ASTR-3415: Astrophysics
Course Lecture Notes
Section IX

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Abstract

These class notes are designed for use of the instructor and students of the course ASTR-3415: Astrophysics. This is the Version 1.2 edition of these notes.

IX. The Milky Way Galaxy

A. Structure of the Milky Way.

1. William Herschel (1783-1822) was the first to seek to penetrate the structure of the galactic system.

   a) Counted the number of stars down to a limiting magnitude in various directions of the sky $\Longrightarrow$ he called it *star gauging*.

   b) If stars were uniformly distributed in space (*i.e.*, **homogeneous**) and this distribution was the same in all directions of the sky (*i.e.*, **isotropic**), then for stars of given absolute magnitude, the apparent luminosity decrease should scale as $1/r^2$ (*i.e.*, the inverse-square law of light).

   c) Stars brighter than m thus occupy a sphere having (refer to Equation II-2 — the distance modulus formula)

   $$\log r = 0.2m + \text{ constant.} \qquad \text{(IX-1)}$$

   d) Their number $N(m)$ will be proportional to the volume of space they reside in hence proportional to r^3, so

   $$\log N(m) = 0.6m + \text{ constant.} \qquad \text{(IX-2)}$$

2. Seares (1928) compared the number of stars per square degree to the above formula and found that there was a much slower increase of $N(m)$ for faint magnitudes than the formula predicted. Two reasons become immediately obvious:

   a) Decrease of star density at large distances.

   b) Interstellar absorption.

3. Stars are not all the same brightness however. Kapteyn (1851-1922) took account of the dispersion of the absolute magnitude of stars when determining the stellar distribution in the Galaxy. The star numbers $N(m)$ are represented by a superposition (*i.e.*, a convolution) of:

a) The **Density Function** $D(r)$ = number of stars per cubic parsec at distance r in a given direction.

b) The **Luminosity Function** $\Phi(M)$ = number of star per cubic parsec in the interval of absolute magnitude from $M - \frac{1}{2}$ to $M + \frac{1}{2}$ (see Figures 23.34, page 1046; 25.1, page 1104; and 25.2, page 1105 in the textbook for examples of luminosity function plots).

c) Taking account of **Interstellar Absorption** of $\gamma(r)$ magnitudes per parsec.

4. Besides understanding the distribution of stars in the Galaxy, we need to understand the motion of the stars. In our Galaxy, there are physically-preferred frames of reference.

a) **Galactocentric system**: Centered at the nucleus of the Galaxy, its reference plane is the galactic plane and reference axis is the galactic rotation axis.

b) **Local Standard of Rest (LSR) system**: Defined by determining the average motions of stars in the solar neighborhood. The center-of-mass <u>motion</u> defines this LSR.

i) The **dynamical LSR** is that reference frame, spatially centered upon the Sun, which moves in a *circular* orbit about the galactic center at a circular speed appropriate to its position in the Galaxy.

ii) The **kinematical LSR** is that reference frame, spatially centered upon the Sun, in which the *space velocities* of all the stars in the solar neighborhood *average* to zero.

5. All stars in the solar neighborhood which are in circular galactic orbits are essentially *at rest* in the dynamical LSR. Any deviations from circular motion, *in the solar neighborhood*, will appear as stellar **peculiar motions**.

6. If we include stars out to about 100 pc from the Sun, the dynamical and kinematic LSR are practically equivalent.

7. The **space velocity** of stars *with respect to the Sun* is composed of 2 components: the radial velocity and the tangential velocity.

a) The line-of-sight or **radial velocity**, v_r, is determined spectroscopically using the **Doppler Effect**:

$$z = \frac{\Delta\lambda}{\lambda_\circ} = \frac{\lambda_{\rm obs} - \lambda_\circ}{\lambda_\circ} = \frac{v_r}{c}, \qquad \text{(IX-3)}$$

where $\lambda_\circ$ is the rest (*i.e.*, lab) wavelength, $\lambda_{\rm obs}$ the observed wavelength (typically the observed wavelength is left 'unsubscripted,' λ — see pages VIII-14 and VIII-15 in the last section), z is called the redshift parameter, and c the speed of light.

b) $v_r < 0$ ($z < 0$) are called *blueshifts* and mean that the object is approaching us.

c) $v_r > 0$ ($z > 0$) are called *redshifts* and mean that the object is receding from us.

