core becomes stable due to **degenerate electron pressure**.

c) When this happens, the core is now called a **white dwarf** (WD).

d) White dwarfs range in mass from 0.6 $M_\odot$ to 1.4 $M_\odot$.

e) The radius of a WD scales as $R \propto 1/M^{1/3} \Longrightarrow$ the more massive a white dwarf, the smaller it is.

f) Most of the WDs are about the size of the Earth giving an enormous density for these objects ($\rho = 10^9$ kg/m^3) $\Longrightarrow$ 1 teaspoon of WD material on the surface of the Earth would weight 5.5 tons – as much as an elephant!

5. White dwarfs are in HSE since electron degeneracy pressure balances gravity. However, if the mass of a WD exceeds 1.4 $M_\odot$, this degeneracy pressure will be too weak to counterbalance gravity!

a) This mass limit is called the **Chandrasehkar limit**.

b) Due to this limit, all white dwarfs have $M < 1.4 M_\odot$.

c) If $M > 1.4 M_\odot$, the core will continue to collapse to the next type of stellar corpse — the neutron star.

6. The nearest white dwarfs to the solar system are Sirius B and Procyon B, each a companion of the brighter star we see in the night sky.

B. Neutron Stars.

1. Stars more massive than 8 $M_\odot$ on the main sequence will be able to successively burn heavier and heavier elements as their core collapses.

2. The final type of stellar burning to go on in a massive core is Si (silicon) burning. The ash from this burning is Fe (iron).

a) Fe is the most stable of all the chemical elements $\Longrightarrow$ you can neither get energy out of an Fe nucleus from either **nuclear fusion** (*i.e.*, bringing lighter elements together to form heavier ones) or **nuclear fission** (*i.e.*, the breaking apart of heavier elements into lighter ones).

b) Once an Fe core forms, its collapse cannot be halted by further nuclear reactions.

c) The Fe core become degenerate as it collapses but is too massive for the electron degeneracy pressure to hold up the weight of the star.

d) Since the Pauli Exclusion Principle dictates that electrons cannot exist in the same state, the only place for them to go is into the Fe nuclei!

3. The electrons interact with protons in an inverse β-decay:

$$e^- + p \longrightarrow n + \nu,$$

forming a stellar core of *pure neutrons*!

4. Once the neutrons are formed, they stop the collapse of the core via the strong nuclear force $\Longrightarrow$ neutron degeneracy pressure holds up the weight of the core $\Longrightarrow$ a **neutron star** (NS) is born!

a) This halting of the collapse of the core is rather sudden and causes the core to bounce and rebound a bit. This bounce sets up a shock wave which propagates outward and blows apart the outer envelope of the star in a Type II **supernova** explosion (see §VII.D).

b) A tremendous amount of energy is released during this explosion over a fraction of a second, increasing the luminosity of the star by a factor of 10^8!

c) However this luminosity increase only corresponds to 1% of the total energy released, most of the energy comes out in the form of neutrinos!

d) As we have seen in the last section, after the explosion, the outer envelope continues to expand, creating a **supernova remnant** which can last for a few million years (*e.g.*, the Crab nebula).

5. There is another type of lower-energy stellar explosion associate with neutron stars. These are the **X-ray bursters** which are similar to classical novae, except matter is dumped on a neutron star instead of a white dwarf (as is the case for a nova).

6. All stars rotate while on the main sequence. As the core of a star collapses, it spins faster and faster due to the conservation of angular momentum.

a) By the time a stellar core reaches NS size (*e.g.*, $R_{\rm NS}$ = 30 km, the size of a city), it is spinning hundreds of times a second!

b) **Pulsars** are rapidly spinning neutron stars whose magnetic axis is not aligned with its polar axis.

c) Pulsars flash very rapidly as a result due to the *lighthouse effect* of the beamed light coming from the magnetic axes sweeping in the line-of-sight towards Earth.

d) Pulsars were first discovered in radio waves in 1967 $\Longrightarrow$ first thought to be signals from extraterrestrial intelligence — they were initially called LGM's for *Little Green Men* as a result.

7. The small size and high mass ($1.4M_\odot < M_{\rm NS} < 3M_\odot$) for neutron stars give them an exceedingly large density $\Longrightarrow \rho = 10^{17}$ kg/m^3, 1 teaspoon of NS material on Earth would weight over 500 million tons!

