ASTR-3415: Astrophysics
Course Lecture Notes
Section I

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Abstract

These class notes are designed for use of the instructor and students of the course ASTR-3415: Astrophysics. This is the Version 1.2 edition of these notes.

I. Radiative Transfer and Atomic Structure

A. Radiation Theory

1. In a radiation field, the radiant energy flowing per unit time through a surface element $d\sigma$ within a small solid angle $d\Omega$ about a direction defined by the polar angles θ, ϕ (see Figure I-1) within the frequency interval ν to $\nu + d\nu$ is

$$dE = I_\nu(\theta, \phi)\, d\nu\, \cos\theta\, d\sigma\, d\Omega. \qquad \text{(I-1)}$$

2. The **intensity** $I_\nu(\theta, \phi)$ denotes the energy flow per unit time per unit frequency interval per unit solid angle about the direction θ, ϕ across the unit area $\perp$ to this direction [erg/s/cm^2/Hz /sr, where sr $\equiv$ steradian].

   a) I_λ as a function of wavelength [erg/s/cm^3/sr, or in terms of Ångstroms (1 Å $= 10^{-8}$ cm), erg/s/cm^2/Å/sr] is related to I_ν via

$$I_\lambda\, d\lambda = I_\nu\, d\nu \qquad \text{(I-2)}$$

or

$$I_\lambda = (c/\lambda^2)\, I_\nu\ , \qquad \text{(I-3)}$$

   since $\nu = c/\lambda$.

   b) The total intensity is

$$I = \int_0^\infty I_\nu\, d\nu = \int_0^\infty I_\lambda\, d\lambda. \qquad \text{(I-4)}$$

   c) Once a photon is emitted into a vacuum (say from a star's surface), its intensity remains the same at all points along its flight path $\longrightarrow I_\nu$ independent of r!

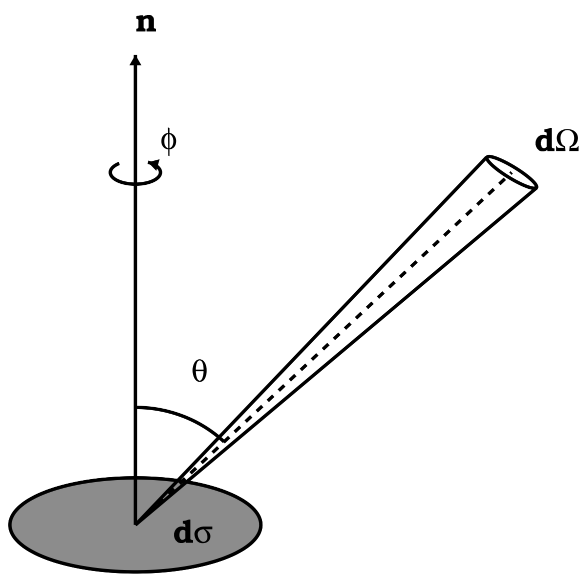

Figure I–1: Definition of the intensity of the radiation.

3. The *brightness* or *strength* of light corresponds to the **radiation flux**:

$$\mathcal{F}_\nu = \pi\, F_\nu = \int_0^{2\pi}\int_{-1}^{1} I_\nu(\mu,\phi)\,\mu\,d\mu\,d\phi. \qquad \text{(I-5)}$$

a) $\mu = \cos\theta$ and $d\Omega = \sin\theta\, d\theta\, d\phi = d\mu\, d\phi$.

b) F_ν is called the **astrophysical flux**.

c) In an **isotropic** (*i.e.*, same in all directions), I_ν is independent of θ and $\phi \Longrightarrow F_\nu = 0$.

d) It is often useful to separate F_ν [erg/s/cm^2/Hz] into an outward and an inward component in a stellar atmosphere:

$$\pi\, F_\nu^+ = \int_0^{2\pi}\int_0^{1} I_\nu\,\mu\,d\mu\,d\phi \qquad \text{(I-6)}$$
$$(0 \le \mu \le 1) \quad \text{outward flux}$$
$$\pi\, F_\nu^- = \int_0^{2\pi}\int_{-1}^{0} I_\nu\,\mu\,d\mu\,d\phi \qquad \text{(I-7)}$$
$$(-1 \le \mu \le 0) \quad \text{inward flux,}$$

from which we can write

$$F_\nu = F_\nu^+ + F_\nu^- . \qquad \text{(I-8)}$$

e) If I_ν is axisymmetric (*i.e.*, independent of ϕ), then the flux equation becomes:

$$\mathcal{F}_\nu = 2\pi \int_{-1}^{1} I_\nu \, \mu \, d\mu. \tag{I-9}$$

f) The total radiation flux is then given by

$$\mathcal{F} = \int_0^\infty \mathcal{F}_\nu \, d\nu = \int_0^\infty \mathcal{F}_\lambda \, d\lambda \; . \tag{I-10}$$

g) Note that $4\pi R^2 \mathcal{F}_\nu = L_\nu$ is the monochromatic luminosity, where R is the radius of a star.

h) $\mathcal{F}_\nu = \pi I_\nu$ if I_ν is isotropic outward and zero inward.

4. Unlike intensity, flux does scale with distance $\Longrightarrow$ an object gets fainter the farther away it is:

$$f_\nu = \left(\frac{R_\star}{d}\right)^2 \mathcal{F}_\nu = \frac{1}{4}\, \alpha_\star^2 \, \mathcal{F}_\nu. \tag{I-11}$$

a) f_ν is the observed flux of a star of radius $R_\star$ at a distance d.

b) $\alpha_\star$ is the angular diameter (in radians) of the star as seen at distance d.

