ASTR-3415: Astrophysics
Course Lecture Notes
Section VI

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Abstract

These class notes are designed for use of the instructor and students of the course ASTR-3415: Astrophysics. This is the Version 1.2 edition of these notes.

VI. Stellar Evolution: Main Sequence and Post-Main Sequence Evolution

A. Life on the Main Sequence.

1. Main sequence (MS) stars are also called *hydrogen-core burners*. Approximately 90% of a star's nuclear burning lifetime is spent in this stage (hence 90% of all stars fall on the main sequence on the HR diagram). When a star first ignites hydrogen in its core, it resides on the **zero-age main sequence** (ZAMS).

2. This *main sequence lifetime* corresponds to the **nuclear time scale**:

$$
\begin{aligned}
t_{\text{nuc}} &\equiv \frac{\text{Energy available from nuclear fuel}}{\text{Rate at which fuel is used up}} \\
&= \frac{\eta\, f\, X_{\text{init}}\, M\, c^2}{L}, \qquad \text{(VI-1)}
\end{aligned}
$$

where $\eta = 0.0071$ is the 4H→He reaction efficiency introduced in §IV.C.3.b, X_{init} is the initial mass fraction of H, and f is the fraction of H actually burned on the MS.

a) So $t_{\text{MS}} = t_{\text{nuc}}$ is the amount of time the star remains on the MS.

b) $f \approx 0.1$ for $1 M_{\odot}$ stars, which gives

$$t_{\text{MS}} \approx 10^{10}\ \text{yr}\ \frac{M/M_{\odot}}{L/L_{\odot}}. \qquad \text{(VI-2)}$$

c) f increases up to 0.3 for upper main sequence stars.

3. The **mass-luminosity relation** for stars on the main sequence.

a) As shown in Eq. (V-12), Eddington showed in 1924, using a radiative diffusion approximation, that a spherical sphere of ideal gas which is in radiative equilibrium should follow a mass-luminosity relationship of the form:

$$\frac{L}{L_\odot} = \left(\frac{M}{M_\odot}\right)^{3.5} , \qquad \text{(VI-3)}$$

which means that more massive stars are much more luminous than lower mass MS stars.

b) Later, based on main sequence stars in binary star systems in the solar neighborhood, a mass-luminosity relationship was empirically found for main sequence stars of

$$\frac{L}{L_\odot} \approx \left(\frac{M}{M_\odot}\right)^{4} \quad \text{for } L > L_\odot \qquad \text{(VI-4)}$$

$$\frac{L}{L_\odot} \approx \left(\frac{M}{M_\odot}\right)^{2.8} \quad \text{for } L \leq L_\odot \; . \qquad \text{(VI-5)}$$

4. Using the Eddington M-L relationship, the MS lifetime then can be written as

$$t_{\rm MS} \approx \left(\frac{M}{M_\odot}\right)^{-2.5} \times 10^{10} \text{ yr} \; . \qquad \text{(VI-6)}$$

5. The energy transport mechanism that dominates the interior of a star changes along the MS as was described in the last section on stellar formation.

a) **High-mass stars** ($M \gtrsim 1.3\, M_\odot$, which correspond to stars earlier than F5 V).

i) As mentioned in §IV.D of the course notes, for stars more massive than 1.2–1.3 $M_\odot$, the CNO cycle dominates over the PP chain in the main sequence star's energy production.

ii) Since the CNO cycle has such a large temperature dependence (anywhere from T^{13} to T^{20}) in comparison to the PP chain, temperature gradients are very large in the cores of these stars.

iii) Due to this large temperature gradient, convective instability is established and energy in the core is transported outward by convection over radiation transport $\Longrightarrow$ hence these stars have **convective cores**.

iv) The temperatures in the outer envelope keep most of the hydrogen completely ionized all of the way to the photosphere of these stars. As such, the dominant opacities in the envelope of these stars is electron scattering, H I bf, and H II ff.

v) The cross sections of these opacity sources is small in comparison to H^- bf and H^- ff. As such, photons are able to flow outward much easier in the envelope of these stars with respect to low mass stars.

vi) As a result, these stars have **radiative envelopes**.

b) **Low mass stars** ($\sim 0.4\,M_\odot < M < \ \sim 1.3\,M_\odot$) stars.

i) The PP chain is the dominant energy production source in these stars. With its lower temperature dependence (anywhere from $T^{3.5}$ to T^6) the temperature gradient in the core of these stars is not high enough to cause convective instability $\Longrightarrow$ energy is transported via radiation.