8. Neutron stars also have a maximum mass, if the mass of a stellar remnant exceeds $3M_\odot$, not even neutron degeneracy can hold up the stellar core. It collapses further to a black hole!

C. Einstein's Theory of Relativity.

1. In the late 19th century, 2 physicists, Michelson and Morley set out to measure the Earth's motion through a then hypothesized **Ether** which was thought to fill the Universe.

a) The existence of the Ether was speculated on in order to explain how light was able to propagate across the Universe $\Longrightarrow$ water waves need water to propagate, sound needs air to propagate, hence light waves (so it was thought) must require the Ether to propagate.

b) The Michelson-Morley experiment did not detect any evidence that the Ether existed — either the Ether moves with the Earth or it did not exist!

c) Their experiment also showed that the speed of light remained the same no matter what direction they were looking in with respect to the Earth's motion.

d) The theory of electromagnetism shows us that light (*i.e.*, photons) can self-propagate without any need for a medium (*i.e.*, the Ether).

2. Einstein was intrigued by this experiment and used it to prove that there was no Ether through his development of the **theory of relativity**. The theory of relativity is separated into 2 parts. The **special theory** was developed in 1905 and is based upon 2 postulates:

a) The laws of physics are the same in all inertial (*i.e.*, non-accelerating) reference systems (or frames).

b) The speed of light in a vacuum is constant ($c = 3.00 \times 10^5$ km/s) and independent of the motion of the observer or the motion of the light source.

3. Einstein's special theory of relativity is essentially a theory of motion and the relationship between mass and energy (through $E = mc^2$). It superseded Newton's laws of motion (although the special theory reduces to the 3 laws of motion when $v \ll c$). This theory changes Newton's concept that time, length (*i.e.*, distance), and mass are constant. Instead it makes them a function of velocity through the **Lorentz transformations**:

a) **Time Dilation**: Time slows as one approaches the speed of light via

$$\Delta t_\circ = \frac{\Delta t}{\sqrt{1 - (v/c)^2}}, \qquad \text{(VIII-1)}$$

where $\Delta t_\circ$ is the elapsed time measured by a stationary observer, Δt is the time interval as measured by the observer in motion, and v is the velocity of the object.

b) **Length Contraction**: If you, the stationary observer, was to measure a ruler of length $L_\circ$ traveling at $v = 0.99c$, you would *measure* its length L following the **Fitzgerald-Lorentz contraction** equation:

$$L = L_\circ \sqrt{1 - (v/c)^2}. \qquad \text{(VIII-2)}$$

or $L = 1$ ft $\times\sqrt{0.0199} = 0.14$ ft $= 1.7$ in. This is not due to an optical illusion, the *ruler* has actually contracted by this amount.

c) **Mass Increase without Bound**: Mass also increases for the object that is traveling close to the speed of light! Einstein showed that

$$M = \frac{M_\circ}{\sqrt{1 - (v/c)^2}}. \qquad \text{(VIII-3)}$$

Note that as $v \rightarrow c$, that $M \rightarrow \infty$! One cannot accelerate mass to the speed of light since it would require an infinite amount of energy to push an infinite amount of mass.

4. These effects have been observed in the laboratory with subatomic particles!

Example VIII–1. The Starship Enterprise has a mass of 2.0×10^6 kg and a length of 500 m. What is the length and mass of the Enterprise as it accelerates from 1/2-sublight ($v = 0.5c$), 90%-sublight ($v = 0.9c$), 99%-sublight ($v = 0.99c$), 99.9%-sublight ($v = 0.999c$) as viewed by stationary observer on Starbase 12 as the Enterprise departs? For every day that passes on the Enterprise, how much time passes on the Starbase?

For the passage of time:

$$\begin{aligned}
\Delta t_{(0.5c)} &= \frac{1 \text{ day}}{\sqrt{1-(0.5c/c)^2}} = \frac{1 \text{ day}}{\sqrt{1-(0.5)^2}} \\
&= \frac{1 \text{ day}}{\sqrt{1-0.25}} = \frac{1 \text{ day}}{\sqrt{0.75}} = \frac{1 \text{ day}}{0.866} = 1.15 \text{ days}
\end{aligned}$$