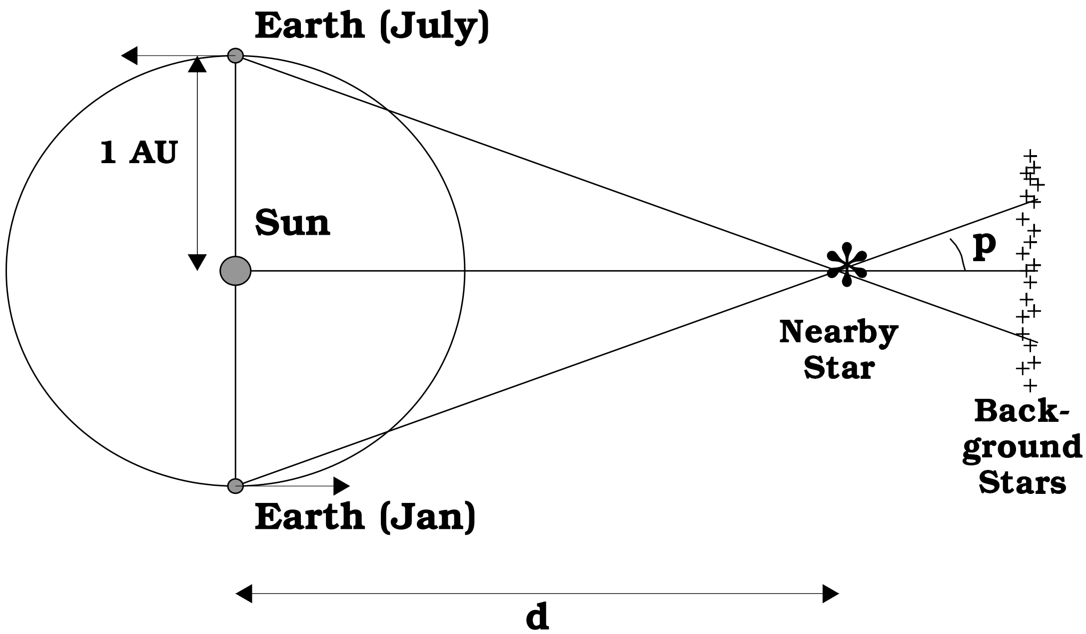

Figure IX–1: Trigonometric Stellar Parallax.

8. The **proper motion** μ (*i.e.*, positional change on the celestial sphere measured in arcsecs/yr) of a star results from its tangential velocity v_t.

a) The farther away a star, typically the smaller the proper motion.

b) The most accurate way to determine a stellar distance is through the use of **trigonometric stellar parallax** resulting from the Earth's orbital motion (see Figure XI-1).

i) Over half a years time, a star will exhibit a change in position p (measured in arcseconds) based upon its distance d (measured in parsecs).

ii) The position of the star in question is measured with respect to faint background stars, which are much more distant, hence do not shift much due to parallax effects.

$$d = 1/p. \qquad \text{(IX-4)}$$

iii) An object with a parallax angle of 1" would be at a distance of 1 pc (note 1 pc = 3.259 ly = 206,265 AU = 3.086×10^{18} cm).

iv) The nearest star to the Sun is Proxima Centauri, a red dwarf companion to the α Cen A & B binary components (*i.e.*, α Cen is a triple-star system), has a parallax of 0.776 arcsecs which corresponds to 1.29 pc (4.2 ly — note α Cen A & B are 4.3 ly distant).

c) The tangential velocity is then

$$v_t = 4.74\,\mu\,d = 4.74\,\mu/p \quad \text{(km/s)}, \qquad \text{(IX-5)}$$

where d is measured in parsecs, p in arcsecs, and μ in arcsecs/yr.

9. The space velocity $v_\star$ is then found by

$$v_\star = \sqrt{v_r^2 + v_t^2}. \qquad \text{(IX-6)}$$

The angle θ between the space velocity and the sight line is determined by

$$\tan\theta = v_t/v_r. \qquad \text{(IX-7)}$$

10. The Sun's galactic orbit is not perfectly circular. Therefore, with respect to the LSR, there is a **solar motion** of $v_\odot = 20 \pm 1$ km/s toward the **solar apex** in the direction RA: $18^h 4^m \pm 7^m$, DEC: $+30° \pm 1°$ [epoch 1950] (constellation of Hercules).

11. The stars in the solar neighborhood exhibit *peculiar motions* with respect to the LSR $\Longrightarrow$ they swarm about like sluggish bees.