- **ii)** As a result, these stars have **radiative cores**.

- **iii)** The outermost regions of the envelopes of these stars have temperatures low enough for neutral hydrogen to exist in sufficient quantities to allow large H^- bf and ff opacities (due to there large cross sections).

- **iv)** These large opacity sources are very efficient in blocking photon flow which sets up convective instabilities.

- **v)** As a result, these stars have **convective envelopes**.

- **vi)** As we go to cooler and cooler main sequence stars, the convective envelope gets deeper and deeper due to the ability of H^- to form from the underionization of hydrogen.

c) **Very low mass stars** ($M \lesssim 0.4\, M_\odot$, which corresponds to M4-M5 V stars) are completely convective due to the fact that sufficient H^- opacity exists throughout much of the envelope to effectively block much of the energy trying to escape from below. The core still stays convective since the PP chain dominates in these stars too.

d) Figure VI-1 shows a cartoon on how the interior structure of main sequence stars change with mass.

6. One thing that is rather interesting is that the appearance of chromospheres in main sequence stars appears in the mid- to

Figure VI–1: Convective (*curved arrows*) zones and radiative (*straight arrows*) zones in main sequence stars as a function of mass.

early-F spectral classes (*i.e.*, stars earlier (hotter) than F1-F5 do not show evidence of chromospheres and coronae, while those later (cooler) than these spectral classes do show evidence for these atmospheric layers — see Figure III-10). The onset of surface convection produces chromospheric and coronal heating in the outer atmospheres of these stars.

7. Stellar Masses.

- **a)** The high mass limit on the main sequence is about 60 $M_\odot$ (O5 V).
  - **i)** Your textbook claims that this limit is about 90 $M_\odot$ since for masses larger than this, thermal oscillations in their cores lead to dramatic variations in nuclear energy production rates which prohibits the formation of stable stars.

ii) However, gravitational fragmentation typically does not produce protostars larger than about 50-60 $M_\odot$. As such, 90 $M_\odot$ stars typically will not form in the first place.

iii) One final thing can occur that prevents us from seeing high mass stars — collapse is so rapid from protostar state that they either:

- Collapse to a black hole before there is even a chance to ignite H.

- Should these objects ever make it to the main sequence, they supernova within a few 100 to a thousand years!

b) The low mass limit is about $0.08\, M_\odot$ (M8 – M10 V).

i) Temperatures never get high enough to ignite H at an equilibrium rate to produce He.

ii) Objects with $M < 0.08\, M_\odot$ are called **brown dwarfs**.

iii) Objects with $M < 10\, M_{\rm Jupiter} (= 0.01\, M_\odot)$ are called planets.

B. Fundamental Time Scales on which Stars Evolve.

1. There are 3 basic time scales that are important to stars during their evolution:

NAME	CONDITIONS	SOLAR
dynamic time	non-HSE	∼1/2 hour
thermal (K-H) time	HSE, non-TE	∼30 million years
nuclear time	HSE, TE	∼10 billion years

2. While on the MS, a star's luminosity will slowly increase and temperature slowly decrease. This gives the main sequence a *thickness* when viewed on the HR diagram.

