ASTR-3415: Astrophysics
Course Lecture Notes
Section V

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Abstract

These class notes are designed for use of the instructor and students of the course ASTR-3415: Astrophysics. This is the Version 1.2 edition of these notes.

V. Stellar Evolution: Birth

A. Stellar Nurseries.

1. Stars form from the dust and gas that lie between the stars $\Longrightarrow$ the **interstellar medium**.

2. Stellar nurseries are found within **giant molecular clouds** (GMC) as shown in Figure V-1. The nearest GMC to the solar system is the Orion complex.

3. We will discuss the ISM in detail when we discuss the Milky Way galaxy as a whole.

B. The Jeans' Length and Jeans' Mass.

1. J.H. Jeans first analyzed the gravitational instability of gas clouds in 1902 $\Longrightarrow$ the minimum size of a cloud which will *collapse* under its own gravity is called the **Jeans' Length**.

2. It is nothing more than the conservation of energy. In an interstellar gas cloud, there are two energy sources: the thermal energy of the gas, $E_{\rm th}$, and the gravitational energy of the clouds gravitational field, $E_{\rm g}$. For a static cloud, energy conservation gives:

$$E_{\rm th} + E_{\rm g} = 0. \tag{V-1}$$

a) The thermal energy is just the pressure multiplied by the volume of the material:

$$E_{\rm th} = P \cdot V = \frac{\rho\, k_B\, T}{\mu\, m_H} \cdot \frac{4}{3}\pi\, R^3\ . \tag{V-2}$$

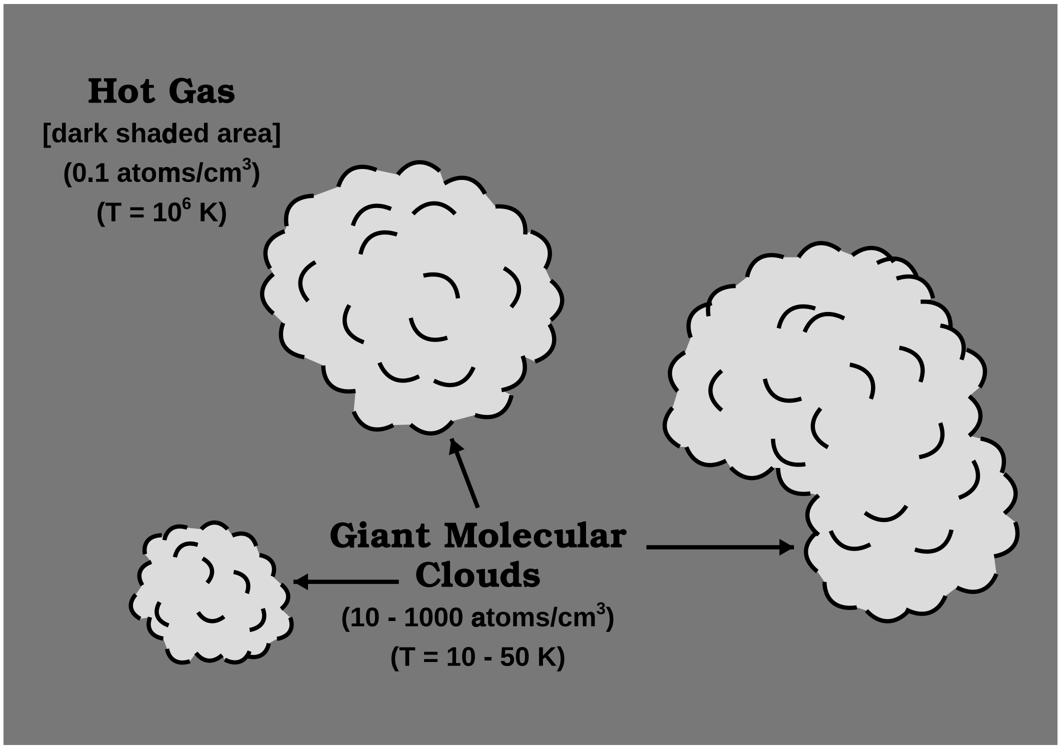

Figure V–1: Structure of the Interstellar Medium.

b) The gravitational potential energy is

$$E_{\rm g} = -\frac{G\,M^2}{R} = -16\pi^2\,G\,\rho^2\,R^5 \ . \qquad \text{(V-3)}$$

c) For a constant density ρ and temperature T in the cloud, $E_{\rm th}$ increases with radius R like R^3, while $E_{\rm g} \propto R^5$, a much stronger dependence on R.