$$\begin{aligned}
\Delta t_{(0.9c)} &= \frac{1 \text{ day}}{\sqrt{1-(0.9c/c)^2}} = \frac{1 \text{ day}}{\sqrt{1-0.81}} \\
&= \frac{1 \text{ day}}{\sqrt{0.19}} = \frac{1 \text{ day}}{0.436} = 2.29 \text{ days}
\end{aligned}$$

$$\begin{aligned}
\Delta t_{(0.99c)} &= \frac{1 \text{ day}}{\sqrt{1-(0.99c/c)^2}} = \frac{1 \text{ day}}{\sqrt{1-0.98}} \\
&= \frac{1 \text{ day}}{\sqrt{0.02}} = \frac{1 \text{ day}}{0.141} = 7.07 \text{ days}
\end{aligned}$$

$$\begin{aligned}
\Delta t_{(0.999c)} &= \frac{1 \text{ day}}{\sqrt{1-(0.999c/c)^2}} = \frac{1 \text{ day}}{\sqrt{1-0.998}} \\
&= \frac{1 \text{ day}}{\sqrt{0.002}} = \frac{1 \text{ day}}{0.0447} = 22.4 \text{ days}
\end{aligned}$$

For every day that passes on the Enterprise, the people on the Starbase age at 1.15, 2.29, 7.07, and 22.4 days as the Enterprise travels at velocities $0.5c$, $0.9c$, $0.99c$, and $0.999c$, respectively.

For the length contraction:

$$\begin{aligned}
L_{(0.5c)} &= (500 \text{ m})\sqrt{1-(0.5c/c)^2} = (500 \text{ m})\sqrt{1-(0.5)^2} \\
&= (500 \text{ m})\sqrt{1-0.25} = \sqrt{0.75}(500 \text{ m}) \\
&= 0.866 \times 500 \text{ m} = 433 \text{ m}
\end{aligned}$$

$$\begin{aligned}
L_{(0.9c)} &= (500 \text{ m})\sqrt{1-(0.9)^2} = (500 \text{ m})\sqrt{1-0.81} \\
&= \sqrt{0.19}(500 \text{ m}) = 0.436 \times 500 \text{ m} = 218 \text{ m}
\end{aligned}$$

$$\begin{aligned}
L_{(0.99c)} &= (500 \text{ m})\sqrt{1-(0.99)^2} = (500 \text{ m})\sqrt{1-0.98} \\
&= \sqrt{0.02}(500 \text{ m}) = 0.141 \times 500 \text{ m} = 70.7 \text{ m}
\end{aligned}$$

$$\begin{aligned} L_{(0.999c)} &= (500 \text{ m}) \sqrt{1 - (0.999)^2} = (500 \text{ m}) \sqrt{1 - 0.998} \\ &= \sqrt{0.002}(500 \text{ m}) = 0.0447 \times 500 \text{ m} = 22.4 \text{ m} \end{aligned}$$

The Enterprise's length would be 433 m, 218 m, 70.7 m, and 22.4 m as measured from the Starbase as it traveled at $0.5c$, $0.9c$, $0.99c$, and $0.999c$, respectively.

Finally, the mass of the Enterprise would increase to:

$$\begin{aligned} M_{(0.5c)} &= \frac{2.0 \times 10^6 \text{ kg}}{\sqrt{1 - (0.5c/c)^2}} = \frac{2.0 \times 10^6 \text{ kg}}{\sqrt{1 - (0.5)^2}} \\ &= \frac{2.0 \times 10^6 \text{ kg}}{\sqrt{1 - 0.25}} = \frac{2.0 \times 10^6 \text{ kg}}{\sqrt{0.75}} = \frac{2.0 \times 10^6 \text{ kg}}{0.866} \\ &= 1.15 \times 2.0 \times 10^6 \text{ kg} = 2.3 \times 10^6 \text{ kg} \end{aligned}$$

$$\begin{aligned} M_{(0.9c)} &= \frac{2.0 \times 10^6 \text{ kg}}{\sqrt{1 - (0.9)^2}} = \frac{2.0 \times 10^6 \text{ kg}}{\sqrt{1 - 0.81}} \\ &= \frac{2.0 \times 10^6 \text{ kg}}{\sqrt{0.19}} = \frac{2.0 \times 10^6 \text{ kg}}{0.436} = 4.6 \times 10^6 \text{ kg} \end{aligned}$$