5. The magnitude, ($m_{\delta\lambda}$), of a star can be related to the integrated flux as seen from Earth ($f_{\delta\lambda}$) by

$$m_{\delta\lambda} = q_{\delta\lambda} - 2.5 \log \, f_{\delta\lambda}. \tag{I-12}$$

a) $f_{\delta\lambda}$ ($= \int_{\delta\lambda} f_\lambda \, d\lambda$) is the observed flux at the top of the Earth's atmosphere in wavelength band $\delta\lambda$.

b) The magnitude that corresponds to this flux is called the *apparent magnitude.*

Table I–1: Broad Band Filters

Filter	λ (Å)	$\delta\lambda$ (Å)	$q_{\delta\lambda}^{\dagger}$
U	3650	700	–38.40
B	4400	1000	–37.86
V	5500	900	–38.52
R	7000	2200	–39.39
I	8800	2400	–40.20
J	12,500	3800	–41.20
K	22,000	4800	–43.50
L	34,000	7000	–45.20
M	50,000	12,000	–46.60
N	104,000	57,000	–49.80

† *absolute zero-point constant when* f_λ *is in units of watts/cm²/Å.*

c) Table I-1 lists the most commonly used photometric magnitudes and their absolute zero-point constant ($q_{\delta\lambda}$).

6. Ground-based observations must be corrected for atmospheric extinction, filter transparency, *etc.* The observed flux, f_λ^{obs}, is related to the apparent (*i.e.*, above the Earth's atmosphere) flux, f_λ, by the following convolution:

$$f_\lambda^{obs} = \int_0^\infty \phi_A(\lambda)\phi_T(\lambda)\phi_F(\lambda)\phi_D(\lambda) f_\lambda d\lambda, \qquad \text{(I-13)}$$

where

$\phi_A(\lambda)$ = fractional transmission of the Earth's atmosphere,

$\phi_T(\lambda)$ = fractional transmission of the telescope,

$\phi_F(\lambda)$ = fractional transmission of the filter, and

$\phi_D(\lambda)$ = efficiency of the detector (1.0 corresponds to 100%).

a) We can eliminate ϕ_T, ϕ_F, ϕ_D, by observing a (flux) *standard* star, with the same instrument set-up, for comparison with the object (Vega is an example of a standard star).

b) ϕ_A also can be determined with a standard star, but it takes a little more work, since the standard is usually in a different part of the sky with respect to the object (hence sky transparency will be different).

B. Blackbody Radiation

1. Matter that is in **thermodynamic equilibrium** (TE) emits thermal radiation $\Longrightarrow$ **blackbody (BB) radiation**.

   a) TE will be discussed in §4 of the notes (*i.e.*, Stellar Interiors).

   b) Max Planck first to derive the law of BB radiation — intensity as a function of frequency (or wavelength), I_ν (or I_λ).

   c) A BB is an opaque body that is a perfect absorber (and perfect radiator) $\Longrightarrow$ any energy incident on a BB will be radiated away at the temperature of the body.

   d) A BB radiator emits light that is characterized only by its temp with a spectral shape called a **Planck curve** (see Figure I-2). Its **monochromatic specific intensity** is given by **Planck's Law** and is called the **Planck Function**:

$$B_\nu = \frac{2h\nu^3/c^2}{e^{h\nu/k_BT} - 1} \quad (\text{erg/s/cm}^2\text{/Hz/sr}), \qquad \text{(I-14)}$$

   or in wavelength space:

$$B_\lambda = \frac{2hc^2/\lambda^5}{e^{hc/\lambda k_BT} - 1} \quad (\text{erg/s/cm}^3\text{/sr}), \qquad \text{(I-15)}$$

2. The temperature of an object can be deduced if the object emits a thermal spectrum.

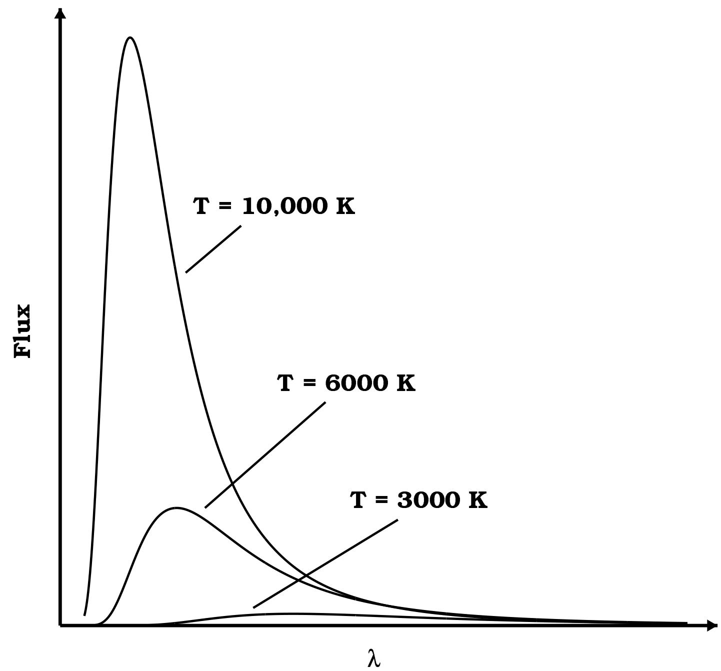

Figure I–2: The Planck curve as a function of wavelength.

a) The monochromatic (*one color*) flux of a blackbody is $\mathcal{F}_\nu = \pi B_\nu$ (or $\mathcal{F}_\lambda = \pi B_\lambda$), since $I_\nu = B_\nu$.

b) The integrated or total flux of a BB can be found by integrating the Planck function over the whole spectrum:

$$\mathcal{F} = \sigma T^4, \tag{I-16}$$

which is called the **Stefan-Boltzmann Law** and $\sigma = 5.6696 \times 10^{-5}$ erg/s/cm^2/K^4 is called the *Stefan-Boltzmann constant*.

c) The wavelength where the peak BB flux arises can be found by taking the derivative of the Planck function and finding the maxima:

$$\lambda_{max} = \frac{0.2898\,\text{cm K}}{T} = \frac{2.898 \times 10^7\,\text{Å K}}{T}, \tag{I-17}$$

which is called the **Wien Displacement Law**.

d) As can be seen by these 2 laws, the hotter a BB, the bluer the light and the brighter it becomes.