3. Using Eqs. (V-2) and (V-3) in Eq. (V-1) gives

$$R_J = \left(\frac{k_B}{12\pi\,G\,\mu\,m_H}\right)^{1/2} \frac{T^{1/2}}{\rho^{1/2}} \ , \qquad \text{(V-4)}$$

which is the **Jeans' Length**. If the radius of a gas cloud is $R > R_J$ (keeping ρ and T constant), the cloud becomes unstable and will collapse.

4. Defining $M_J \equiv \frac{4\pi}{3}\,\rho\,R_J^3$ as the **Jeans' Mass** gives

$$M_J = \frac{4\pi}{3}\left(\frac{k}{12\pi\,G\,\mu\,m_H}\right)^{3/2} \frac{T^{3/2}}{\rho^{1/2}} \qquad \text{(V-5)}$$

and if $M > M_J$ for a gas cloud, the cloud will collapse due to gravitational instabilities.

5. One should note that this **Jeans' instability criterion** can also be deduced from the virial theorem (see Eq. IV-39) as done in your textbook on pages 447-449. In this case, collapse will take place when

$$\text{KE} < \frac{1}{2}\,\text{PE} . \tag{V-6}$$

6. During this stage of star formation, the gas cloud is in *free fall.*

a) Throughout this free-fall stage, the temperature of the gas stays relatively constant $\Longrightarrow$ the collapse is said to be *isothermal.*

b) Equating Newton's 2nd Law of Motion to his Law of Gravitation, your textbook authors show (on pages 449-451) that the **free-fall time** is

$$t_{\rm ff} = \left(\frac{3\pi}{32}\,\frac{1}{G\rho_\circ}\right)^{1/2} , \tag{V-7}$$

where $\rho_\circ$ is the initial mass density of the cloud.

c) Since this free-fall time depends only on density, all parts of the cloud will collapse at the same as long as the cloud has uniform density $\Longrightarrow$ **homologous collapse**.

7. A GMC will not necessarily collapse as a single unit, typically just a portion of it will collapse based upon the triggering mechanism (see below).

a) The density of the collapsing cloud will increase by orders of magnitude during the free-fall time.

b) Though we have developed the collapse criteria under the assumption of constant density, in reality, there will be pockets of inhomogeneities in the cloud.

c) As a result, sections of the cloud will independently satisfy the Jeans' mass limit and begin to collapse locally, producing smaller features within the original cloud.

d) This cascading collapse could lead to the formation of large numbers of smaller objects.

C. Triggers of Star Formation (SF).

1. Observations (see Figure V-2).

a) In the Milky Way Galaxy, SF occurs in GMCs.

b) OB stars form in associations at edges of GMCs.

c) OB association ionizes the surrounding gas producing an **H II region**.

d) Lower mass (T Tauri-type) stars form throughout the volume of the GMC.

e) SF defines the optical spiral arms of the Milky Way.

2. What is the trigger? Any process that can cause a stable ($M < M_J$) cloudlet to become unstable ($M > M_J$).

a) **Agglomeration**: Component cloudlets of GMC's collide and sometime coallace until $M > M_J$.

b) **Shock Wave Compression**: A shock can be the trigger $\Longrightarrow$ it acts like a *snow plow* causing ρ to increase, and as a result, M_J drops (see Figure V-3).

Figure V–2: Model of a Stellar Nursery.

i) **Spiral Density Wave**: As Milky Way Galaxy rotates, its two spiral arms can compress a GMC, which then leads to star formation.

ii) **Ionization Front**: O & B stars form very quickly once cloud collapse has started (see below). These produce H II regions from their strong ionizing UV flux, which initially expand outward away from the OB association. This ionization front heats the gas causing a shock to form. The shock can compress the gas such that $M > M_J$, which once again, leads to star formation.

iii) **Supernova Shocks**: O & B stars evolve very quickly on the main sequence and die explosively as supernovae. The shock sent out by such a supernova can excite further star formation.