$$\begin{aligned} M_{(0.99c)} &= \frac{2.0 \times 10^6 \text{ kg}}{\sqrt{1 - (0.99)^2}} = \frac{2.0 \times 10^6 \text{ kg}}{\sqrt{1 - 0.98}} \\ &= \frac{2.0 \times 10^6 \text{ kg}}{\sqrt{0.02}} = \frac{2.0 \times 10^6 \text{ kg}}{0.141} = 1.4 \times 10^7 \text{ kg} \end{aligned}$$

$$\begin{aligned} M_{(0.999c)} &= \frac{2.0 \times 10^6 \text{ kg}}{\sqrt{1 - (0.999)^2}} = \frac{2.0 \times 10^6 \text{ kg}}{\sqrt{1 - 0.998}} \\ &= \frac{2.0 \times 10^6 \text{ kg}}{\sqrt{0.002}} = \frac{2.0 \times 10^6 \text{ kg}}{0.0447} = 4.5 \times 10^7 \text{ kg} \end{aligned}$$

The Enterprise's mass would increase from 2.0×10^6 kg to 2.3×10^6 kg (15% more massive), 4.6×10^6 kg (130% more massive), 1.4×10^7 kg (600% more massive), and 4.5×10^7 kg (2150% more massive) as it traveled at a velocity of $0.5c$, $0.9c$, 0.99c, and 0.999c, respectively.

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5. Whereas the special theory deals with non-accelerating motion, part 2 of the theory, the **general theory of relativity** (published in 1915) deals with accelerating objects in a gravitational field — it essentially rewrites Newton's Universal Law of Gravitation.

6. Whereas Newton envisioned gravity as some *magical* force, Einstein describes gravity as a curvature in space and time (called the **space-time continuum**).

a) The general theory is based upon the **principle of equivalence**. You cannot distinguish between being at rest in a gravitational field and being accelerated upward in a gravity-free environment.

b) Space and time are interrelated — you cannot have one without the other!

c) The Universe can be considered to consist of a *fabric* of space-time (*i.e.*, the vacuum) with pockets of matter that exist upon this fabric, where this matter *bends* the fabric of space-time. The more mass density, the greater the bending or **warping** of space-time (see Figure VIII-1).

7. Objects still follow straight lines as Newton said in his first law of motion, but in a gravitational field, the lines are now curved, so the object's motion curves around a massive body. Even light bends around massive objects $\Longrightarrow$ gravitational lenses.