3. A BB spectrum contains no lines — it is a **continuum** → a continuous band of colors.

C. The Equation of Radiative Transfer

1. If a photon is traveling within a medium (*i.e.*, non-vacuum), its intensity <u>does</u> change as it propagates through the gas depending on the opacity of the gas.

a) The **opacity**, κ [cm^{-1}], of a gas measures how opaque the gas is.

b) It is the inverse of the *mean-free-path*, L [cm], of the photon $\Longrightarrow$ the distance a photon travels before it interacts with another particle.

c) The opacity dictates how *deep* we can see into a gas. As such, the **optical depth** along depth s (s increasing outward) of a gas is defined by

$$d\tau_\nu = -\kappa_\nu \, ds, \qquad \text{(I-18)}$$

τ_ν increases in the opposite direction to s, $\tau_\nu = 0$ at the top and getting larger as you go inward.

d) One typically does not see deeper into a gas than $\tau_\nu \approx 1$.

2. As a photon travels through a gas along a small length ds in direction θ ($\mu = \cos\theta$), I_ν is attenuated by the following expression:

$$\mu \frac{dI_\nu}{ds} = -\kappa_\nu \, I_\nu . \qquad \text{(I-19)}$$

3. Besides this absorption process, particles in the gas can also emit photons (*i.e.*, emission). Hence, I_ν can increase along this path by

$$\mu \frac{dI_\nu}{ds} = \epsilon_\nu , \qquad \text{(I-20)}$$

where ϵ_ν [erg/s/cm^3/Hz/sr] is the **emissivity** of the gas.

4. Summing these 2 equations gives the manner in which photons travel through a gas $\Longrightarrow$ the **Equation of Radiative Transfer**:

$$\mu \frac{dI_\nu}{ds} = \epsilon_\nu - \kappa_\nu \, I_\nu . \tag{I-21}$$

5. We can divide each term by κ_ν and get

$$\mu \frac{dI_\nu}{d\tau_\nu} = I_\nu - S_\nu . \tag{I-22}$$

a) S_ν is called the **source function** and is the ratio of the number of photon creation events divided by the number of photon destruction events ($S_\nu = \epsilon_\nu/\kappa_\nu$) for a given region of gas.

b) As such, you can view the transfer equation as the change in intensity of a photon equals the incident intensity on a volume of gas plus any photons created by the gas minus any photons destroyed by the gas.

c) Often S_ν will be separated into a *continuum* component and a *line* component: $S_\nu = S_\nu^\ell + S_\nu^c$.

6. For gas in thermodynamic equilibrium, the emissivity of the gas is given by

$$\epsilon_\nu = \kappa_\nu \, B_\nu(T), \tag{I-23}$$

hence the source function becomes the Planck function for this gas and the transfer equation is somewhat easy to solve:

$$I_\nu = I_{\nu,\circ} \, e^{-\tau_\nu/\mu} + \frac{1}{\mu} \int_{\tau_\nu} B_\nu(\tau_\nu) \, dt \tag{I-24}$$

7. For nonthermodynamic equilibrium gas, however, S_ν is much more difficult to ascertain, since it will depend upon both the mean intensity ($J_\nu = \frac{1}{2} \int I_\nu \, d\mu$) of the photons and the thermal nature of the gas in the volume of interest $\Longrightarrow$ the equation of transfer becomes a *integral-differential equation* in I_ν!

D. Opacity

1. The opacity of gas is a measure of the resistance to photon flow.

2. The concept of opacity is what links radiative transfer to atomic physics. The opacity of the gas is a function of atomic transition rates, which are related to transition cross sections.

3. There are 2 basic types of opacity in gas: (1) **continuous opacity** and (2) **line opacity**.

4. There are 4 basic continuum processes involving the interaction of radiation and matter.

a) **Bound-free** (ionization or b-f) transitions $\Longrightarrow$ e^- liberated from atom, ion, or molecule. The inverse process (free-bound) is called recombination.

i) *Photoionizations* $\Longrightarrow$ photon absorption liberates e^-. The reverse process is called *radiative recombination.*

ii) *Collisional ionizations* $\Longrightarrow$ atom, ion, or molecule collides with an electron (typically) or some other particle (*i.e.*, H I, H_2, He I, etc.). The reverse process is called *collisional recombination.*

b) **Free-free** (f-f) transitions.

i) *f-f absorption* $\equiv$ free e^- absorbs photon in vicinity of an ion $\Longrightarrow$ KE of e^- is altered.

ii) *Bremsstrahlung* $\equiv$ free e^- altered by E-field of an ion giving up a photon (inverse f-f).

c) **3-body ionizations and recombinations** $\Longrightarrow$ photoexcitation (*i.e.*, e^- jumps from a lower to upper bound state through photon absorption) followed immediately by a collisional ionization (3 particles = atom, photon, colliding particle) and the inverse process is called 3-body recombination.

d) **Scattering** $\Longrightarrow$ photon redirected in its flight path and <u>not</u> absorbed by the gas.

i) *Thomson* or *electron scattering* results from a free e^- (or ion) oscillating in an EM field $\rightarrow$ important in hot stars.

ii) *Compton scattering* results from an actual collision between a photon and electron (or some type of charged particle) $\rightarrow$ generally not important in stellar atmospheres (requires high energy [*i.e.*, X-ray] photons).

iii) *Rayleigh scattering* results when the oscillating EM field of a photon perturbs, in an oscillating manner, the bound e^- in the atom or molecule as the photon passes. This in turn affects the EM field of the photon $\rightarrow$ important in cool stars, causes the Earth's sky to be blue.

iv) *Mie scattering* results from photons scattering off of dust particles.