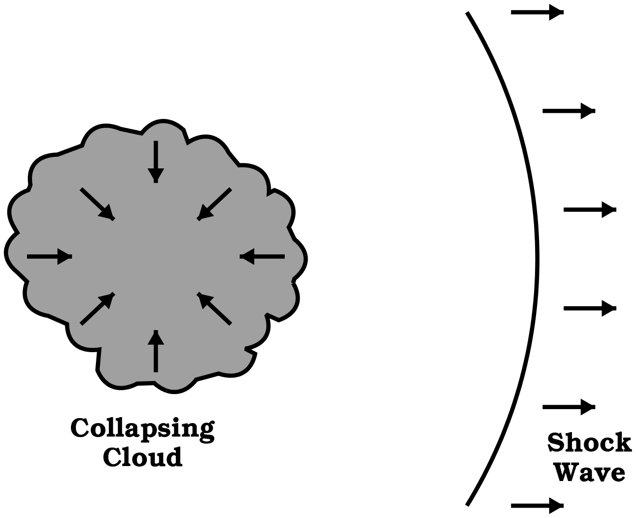

Figure V–3: Shock wave induced collapse.

D. The Free-Fall Stage of Stellar Birth.

1. As a portion of a GMC begins to contract, cloud complexes with masses greater than $\sim 50\ M_\odot$ become unstable and fragment into smaller cloudlets (see Figure V-4). Each little cloudlet continues to collapse as described above.

2. As a large portion of the GMC collapses, many internal eddies and turbulent motions can exist within the cloud. As a result, when fragmentation to stellar-mass sizes occur, each little cloudlet has a rotation associated with it that was induced from one of these eddies as shown in Figure V-5.

3. As the cloudlet contracts, it spins faster due to the conservation of angular momentum $L = M\,R^2\,\omega$, where ω is the angular velocity of the protostellar *cloudlet* $\Longrightarrow$ since M is constant, as R gets smaller, ω gets larger. This increased spin causes the equatorial region to bulge outward which flattens the cloudlet. This continues until a central bulge with an equatorial disk forms as shown in Figure V-6.

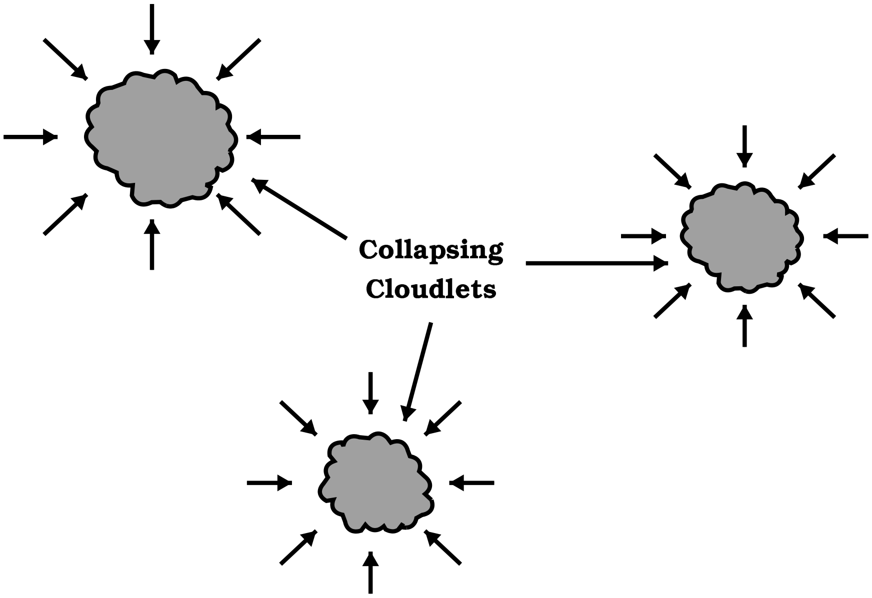

Figure V–4: Collapsing cloud fragmentation to smaller collapsing “cloudlets.”