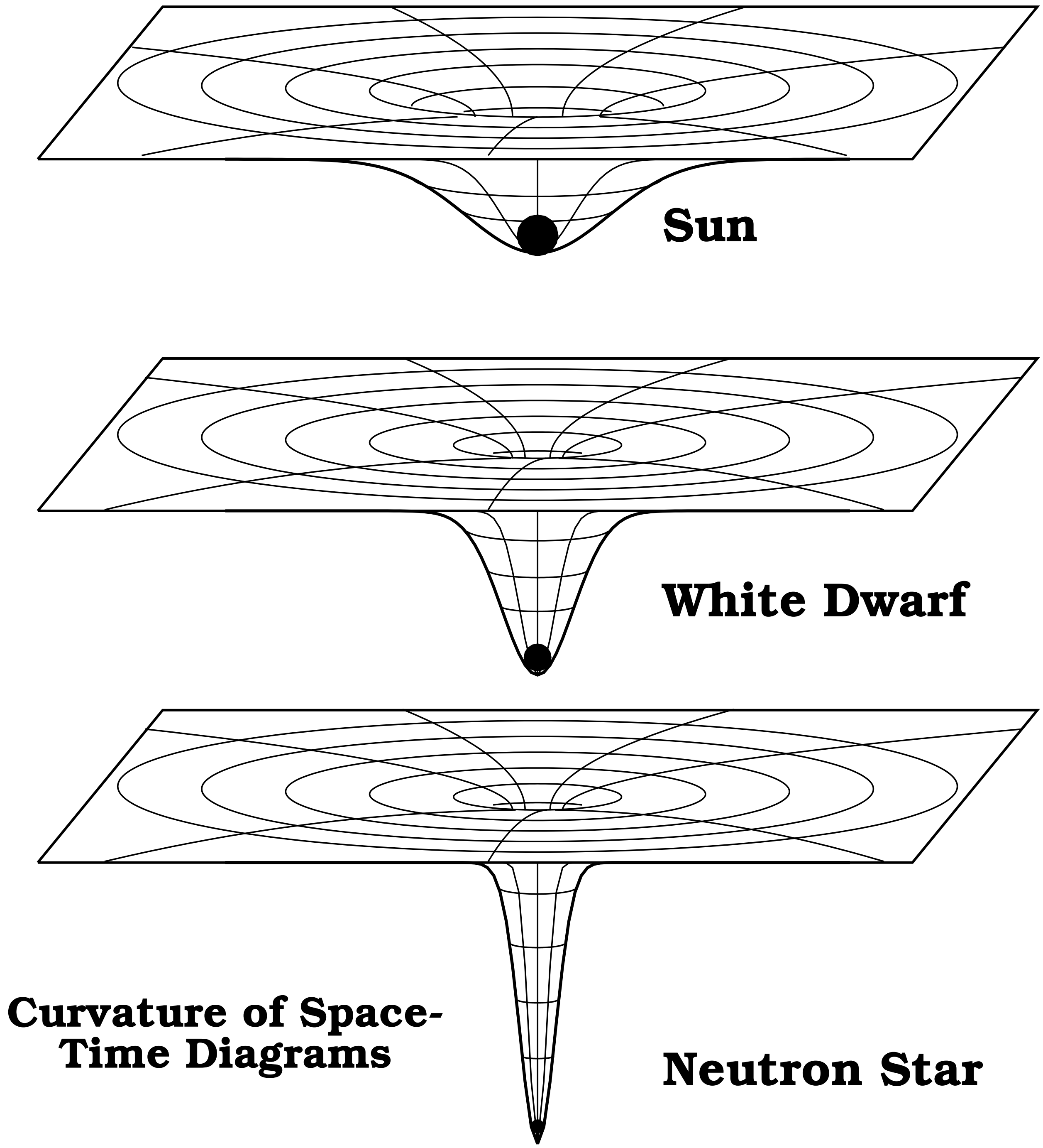

Figure VIII–1: A 2-dimensional representation of space-time. The curvature of space-time gets greater and greater as a stellar core gets denser and denser as it collapses.

8. Starlight bending around the Sun during a solar eclipse in the 1920's proved the validity of general relativity.

D. Black Holes.

1. As a stellar core collapses, it gets denser and denser, and the escape velocity from the surface of the star goes up following:

$$v_{\rm esc} = \sqrt{\frac{2GM}{R}}, \qquad \text{(VIII-4)}$$

where G is the gravitational constant, M the mass of the core, and R the radius of the core (or star).

a) At the Earth's surface: $R = R_\oplus, M = M_\oplus$, so $v_{\rm esc} =$ 11 km/s.

b) At the Sun's photosphere: $R = R_\odot, M = M_\odot$, so $v_{\rm esc} =$ 620 km/s.

c) At a WD surface: $R = R_\oplus, M = M_\odot$, so $v_{\rm esc} = 6500$ km/s $= 0.02\ c$.

d) At a NS surface: $R = 30$ km, $M = 2M_\odot$, so $v_{\rm esc} =$ $230,000$ km/s $= 0.77\ c$.

2. A **black hole** (BH) is defined when an object reaches a size such that its escape velocity equals the speed of light.

Example VIII–2. How big would the Sun have to be in order to become a black hole?