- **a)** As such, stars do not stay at their ZAMS position for their entire H-core burning life.
- **b)** As the hydrogen is fused into helium, μ (the mean molecular weight) in the core increases (see Eq. IV-22).
  - **i)** This will cause the pressure to drop (assuming ideal gas, see Eq. IV-23).
  - **ii)** Following the dynamic time scale, the core compresses as a result which increases the density and temperature.
  - **iii)** This in turn increases the thermonuclear reaction rates which increases the surface luminosity.
- **c)** For stars of 1.2-1.3 $M_\odot$ or less, this increased luminosity, increases the surface temperature of the star. These stars get brighter and hotter during their main sequence lifetime (see Fig. 13.1 in your textbook).
- **d)** For higher mass stars where H is ionized throughout the entire interior, this added flux increases the photon momentum on the free electrons in the outer layers, which expands these layers and results in a cooling in the photosphere. These stars get brighter and cooler while on the main sequence (see Fig. 13.1 in your textbook).
- **e)** Note that these changes are extremely gradual, and as such, stars on the main sequence can be considered to be in hydrostatic and thermal equilibrium.

Figure VI–2: Evolutionary track for stars on the main sequence.

f) Figure VI-2 shows how the main sequence thickens as a result of this gradual change in thermonuclear reaction rates based on the data from Fig. 13.1 in the textbook. Table VI-1 lists specific parameters for stars on the MS.

3. Once the nuclear time scale has been exceeded, hydrogen in the region of thermonuclear reactions (*i.e.*, the core) becomes exhausted. This core goes out of TE and HSE and begins to collapse.

C. Ascent on the Red Giant Branch — Marching Towards Helium Ignition.

1. For this section, we will investigate stellar evolution from main sequence up until the ignition of helium in the core for stars in three mass classifications. **Very Low-Mass Stars** ($M \lesssim 0.4 M_\odot$):

Table VI–1: Stellar Parameters for ZAMS Stars.

Spectral Class	$M/M_\odot$	$R/R_\odot$	$L/L_\odot$	$T_{\rm eff}$ **(K)**	$t_{\rm MS}$ **(years)**
O5	60	12	7.9×10^5	44,500	7.6×10^5
B0	17.5	7.4	5.2×10^4	30,000	3.4×10^6
A0	2.9	2.4	54	9520	5.4×10^8
F0	1.6	1.5	6.5	7200	2.5×10^9
G0	1.05	1.1	1.5	6030	7.0×10^9
K0	0.79	0.85	0.42	5250	1.9×10^{10}
M0	0.51	0.61	7.7×10^{-2}	3850	6.6×10^{10}
M5	0.21	0.27	1.1×10^{-2}	3240	1.9×10^{12}

a) These stars are completely convective while on the main sequence. As such, since H is continuously being replenish into the core from the outer envelop, these stars have $f = 1$ in Eq. (VI-1) and burn H in their cores until the star is completely exhausted of H.

b) Once the PP chain runs out of fuel, the He-rich star goes out of HSE and beings to collapse.

c) As the star gets smaller, is gets hotter, ionizing much of the interior of the star.

d) The star collapses until the free electrons start to "feel" the Pauli Exclusion Principle — two electrons cannot exist in the same quantum state in the same place at the same time (see §IV.B.4.d).

i) Once this occurs, the equation of state is no described by the ideal gas law.

ii) Instead, *degenerate electron pressure* describes the equation of state:

$$P_e = K\,\rho^{5/3}\ , \qquad \text{(VI-7)}$$

where K is a constant $\Longrightarrow$ pressure is a function of density alone and is independent of temperature.

iii) This degenerate electron pressure is strong and causes the gravitational contraction to cease.

iv) The star will exist from this point forward until the end of time as a **helium-rich white dwarf** (see §VIII.A for further details).