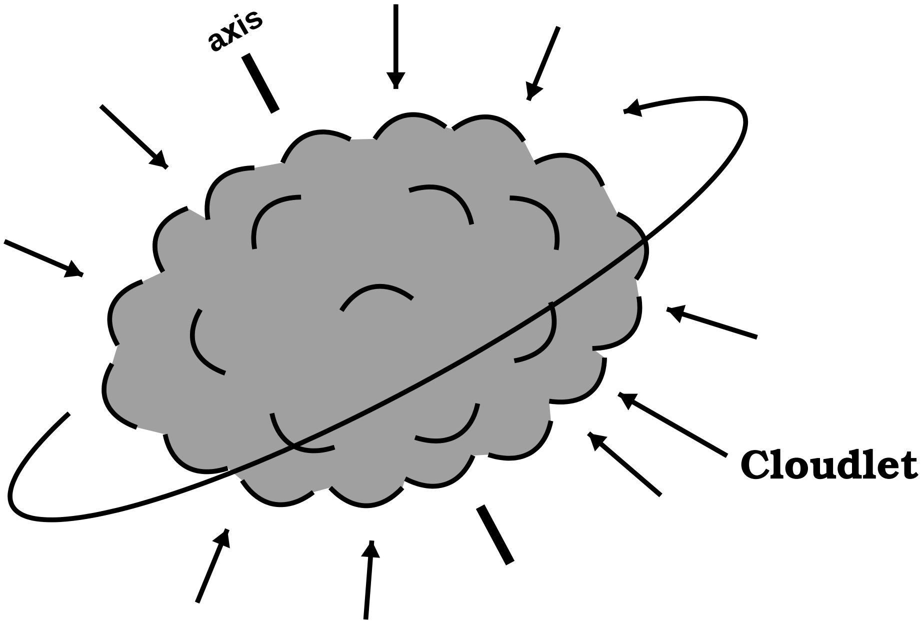

Figure V–5: Cloudlet contraction and spin as a result of internal eddies.

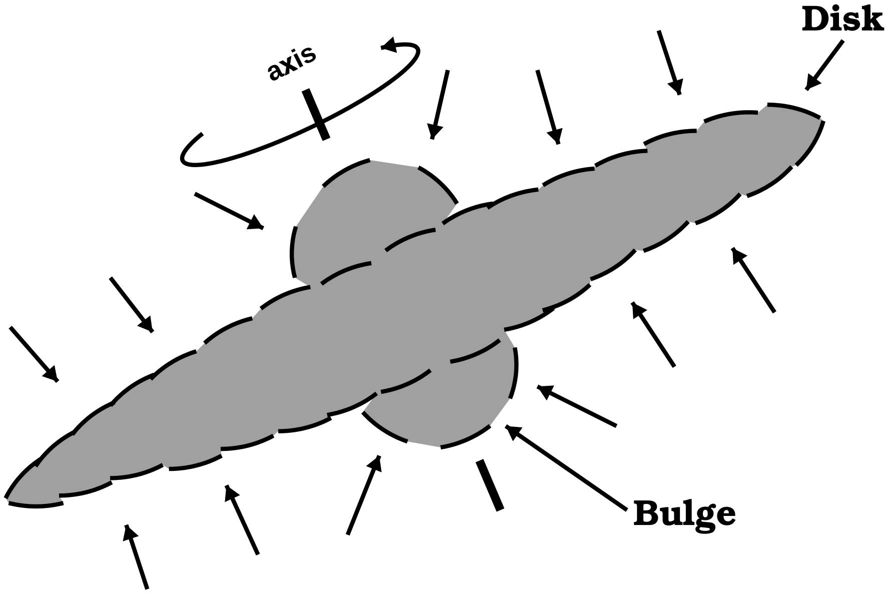

Figure V–6: Formation of protostar and protoplanetary disk.

a) The equatorial disk flattens rather quickly (hundreds of years) during this stage. At this point, the disk is now referred to as a **proplyd** or **protoplanetary disk**.

b) In this protoplanetary disk, dust grains begin to stick together from condensation and accretion, building in size to form planetesimals. These planetesimals conglomerate further into protoplanets (see Figure V-7).

  i) In the outer protoplanetary disk, ice crystals condense out of the gas along with some dust grains.

  ii) In the inner protoplanetary disk, it is too hot for ice crystals to form, only dust condenses out.

  iii) The dust (and ice) begin to conglomerate together in a process known as **advection** (similar to building a snowman), building bigger and bigger particles. This processed continued until boulder to mountain sized objects existed $\Longrightarrow$ the **planetesimals**.