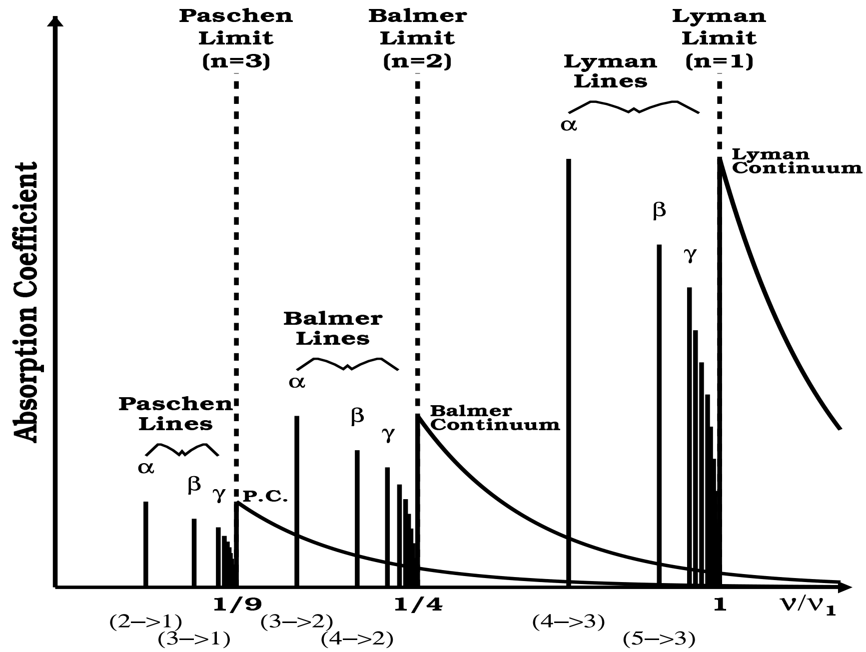

Figure I–3: Schematic of the b-f and b-b absorption coefficients of hydrogen.

5. **H^- opacity** (H I$+e^-$) is a special type of b-f opacity and dominates all continuous opacities at visual λs in many stars (including the Sun) $\Longrightarrow$ only 1 bound state. The extra e^- is bound to the H atom with an energy of 0.754 eV. There is also f-f opacity from H^- $\Longrightarrow$ important at IR wavelengths.

6. **H b-f opacities** are a very strong continuous opacity source in all stars shortward of 912 Å and in B, A, and F stars at UV and visual wavelengths.

   a) Figure I-3 shows the functional form of the hydrogen absorption coefficients. The b-f opacity ($\kappa_\nu = n_{i\equiv n}\alpha_\nu$) of H (and H-like ions) varies as ν^{-3} for $\nu > \nu_{threshold}$:

$$\begin{aligned}\alpha_n(\nu) &= \frac{64\,\pi^4\,m\,e^{10}\,Z^4}{3\,\sqrt{3}\,c\,h^6\,n^5\,\nu^3}\;g_n^{bf}(\nu) \\ &= 2.8154\times 10^{29}\frac{Z^4}{n^5\,\nu^3}\;g_n^{bf}(\nu) \quad \text{cm}^2, \text{(I-25)}\end{aligned}$$

   where $e =$ the electrostatic constant, $Z =$ nuclear charge (*i.e.*, Z=1 for H), n = the principal quantum number,

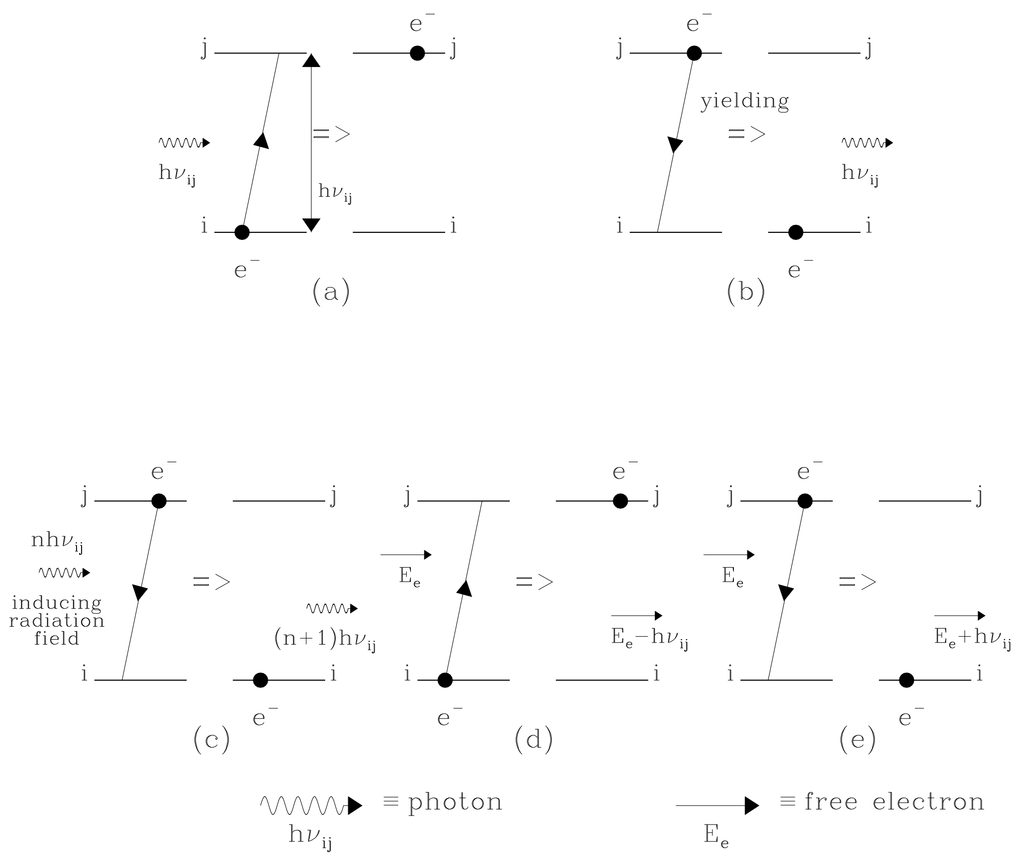

Figure I–4: The 5 processes involving bound-bound transitions. For each subframe, the left side represents "before" the interaction, and the right side, "'after" the interaction.

and $g_n^{bf}(\nu)$ is the Gaunt factor (quantum correction to the classical physics formula) of order unity (see Karsas, W.J. and Latter, R. 1961, *Astrophys. Journ. Suppl.*, **6**, 167).