2. Low-Mass Stars ($0.4 M_\odot \lesssim M \lesssim 4 M_\odot$):

a) Once again, the Carroll and Ostlie textbook goes into far greater detail of the post-main sequence evolution of these stars than these notes. Here we just mention a few key points about the evolution of these stars from H-core exhaustion to He-core ignition.

b) Note that the upper limit of this mass regime is a bit uncertain and is very sensitive to the metal abundance of a given star. Some authors put the upper limit for this regime at around 3 $M_\odot$.

c) Once the hydrogen abundance (X) drop below ~1-2%, the PP chain/CNO cycle can not longer sustain itself. The core is now dominated by helium "ash," however, because the core is still hot, the second law of thermodynamics dictates that this heat energy has to flow outward to cooler regions.

d) Due to this, the core goes out of TE and HSE and begins to contract. This contraction causes the density, pressure, and temperature to increase in the core.

e) This contraction heats the core (though initially not enough to start helium fusion) and also heats the H-rich shell just above it.

  i) As soon as $T \gtrsim 10^7$ K, H fusion starts in that shell.

  ii) The star is now a *H-shell burner*.

  iii) Due to the high opacities inside the star, it takes a while for this core-collapse, H-shell burning information to get to the outer envelope of the star.

f) This H-shell burning increases the energy input into the outer envelope.

  i) However, most of the work from this added energy goes towards increasing the pressure in the outer envelope $\Longrightarrow$ causing the envelope to expand.

  ii) As it expands, the extreme outer layers of the star cool since the energy flux must be conserved and the surface area is getting larger. From the laws of blackbody radiation, as the surface cools, it gets redder and redder.

  iii) The star gets larger (and more luminous) and cooler producing an orange **subgiant** star. The F5 star *Procyon* is at this stage in its evolution (although Procyon, being a bit more massive than the Sun, is moving from the white region of the main sequence to the yellow region of the subgiant luminosity class).

- **iv)** The star is now said to be ascending the **red giant branch**.

g) While the outer envelope is expanding, the core continues to contract and heat. The H-burning shell is producing more helium ash which falls on the core, increasing the core's mass as a result.

- **i)** Helium fusion will not begin until $T > 10^8$ K (see §IV.D.5).

- **ii)** Prior to these temperatures being obtained in the core, electrons are forced closer and closer together until the gas becomes degenerate.

h) These star quickly experience a run-away thermonuclear event in their cores know as the "helium flash," or to be more specific, the **Helium-Core Flash**.

- **i)** The electron degeneracy pressure counterbalances gravity (as described above), hence resists compression due to gravity.

- **ii)** As shown in Eq. (VI-7), pressure (P) in a degenerate gas is independent of temperature (T).

- **iii)** The temperature of the helium atoms continues to rise as matter is dumped from the H-burning shell above — but pressure remains constant

  $\Longrightarrow$ the normal pressure-temperature-gravity thermostat is off since the ideal gas law is no longer in effect!

iv) T reaches 10^8 K so helium starts fusing which increases T more (without increasing P) $\Longrightarrow$ increase of T increases reaction rates, which further increases T!

$\Longrightarrow$ Resulting in runaway thermonuclear reactions!

$\Longrightarrow$ Hence, the **Helium-Core Flash**.

At this time, the core luminosity increases to $10^{11} L_\odot$, comparable to that of an entire galaxy in just a few seconds time! Most of this energy never reaches the surface however, instead it is absorbed by the gas in the envelope, possibly causing some mass to be lost as this time from the envelope.

v) Soon, T gets high enough to "lift" degeneracy $\Longrightarrow$ HSE is reinstated $\Longrightarrow$ the (now) red giant becomes stable and is called a **red giant clump star** (if a Population I star — see §IX) or a **horizontal branch star** (if a Population II star).

vi) The star now has two energy sources, H-shell burning (primarily from the CNO cycle) and He-core burning (via the triple-α process) and will remain in this state for the final 10% of its thermonuclear life.

3. **High-Mass Stars** ($M \gtrsim 4\ M_\odot$) follow the same evolutionary sequence as the Low-Mass Stars, except the core never becomes degenerate during collapse. Once temperatures get above 10^8 K, the triple-α process begins gradually until the rates obtain an equilibrium value — no runaway occurs in these stars, hence no He-core flash.