$$v_{\rm esc} = c, \quad M = M_\odot \quad v_{\rm esc}^2 = \frac{2GM}{R}$$

$$R = \frac{2GM}{c^2} = 3 \times 10^3 \text{ m} = 3 \text{ km}$$

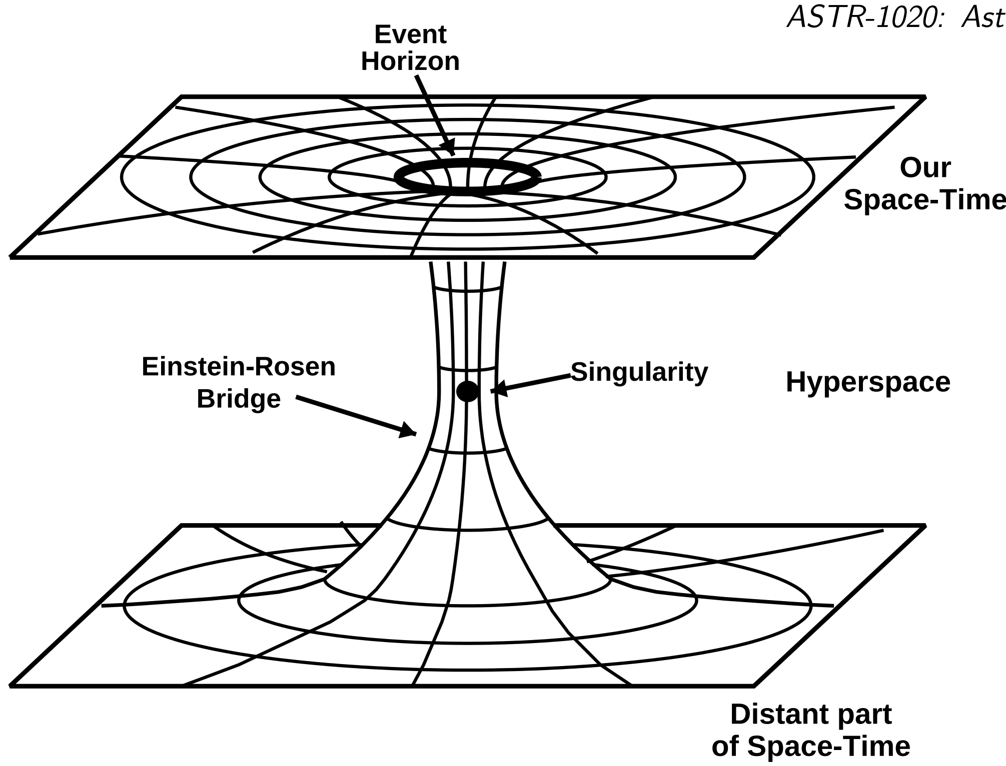

Figure VIII–2: The space-time continuum in the vicinity of a black hole.

a) A black hole is defined as a region in space-time where $v_{\rm esc} \geq c$. The outer boundary of this region where $v_{\rm esc} = c$ is called the **event horizon**.

b) The event horizon has a radius of

$$R_{\rm S} = 2GM_{\rm BH}/c^2, \qquad \text{(VIII-5)}$$

which is called the **Schwarzschild Radius**.

c) The vast majority of the region within a black hole (*i.e.*, within the event horizon) is empty space. The mass that was once a stellar core has no size (*i.e.*, zero volume). This infinitely dense object at the center of a black hole is called a **singularity**.

3. Stellar cores will become black holes only if $M_c > 3M_\odot$. No known force in nature will prevent its collapse.

4. BH's bend space-time to such an extent that the BH actually rips a hole in the fabric of the Universe and reconnects with a distance part of the Universe or connects to a parallel Universe in a different space-time dimension (see Figure VIII-2).

a) A connecting tunnel forms in space-time called an **Einstein-Rosen bridge** (also called a **wormhole**).

b) Our 3-dimensional Universe bends into a 4th dimension, called **hyperspace**.

c) Time proceeds slower and slower as you approach the event horizon of a BH.

d) Tidal forces increase without bound as you approach the singularity $\Longrightarrow$ anything with size, even nuclear particles, get ripped apart from these tidal forces as matter falls into the wormhole.

5. The only way to detect a BH is to observe the effect one has on a companion star in a binary star system (see the figure on Page VII-12 of these notes).

a) As material is stripped away from a normal star by the BH, it spirals down to the BH and heats up to very high temperatures (due to friction).

b) Before crossing the event horizon, the gas heats to such a high temperature that it emits X-rays.

6. **Black hole candidates** must satisfy the following observational requirements before they are considered black holes.

a) An X-ray source in a binary star system whose X-rays vary in brightness over the time period of seconds (hence emitting region must be very small in size).

b) The unseen companion of the spectroscopic binary has a mass greater than 3 $M_\odot$.

c) The best candidates are the ones where the mass of the unseen companion is greater than the mass of the visible star.

7. The best of the stellar BH candidates are:

a) Cyg X-1: optical component is a 30 $M_\odot$ (spectral type O9.7) supergiant star; X-ray component is a 8.7 $M_\odot$ collapsed object.

b) LMC X-3: optical component is a 5 $M_\odot$ (spectral type B3) main sequence star; X-ray component is a collapsed object between 3 $M_\odot$ and 10 $M_\odot$.

c) V404 Cyg: optical component is a 0.7 $M_\odot$ giant star; X-ray component is a 12 $M_\odot$ collapsed object — this is the best stellar BH candidate.

d) SS 433: optical component is a late A-type main sequence star (with a mass around 2.3 $M_\odot$); X-ray component is a collapsed object with a mass somewhere between 3 $M_\odot$ and 30 $M_\odot$. Jets are seen coming out of the poles of the accretion disk around the collapse object. These jets are traveling at speeds of 0.26 c. As such, this object is often referred to as a *microquasar*.