b) The location of the b-f edges for H-like ions (called *jumps* in flux spectra) can be found by

$$\lambda_n = \frac{c\,h^3}{2\,\pi\,m\,e^4}\frac{n^2}{Z^2} = \frac{n^2}{Z^2}\;911.75358\,\text{Å}. \qquad \text{(I-26)}$$

7. There are 5 distinct physical processes involving **bound-bound** (b-b) transitions or line opacity (see Figure I-4):

a) *Radiative excitation*: e^- jumps from a lower state "i" to an upper state "j" with an energy difference ΔE_{ij} via a photon absorption of energy $h\nu_{ij}$ such that $\Delta E_{ij} = h\nu_{ij}$.

b) *Spontaneous de-excitation* (or *emission*): a bound e^- in an upper state has a finite lifetime in such state and can spontaneously (*i.e.*, without external stimuli) decay to a lower state. This decay leads to a lower energy state of the e^-, which must be conserved by the emission of a photon — again $\Delta E_{ij} = h\nu_{ij}$.

c) *Stimulated de-excitation* (or *emission*): a bound e^- in an upper state can decay to a lower state if the atom/ion is bombarded by radiation of frequency $h\nu_{ij} \Longrightarrow nh\nu_{ij} + E_j = E_i + (n+1)h\nu_{ij}$.

d) *Collision excitation*: e^- jumps from a lower to upper state via a free e^- collision $E_e + E_i = E_j + E'_e$ ($E'_e = E_e - \Delta E_{ij}$).

e) *Collisional de-excitation*: free e^- collision induces an excited bound e^- to decay to a lower state, which adds to the free e^- energy $\Longrightarrow E_e + E_j = E_i + E'_e$ ($E'_e = E_e + \Delta E_{ij}$).

8. A transition will typically occur if certain **selection rules** are satisfied (see next section).

9. The **line opacity** has the following functional form (neglecting stimulated emission):

$$\kappa_\nu = n_i\,\alpha_\nu = n_i\,\frac{\pi e^2}{mc} f_{ij}\phi_\nu, \tag{I-27}$$

or in wavelength units (note that $\phi(\nu)\,d\nu = -\phi(\lambda)\,d\lambda$)

$$\kappa_\lambda = n_i\,\alpha_\lambda = n_i\,\frac{\pi e^2 \lambda_\circ^2}{mc^2} f_{ij}\phi_\lambda, \tag{I-28}$$

where n_i is the number density in the lower state, α = the cross section of the transition, $\pi e^2/mc$ = the *classical oscillator cross section*, f_{ij} = the **oscillator strength**, a quantum correction to the classical cross section, and ϕ = the line profile.

a) Eqs. I-27 & I-28 result from a semi-classical treatment of the matter-radiation interaction (classical EM $\longrightarrow$ harmonic oscillator & quantum mechanics of matter $\longleftarrow$ f_{ij}).

b) $\boxed{\frac{\pi e^2}{mc} = 0.02654 \text{ cm}^2}$.

c) The functional form of ϕ_ν will depend upon the characteristic of the broadening involved (*i.e.*, natural, pressure, Doppler, etc.) — more to come in §II. It is always normalized such that $\int_0^\infty \phi_\nu \, d\nu = 1$.

d) f_{ij}, the **absorption oscillator strength**, is inversely proportional to the probable amount of time an e^- will reside in the jth level. Allowed transitions: $f_{ij} > 10^{-2}$ (strongest: $f_{ij} \sim 1$). Forbidden transitions: $f_{ij} < 10^{-8}$ (with semi-forbidden transitions in between the two).

E. Atomic Structure and Spectroscopic Notation

1. As can be seen, in order to determine the opacity of a gas at a given wavelength, one must know the number of electrons in a given state. In this course, we will only worry about gas that it is thermal equilibrium and in a *steady state*.

a) In this case, statistical mechanics shows us that the *number density* ($n \equiv$ number per unit volume) in state j is related to number density in state i ($j > i$) by **Boltzmann's equation**:

$$\frac{n_j}{n_i} = \frac{g_j}{g_i} \exp[(E_i - E_j)/k_B T]. \qquad \text{(I-29)}$$

i) g is the multiplicity (or **statistical weight**) of the level (see Table I-2).

ii) E is the energy of the level and *exp* is the long-hand notation of the base $e = 2.71828$.

iii) k_B is Boltzmann's constant (1.3806×10^{-16} erg/K) and T the temperature of the gas.

b) Also in such a gas, **ionization equilibrium** is achieved (*i.e.*, ionization rate = recombination rate) and **Saha's equation** applies:

$$\frac{N_{k+1}}{N_k} n_e = 2 \, \frac{u_{k+1}}{u_k} \, \frac{(2\pi \, m \, k_B \, T)^{3/2}}{h^3} \, e^{-\chi_k/k_B T} . \qquad \text{(I-30)}$$

i) N is the total number of atoms in ionization stages k and the next higher ion stage $k+1$.

ii) n_e is the electron number density.

iii) $u = \sum g_i \, e^{-E_i/k_B T}$ is called the **partition function** for ions k and $k+1$.

iv) h is Planck's constant (6.6262×10^{-27} erg s) and χ_k is the ionization potential of ion k.

v) The lowest ion stage are *neutral atoms* with $k = 0$.

c) In astrophysics, ionic stages are labeled with Roman numerals: neutral with "I" (*e.g.*, H I, He I, Fe I), singly ionized atoms with "II" (H II, He II, Fe II), doubly ionized with "III" (He III, C III, Fe III), etc.

2. Each element/ion has an *electronic configuration* associated with it, which is based on the periodic table. Each e^- in that configuration has a characteristic set of quantum numbers.

a) $n \equiv$ *principal quantum number* $\Longrightarrow$ shell ID

n	=	1	2	3	4	5	6	...
shell	:	K	L	M	N	O	P	...

Each shell can contain a maximum of $2n^2$ e^-s.

b) $\ell \equiv$ *orbital angular momentum quantum number* $\Longrightarrow$ subshell ID.