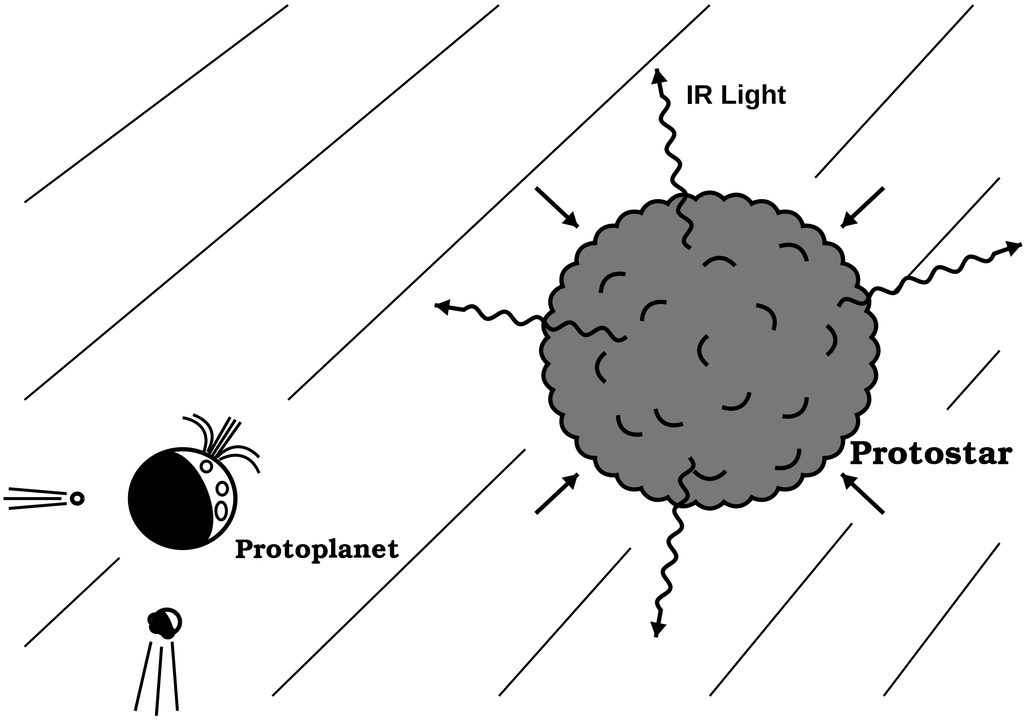

Figure V–7: Inside the protoplanetary disk of the collapsing cloudlet.

iv) Planetesimal is the name given to big rocks in orbit about a "protostar." Due to their small size ($D < 1000$ km), they are not spherical in shape.

- The "rocky" planetesimals are called **asteroids**.
- The "icy" planetesimals are called **comets**.

v) Planetesimals occasionally smash into each other, sometimes destroying each other, sometimes sticking together to form even bigger planetesimals $\Longrightarrow$ this is a process known as **accretion**.

c) Numerous such disks have been seen in stellar nurseries with the *Hubble Space Telescope*. They are especially easy to see at IR wavelengths.

d) Unfortunately, we do not have time to elaborate on the formation of planets in this course.

4. During this time, the cloudlet is in *free-fall* — the so-called **free-fall stage**. Its luminosity continues to decrease as the cloud collapses, while temperature increases slightly. This stage ends when the protostar becomes opaque to it own radiation (primarily IR photons). This happen when the cloudlet's central bulge is about the size of Mercury's orbit (see Figure V-8).

   a) The free-fall time depends upon the density that the cloudlet had at initial formation due to fragmentation as shown in Eq. (V-7).

      i) GMC temperatures are on the order of 50 K. Though the average density of a GMC is on the order of 10^3 cm^{-3}, when cloudlet fragmentation occurs, the particle density is on the order of 10^8 cm^{-3} which corresponds to a mass density of 2×10^{-16} gm cm^{-3}.

      ii) For such initial conditions, the free-fall time to an opaque state is about 4700 years. Increasing the initial density by a factor of 10 would reduce the free-fall time to a mere 1500 years — a blink of an eye to the age of the Universe $\Longrightarrow$ cloudlets collapse rapidly to the protostar stage!

   b) The higher-mass cloudlets have higher densities due to the higher gravitational force associated with them. As such, the more massive the cloudlet, the faster it will collapse.

5. Though there is a slight increase in temperature, this increase is very small, and as such, this free-fall stage is said to correspond to an **isothermal collapse** of the cloudlet.

Figure V–8: Theoretical time dependence of the radius of a 1 $M_\odot$ protostar (from Shklovskii 1978).

E. The Hayashi Stage of Stellar Birth — The Protostars.

1. The contraction now continues at a much slower rate once the gas becomes opaque to IR photons.
    - a) This marks the beginning of the **Hayashi stage**. At this point, the collapsing cloudlet is now called a **protostar**.
    - b) During this stage, the gas becomes unstable to convection, and as a result, the protostar's energy is transported via convection. This efficient energy transport mechanism increases the luminosity of the protostar considerably while the protostar maintains a relatively constant radius.
    - c) Temperatures begin to rise at a much more rapid rate as more material falls onto the protostar (though still not as much as later epochs).