8. Black holes can also result from non-stellar evolutionary processes.

a) The Big Bang may have created an enormous number of **mini-black holes**.

b) All of the large galaxies in the Universe contain **super-massive black holes** (1 million to 1 billion solar masses in size) at their centers (indeed, our home galaxy, The Milky Way, has one)!

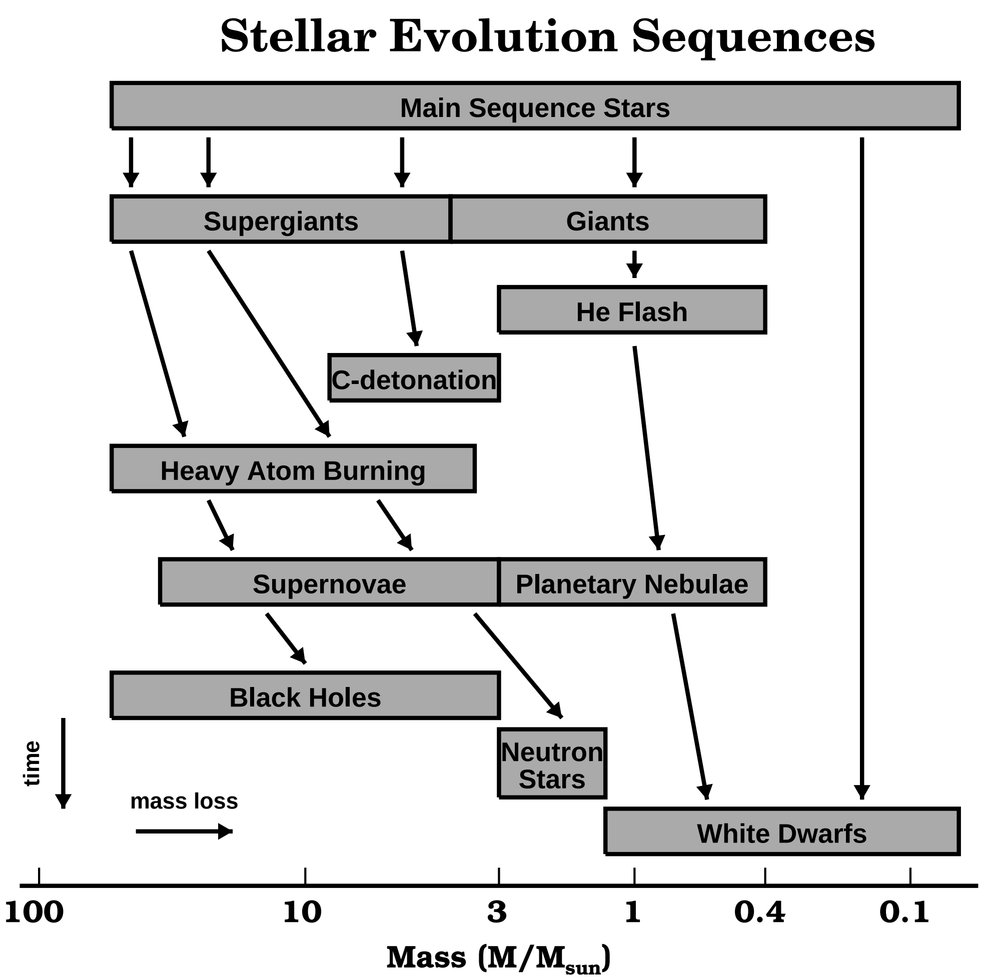

Figure VIII–3: Summary of stellar evolution.

E. Stellar Evolution Summary.

1. We have established that stars change over time — **they evolve**!

2. All elements heavier than helium (that is, everything except H and He) were created in factories we call **stars**. There would be no small rocky planets, no moons, and <u>no people</u>, if it were

not for stars producing the material from which these objects are made.

3. How a star evolves depends upon its initial mass of the collapsing material from which it formed. Figure VIII-3 summarizes the process of stellar evolution for all possible masses of stars.