ℓ	=	0	1	2	3	4	5	...	$(n-1)$
subshell	:	s	p	d	f	g	h	...	

i) Each subshell can contain a max of $2(2\ell+1)$ e^-s.

ii) The orbital angular momentum vector can have $2\ell+1$ orientations in a magnetic field from $-\ell$ to $+\ell$:

$$-\ell \leq m_\ell \leq \ell$$

c) $s \equiv$ *spin angular momentum quantum number* $\Longrightarrow$ spin direction (*i.e.*, up or down).

$$s = \frac{1}{2}$$

The spin angular momentum vector can have $2s+1$ (=2) orientations in a B-field.

$$m_s = \pm\frac{1}{2}$$

d) $j \equiv$ *total angular momentum quantum number.*

$$j = \ell \pm s$$

The total angular momentum vector can have $2j + 1$ orientations ($-j \le m_j \le j$) in a B-field.

e) *Examples*:

i) An e^- with $n = 2$, $\ell = 1$, and $j = 3/2$ is denoted by $2p_{3/2}$.

ii) The lowest energy state of neutral sodium, Na I, has an e^- configuration of $1s^2\,2s^2\,2p^6\,3s$. (NOTE: the exponents indicate the number of e^-s in that subshell, no number $\equiv 1$.) The K- and L-shells are completely filled — the 3s e^- is called a valence e^-.

3. For one e^- atoms (*i.e.*, H I, He II, C VI, Fe XXVI, etc.), the principal (n) levels have energies of

$$E_n = -\frac{2\,\pi^2\,m\,e^4\,Z^2}{n^2\,h^2}, \qquad \text{(I-31)}$$

where Z = charge of the nucleus.

a) Negative energies $\Longrightarrow$ bound states
Positive energies $\Longrightarrow$ free states
Ionization limit ($n \to \infty$) in Eq. I-31 has $E = 0$.

b) In astronomical spectroscopy, we set $E_1 = 0$ and represent atoms in terms of *energy level diagrams* (see Figure I-5), where the energy levels are determined by

$$E_n = 13.6\ Z^2 \left(1 - \frac{1}{n^2}\right)\ \text{eV}. \qquad \text{(I-32)}$$

$n \to \infty$ defines the **ionization potential** (IP) of the atom (or ion), so that for H: IP = 13.6 eV, for He II: IP = 54.4 eV, etc.

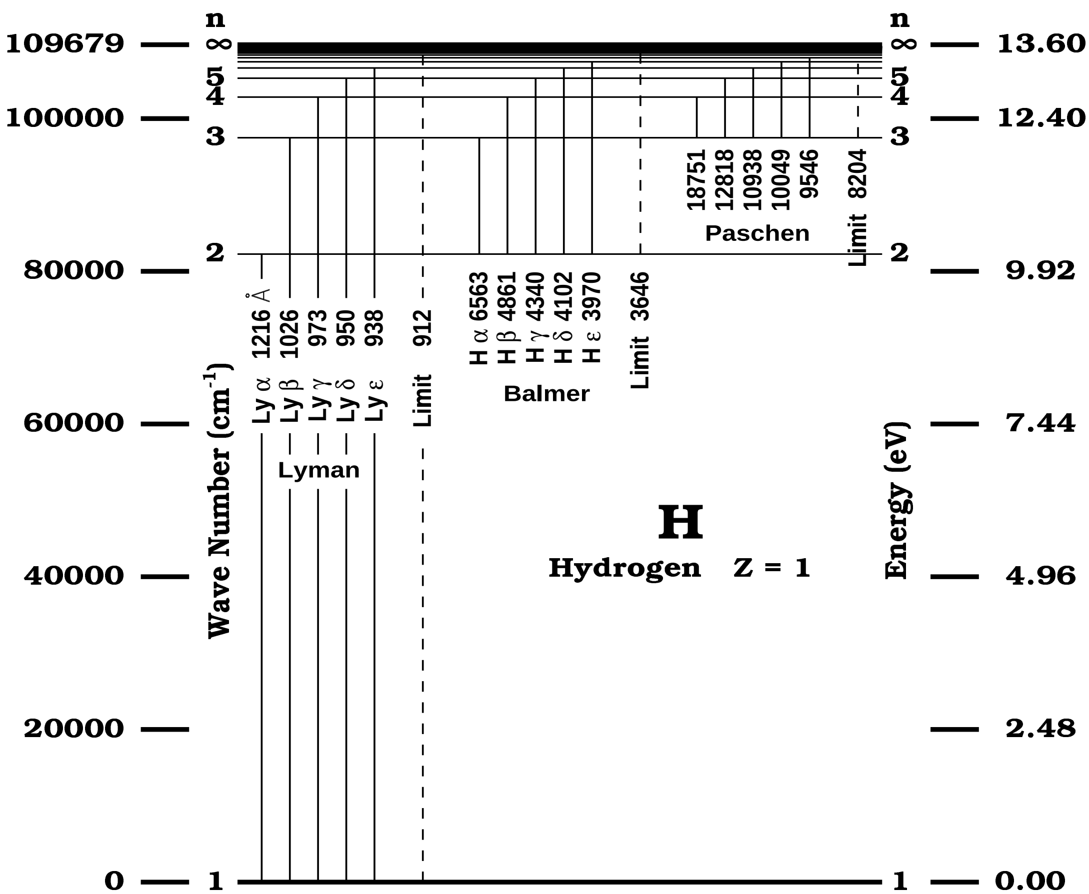

Figure I–5: A partial Grotrian diagram of neutral hydrogen. The lowest 7 levels are shown with various transitions labeled.

c) NOTE: 1 eV = 1.602×10^{-12} erg = 8066 cm^{-1} = 12,398 Å = 11,605 K.

d) The lowest energy state ($E = 0$) is called the **ground state**. States above the ground are said to be **excited**.