- **d)** However, since the gas is opaque to IR radiation (where $\lambda_{\rm max}$ is located), the internal energy is unable to escape. At this point, the collapse is said to proceed **adiabatically**.

- **e)** During this stage of stellar birth, the rate of evolution becomes controlled by the rate at which the protostar can thermally adjust to the collapse $\Longrightarrow$ this is just the **Kelvin-Helmholtz time** given in Eq. (IV-41).

- **f)** For a 1 $M_\odot$ protostar, this **quasi-static collapse** takes about 10^7 years to complete until the next stage of stellar birth.

2. The evolution of the protostar has a variety of *key events* that take place which depend upon the mass of the protostar. These "events" were first described by Iben (1965, *Astrophysical Journal*, **141**, 993). Figure 12.8 in your textbook (Carroll and Ostlie) mark the location of these events on an H-R Diagram for various stellar masses. Here, we list the key events for a **1 $\boldsymbol{M_\odot}$ protostar**. Later, we will describe the evolution of a low-mass and then a high-mass protostar.

- **a)** With the steadying increasing effective temperature of the protostar, metals start to ionize which liberates *free* electrons. Some of these electron can interact with neutral hydrogen to produce the $\rm H^-$ ion.

- **b)** Opacity in the outer layers becomes dominated by the $\rm H^-$ bf- and ff-opacities.

- **c)** This increased opacity causes the convection instability criterion to be met (see Eq. IV-58) and convection begins. This convection envelope becomes very deep since the temperatures are not yet high enough down deep to ionize hydrogen (hence reduce the H^- opacity).

d) When deep convection sets in, luminosity decreases sharply with effective temperature increasing only slightly (see points '1' and '2' on Fig. 12.8 in your textbook and points 'A' and 'B' in Figure V-11).

e) At point '1' on Fig. 12.8, temperatures at the center of the protostar are such that *deuterium burning* commences (*i.e.*, the second step of the PP I chain).

  i) The deuterium (^{2}H) that burns here is deuterium created during the Big Bang of the formation of the Universe (more to come).

  ii) This reaction is favored over the first step of the PP I chain because it has a fairly large cross-section at low temperatures.

  iii) Since ^{2}H is not very abundant, these nuclear reactions have little effect on the overall collapse — they simply slow the rate of contraction slightly.

  iv) At this point, the energy production rate is a combination of a nuclear term and a gravitational term:

$$\varepsilon = \varepsilon_{\rm nuc} + \varepsilon_{\rm grav} \ . \qquad \text{(V-8)}$$

f) As the central temperature continues to rise, increasing levels of ionization of H decrease the total opacity in that region (the cross section of H^- opacity is many orders of magnitude bigger than electron scattering and H bf- and H II ff-opacities).

  i) At this point, a radiative core develops and progressively encompasses more and more of the star's mass.

  ii) By point '3' of Fig. 12.8 ('C' of Fig. V-11), the radiative core allows energy to escape into the convective envelope more readily, causing the luminosity of the star to increase again.

g) The path that a star takes on the H-R Diagram from the initial formation of the surface convection zone when luminosity drops by an order of magnitude to the point where luminosity begins to rise again is referred to as the **Hayashi track**.

h) At about the time that the luminosity begins to increase again, the central temperatures are now high enough for the first two steps of the PP I reaction chain and the steps from ^{12}C to ^{14}N in the CNO cycle to begin in earnest, but not yet at their equilibrium rates. This step is designated by point '4' in Fig. 12.8 ('D' in Fig. V-11). As time progresses, $\varepsilon_{\rm grav}$ has a smaller and smaller impact on L as compared to $\varepsilon_{\rm nuc}$.

i) During the time interval from points '2' to '4', the surface of the protostar becomes hot enough to emit a significant amount of its energy flux as visible light photons (still being powered by gravitational contraction).

  i) The pressure from this light starts to push dust sized particles out of the protoplanetary disk via radiation pressure.

ii) All that remains in the protoplanetary disk are larger planetesimals (rocky ones close in — the 'asteroids,' and icy ones farther out — the 'comets') and the protoplanets.

iii) This *spring-cleaning* phase is called the **T-Tauri stage** of the protostar evolution, named after the prototype of this class, T Tauri (see Figure V-9).

iv) This "spring cleaning" phase can continue well into the epoch when thermonuclear reactions start up in earnest, the so-called main sequence phase.

v) Since so much material exists in the disk, the momentum exchange between the photons and matter is not efficient. However at the poles, the density is much less which results in a strong **bipolar outflow** $\Longrightarrow$ bipolar jets form.

vi) These jets can interact with the surrounding material causing higher density clumps to form. Such clumps seen in a T Tauri star's jets are call **Herbig-Haro objects**.

j) At point '5' in Fig. 12.8 ('E' in Fig. V-11), the rate of nuclear energy production has become so great that the central core is forced to expand somewhat, causing the gravitational energy term in Eq. (V-8) to become negative. This effect is apparent at the surface as the total luminosity decreases towards the main sequence value. This decrease in L forces T to decrease as well.

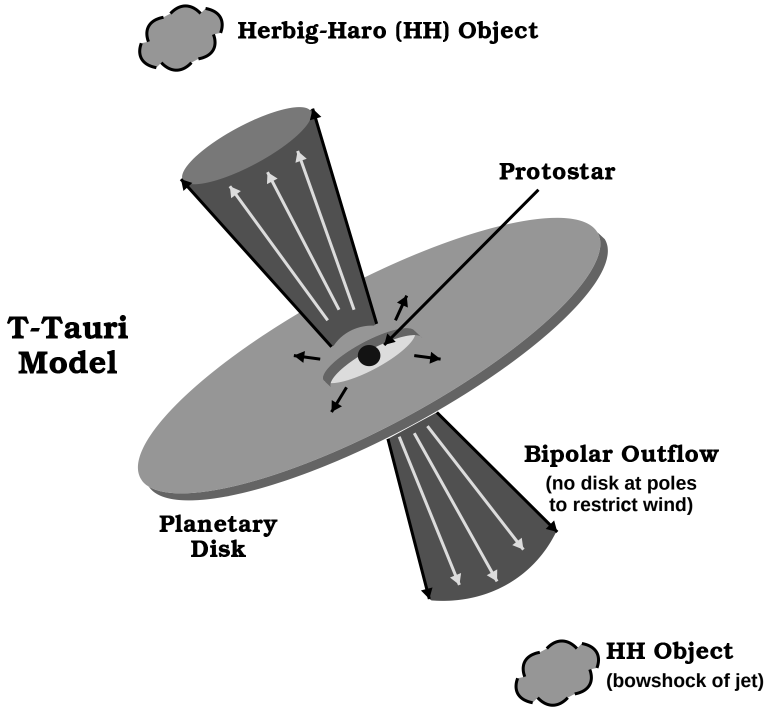

Figure V–9: Model of a T Tauri star. Gas flows easier at the poles than in the disk which results in a bipolar outflow.

k) At point ‘6’ in Fig. 12.8 (‘F’ in Fig. V-11), ^{12}C is completely exhausted from the CN reactions of the CNO cycle which reduces the energy production, the core adjusts by contracting which increases the temperature until the point when the 3rd step of the PP I can start occurring in earnest.

l) At this point, the temperatures become high enough to allow the last stages of the CNO cycle to proceed (though not at a very high rate), which replenishes the core with ^{12}C.

m) When this occurs $\varepsilon_{\rm nuc} \gg \varepsilon_{\rm grav}$, core contraction stops and the PP I chain achieves thermal equilibrium with the gas. At this point, **A STAR IS BORN**, which is marked by point ‘7’ on Fig. 12.8 in the textbook (‘G’ in Fig. V-11).

Figure V–10: Variation of the central density and temperature in a collapsing protostellar cloud of 1 $M_\odot$ and the different physical processes taking place in it.

n) The time involved for each of these steps in these detailed calculations to take place (as tabulate in Table 12.1 in the textbook) is not very different than the Kelvin-Helmholtz time scale (see Eq. IV-41).

o) Figures V-10 and V-11 summarizes the formation of a 1 $M_\odot$ star (*i.e.*, the Sun) as described in the notes above.

3. Protostar Evolution of a **0.4 $M_\odot$** (*i.e.*, Low Mass) **Star**.

a) There are a few major differences between the evolution of a low-mass protostar ($\sim 0.4\ M_\odot$) to that of a solar mass protostar:

b) Due to the lower temperatures in the core of these protostars (as compared to a 1 $M_\odot$ protostar), opacity stays relatively high in the core which prevents a radiative core from developing.