4. For atoms or ions with several e^-s, the angular momentum vectors are normally coupled as follows (**Russell-Saunders** or **LS coupling**):

a) Orbital angular momenta $\vec{\ell}$ are added vectorally: $L = \sum \vec{\ell}$.

b) Ditto for spin: $S = \sum \vec{s}$.

c) L and S combine vectorally to give the total angular momentum J: $|L - S| \leq J \leq L + S$.

d) A particular pair of values for S and L constitutes a **term**.

i) A level or state is designated by its **spectroscopic notation**: ${}^{(2S+1)}L_J$

$$\begin{array}{rcccccccc} L & = & 0 & 1 & 2 & 3 & 4 & 5 & \ldots \\ \text{state} & : & \text{S} & \text{P} & \text{D} & \text{F} & \text{G} & \text{H} & \ldots \end{array}$$

ii) $2S + 1 \equiv$ **multiplicity** of the state $\Longrightarrow$ number of J-levels if $L \geq S$. If $L < S$, $2L+1 \Longrightarrow$ number of J-levels in the state.

iii) $S = 0 \Longrightarrow 2S + 1 = 1 \Longrightarrow$ singlet state
$S = 1/2 \Longrightarrow 2S + 1 = 2 \Longrightarrow$ doublet
$S = 1 \Longrightarrow 2S + 1 = 3 \Longrightarrow$ triplet, etc.

iv) A level can have *odd* or *even* **parity** depending upon whether the arithmetic sum of the ℓ-values of the participating e^-s is odd or even (*i.e.*, ^{1}P is an even parity state while $^1P^\circ$ is an old parity state.

5. **Hund's Rules** (see Figure I-6): For any given electron configuration,

a) Higher $S \Longrightarrow$ lower energy.

b) Higher L (for this S value) $\Longrightarrow$ lower energy.

c) Higher $J \Longrightarrow$ higher energy if subshell is less than half-filled, lower energy if more than half-filled.

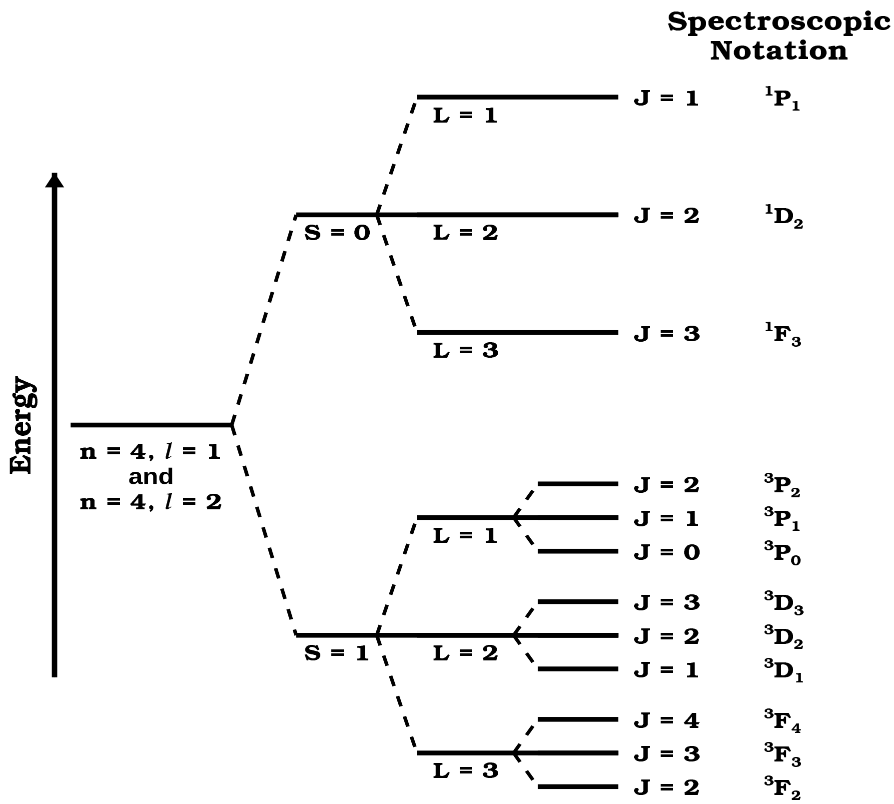

Figure I–6: Hund's rules demonstration: Example of energy splitting in the LS coupling scheme.

6. Various terminology is used to describe transitions in spectroscopy. Table I-2 defines and relates some of these terms (from Allen 1976, *Astrophys. Quant.*, 3rd Ed., p.53).

7. **LS** or **Russell-Saunders coupling**: For lighter atoms with several e^-s outside closed shells, a spin-orbit interaction exists between the e^-s which is small compared to the nuclear Coulomb (*i.e.*, electrostatic) interaction. The LS coupling technique (*i.e.*, the spin-orbit interaction is treated as a perturbation) is used in these cases to describe the electronic states and transition probabilities. The following are LS selection rules governing **dipole** *radiative* transitions:

   a) Only one e^- jumps.

   b) $\Delta\ell = \pm 1$ (parity rule).

Table I–2: Atomic Terminology

Atomic Division	**Specification**	**Statistical Weight** g	**Transition**
State	Specified by L, S, J, M, or L, S, M_L, M_S	1	Component of line
Level	Specified by L, S, J, *e.g.*, ${}^3\mathrm{P}_{1/2}$	$2J+1$	Spectrum line
Term	Group of levels specified by L, S	$(2S+1)(2L+1)$	Multiplet
Polyad	Group of terms from one parent term with same multiplicity or S		Super-multiplet
Configuration	Specified by n and ℓ of all electrons		Transition array

c) For terms:

i) $\Delta S = 0$.

ii) $\Delta L = 0, \pm 1$.

iii) $\Delta J = 0, \pm 1$ (but $J = 0 \not\rightarrow J = 0$).

8. A transition is **allowed** if none of the selection rules are violated. Allowed transitions from the $E = 0$ state are called **resonance lines** (*i.e.*, the Ca II K line is a resonance line from the $4s\,{}^2\mathrm{S}_{1/2}$ state to the $4p\,{}^2\mathrm{P}^\circ_{3/2}$ state, note that the LS dipole selection rules are not violated). Allowed transitions, for example, are labeled as "Ca II K" or "Ca II 3934 Å" or "Ca II λ3934."

9. A transition is said to be **semi-forbidden** if the ΔS rule (spin forbidden) <u>or</u> the ΔL rule (orbit forbidden) are violated. Semi-forbidden transitions are labeled with <u>one</u> right-side square bracket $\Longrightarrow$ "C II] λ2325."

10. A transition is said to be **forbidden** (or completely forbidden) if both the ΔS <u>and</u> ΔL rules are violated and/or the transition involves no change in parity and/or $J = 0 \rightarrow J = 0$. Forbidden transitions are labeled with both square brackets $\Longrightarrow$ "[O III] λ5006."

11. Radiative transitions can occur if one or more of the selection rules are violated via a magnetic dipole interaction or electric quadrapole interaction.

12. Transitions between spectroscopic terms in the same subshell are called **intersystem** (or **intercombination**) **lines** and are always semi-forbidden or completely forbidden (note that the transitions among the ^{3}P, ^{1}S, and ^{1}D terms in the $2p^2$ subshell of N II and O III in Figure I-7 are intersystem lines — the 5006 Å line between $2p^2\,{}^3$P and $2p^2\,{}^1$D is the famed nebular line, which give nebulae that greenish tint to the naked eye).

13. Transitions within the *ground state* multiplet are called **fine-structure lines** and are always forbidden ($\Delta\ell = 0, \Delta J = 1$). They are almost always in the IR or far-IR (*e.g.*, C II 158 μm, ^{2}P$_{3/2}$ $\rightarrow$^{2}P$_{1/2}$). These lines are responsible for cooling low-temperature (30-300 K) gas in space.

14. **Hyperfine lines** are transitions resulting from an electron spin flip in the ground state (*e.g.*, H I 21 cm line).

15. When an excited state is not directly coupled via dipole transitions to the ground state, it is called a **metastable state** since it behaves as an e^- reservoir like the ground state (*i.e.*, the lower level of the He I 10,830 Å line).

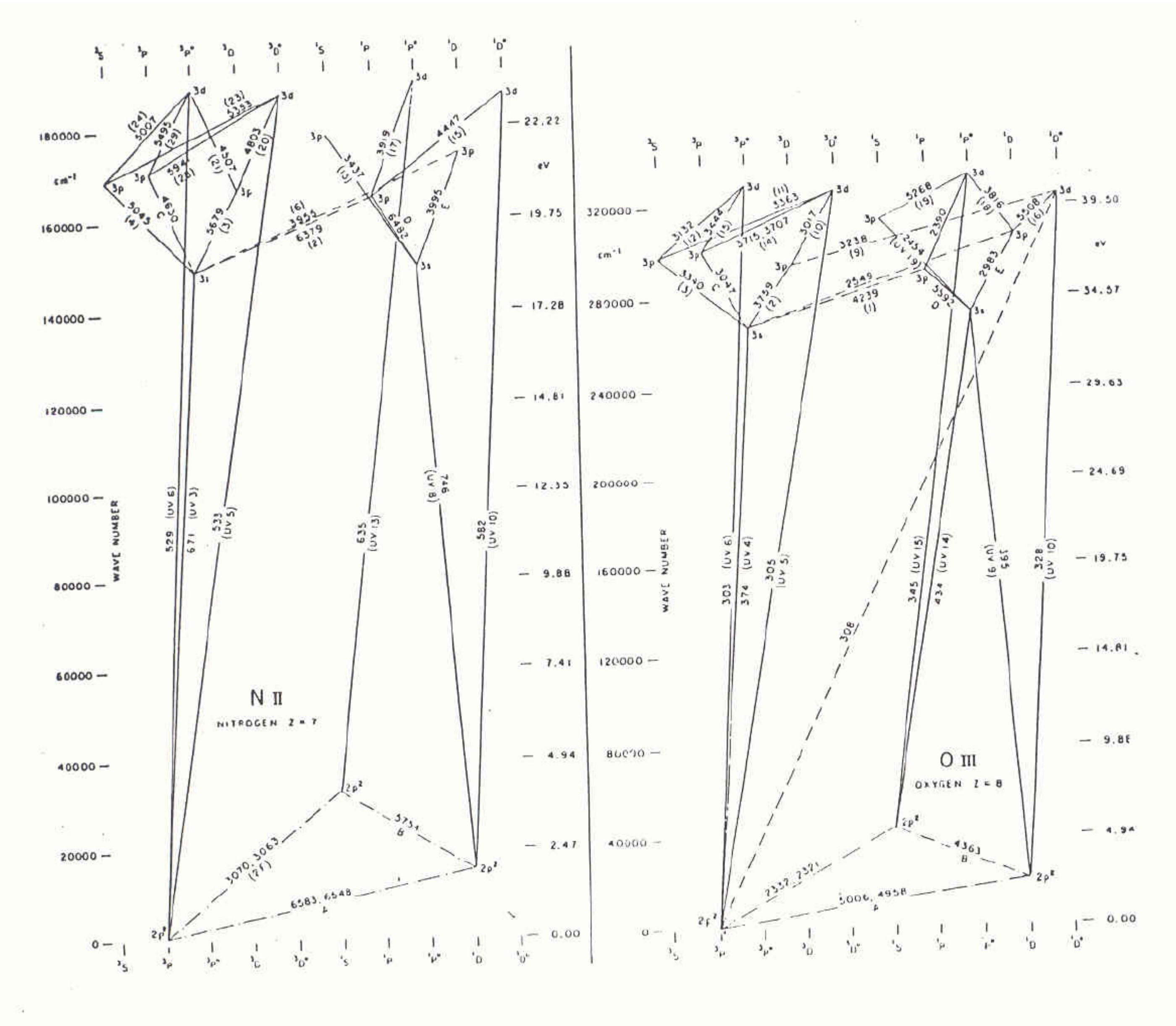

Figure I–7: A partial Grotrian diagram of N II and O III. Note that the ^{3}P term is considered the ground state of these ions since it lies at the lowest energy. The ^{1}S and ^{1}D states are said to be metastable.

16. When elements and ions have the same total number of electrons, they are said to be **isoelectronic sequences** of each other. For instance, Figure I-7 displays a **Grotrian diagram** of the N II and O III ions, which have the same e^- configurations as C I. Hence C I, N II, and O III are all isoelectronic sequences of each other.