Figure V–11: A schematic description of the track of a collapsing 1 $M_\odot$ protostar in the H-R Diagram.

c) The protostar (and later star) stays completely convective throughout.

d) The core never gets hot enough to start the CN portion of the CNO cyle.

e) As a result of these events, the luminosity never turns around and starts to increase prior to nuclear reactions being in earnest $\Longrightarrow$ the protostar "falls" directly onto the main sequence.

f) Should a protostar's mass be $M < 0.08 M_\odot$, the central temperature never gets hot enough for the PP chain to get started (though deuterium burning can take place for a brief period of time). Collapse continues until electron degeneracy sets in (in about 100 billion years!). Such objects are called **brown dwarfs**.

4. Protostar Evolution of a **15 $M_\odot$** (*i.e.*, High Mass) **Star**.

   a) Because high mass stars ($M \gtrsim 5M_\odot$) reach high enough temperatures to ignite the first steps of the PP chain and the CN portion of the CNO cycle so quickly, their Hayashi tracks are very short.

   b) These stars leave the Hayashi track at a much higher luminosity than their solar-type cousins and retain a relatively constant high luminosity as the protostar collapses due to the rapid increase in central and effective temperatures — this can easily be shown with the blackbody radiation relation:

   $$\frac{L}{L_\odot} = \left(\frac{R}{R_\odot}\right)^2 \left(\frac{T}{T_\odot}\right)^4 , \qquad \text{(V-9)}$$

   the effective temperature increases by approximately a factor of 10 as the radius of the protostar decreases by a factor of 100 after the protostar leaves the Hayashi track.

   c) Once the thermonuclear reactions reach their thermal equilibrium rates, the high temperature of the core allows the CNO cycle to completely dominate the PP chain as the primary source of energy.

   d) Due to the strong temperature dependence of the CNO cycle, convective instability remains in effect in the core and the energy is transported outward via convection.

   e) In the envelope, temperatures are high enough to keep H (and He) ionized all of the way to the photosphere which prevent H^- formation from occurring. Due to the lower opacities in the envelope, convective instability does not take place and the envelope becomes radiative from the Hayashi track all of the way through to thermonuclear ignition (*i.e.*, main sequence).

5. In the outer layers of the most massive stars, the radiation pressure from the photon flow from the center of the star is enormous. The **Eddington limit**, $L_{\rm Ed}$, is defined as the maximum luminosity a star can have without blowing itself apart from radiation pressure.

   a) From the radiative transfer equation, the pressure gradient near the surface of a star can be written as

   $$\frac{dP}{dr} \simeq -\frac{k\rho}{c}\,\frac{L}{4\pi r^2} \; .$$

   b) HSE demands that

   $$\frac{dP}{dr} = -\frac{GM\rho}{r^2} \; .$$

   c) Combining these two equation and solving for the luminosity, we get

   $$L_{\rm Ed} = \frac{4\pi Gc}{k}\,M \; , \qquad \text{(V-10)}$$

   where this Eddington luminosity is the maximum luminosity a star can have and still satisfy HSE.

   d) For massive stars, the effective temperatures are 40,000 K or greater which is enough to ionize hydrogen in their photospheres, and as such, electron scattering is the dominant opacity (*i.e.*, k) source. Making use of this and expressing the Eddington luminosity in terms of solar luminosities, we can write

   $$\frac{L_{\rm Ed}}{L_\odot} \simeq 3.8 \times 10^4\,\frac{M}{M_\odot} \; . \qquad \text{(V-11)}$$

e) Eddington also showed that a gas sphere in radiative equilibrium will follow the following mass-luminosity relationship (see next section):

$$\frac{L}{L_\odot} \simeq \left(\frac{M}{M_\odot}\right)^{3.5} \quad . \tag{V-12}$$

f) Equating this radiative-equilibrium luminosity with the Eddington limit gives us the maximum mass a star can have without blowing itself apart from radiation pressure:

$$M \simeq 70\, M_\odot \ . \tag{V-13}$$

g) Since gravitational instabilities tend to prevent stars from forming at such a high mass, there should not be too many stars that exceed the Eddington limit. The Wolf-Rayet stars, however, come very close to this Eddington limit (which is why they have such strong stellar winds — see Table II-1 of these course notes).