12. The solar motion was found by *averaging* stellar peculiar motions *with respect to the Sun.* To find the peculiar motion of a given star *with respect to the LSR*, we must now subtract the reflected solar motion from the *observed* stellar peculiar motion components.

a) Of course we must also subtract off the Earth's rotation velocity and orbital velocity off of any observed stellar velocities. From this point forward, we will assume that this has already been taken into account.

b) A star's peculiar radial speed with respect to the LSR is

$$v_r(\star) = v_r(\text{obs}) + v_\odot \cos\lambda, \qquad \text{(IX-8)}$$

where λ is the angle on the celestial sphere between the solar apex and the star and $v_r(\text{obs})$ is the heliocentric radial speed.

c) The tangential component is a bit more difficult to ascertain. We must separate the tangential velocity into 2 components:

$$v_t^2 = v_u^2 + v_\tau^2, \qquad \text{(IX-9)}$$

where the upsilon component v_u in the plane that intersects with the celestial sphere that is defined by the solar apex and line-of-sight to the star, and the tau component v_τ is perpendicular to that plane.

d) A star's proper motion is then split into these two components such that

$$\mu^2 = \tau^2 + u^2, \qquad \text{(IX-10)}$$

where each of these proper motions are measured in arcsec/year.

e) The tau component is clearly unaffected by the solar motion, so

$$v_\tau(\star) = v_\tau(\text{obs}) = 4.74 \left(\frac{\tau(\text{obs})}{p}\right). \qquad \text{(IX-11)}$$

f) The upsilon component is influenced by a component of the solar motion:

$$v_u(\star) = v_u(\text{obs}) + v_\odot \sin\lambda = 4.74 \left(\frac{u(\text{obs})}{p}\right) + v_\odot \sin\lambda. \qquad \text{(IX-12)}$$

g) Hence we can express the proper motion components as

$$\tau(\star) = \tau(\text{obs}) \qquad \text{(IX-13)}$$

$$u(\star) = u(\text{obs}) \left(\frac{v_\odot \sin\lambda}{4.74}\right) p \qquad \text{(IX-14)}$$

h) The peculiar motion of a star relative to the LSR is then

$$v_\star^2 = v_r^2 + v_t^2 = v_r^2 + v_\tau^2 + v_u^2. \qquad \text{(IX-15)}$$

Note that stellar peculiar motions can only be determined if accurate parallax measurements are known for a star (from the ground, that's stars within 100 pc, with *Hipparchos*, out to 500 pc).

13. Peculiar Motion Parallaxes. Three statistical methods of determining stellar parallax (p) are evident from our discussion of peculiar motions above.

a) Inverting Equation (IX-12), we get

$$p = \left(\frac{4.74}{v_\odot \sin\lambda}\right) [u(\star) - u(\text{obs})]. \qquad \text{(IX-16)}$$

b) For a single star, Eq. (IX-16) tells us nothing since we do not know $u(\star)$. However, for a large number of stars in a particular region of the sky (λ), we may frequently assume that their $u(\star)$'s are *random*, so that they average (indicated by a *bar* on top) to zero:

$$\overline{p} = -\left(\frac{4.74}{v_\odot \sin\lambda}\right) \overline{u(\text{obs})}. \tag{IX-17}$$

c) If the region spans a significant range of angular distances λ from the solar apex, we must be more subtle. Let us rewrite Eq. (IX-16) as

$$u_i(\text{obs}) = u_i(\star) - \left(\frac{v_\odot}{4.74}\right) p_i \sin\lambda_i, \tag{IX-18}$$

for the ith star being considered. Multiplying through on both sides by $\sin\lambda_i$, and summing over all stars, we find

$$\overline{u(\text{obs})\sin\lambda} = -\left(\frac{v_\odot}{4.74}\right) \overline{p_i}\overline{(\sin^2\lambda_i)}, \tag{IX-19}$$

or

$$\boxed{\overline{p} = -\left(\frac{4.74}{v_\odot}\right)\left[\frac{\overline{u(\text{obs})\sin\lambda}}{\overline{\sin^2\lambda}}\right]} \; . \tag{IX-20}$$

d) Eq. (IX-20) is called the **mean parallax** [so is Eq. (IX-17)] and results only because of two extremely important facts:

i) $\overline{u(\star)\sin\lambda}$ vanishes since the random projections of $u(\star)$ are also random with a mean of zero.

ii) $\overline{p(\sin^2\lambda)} = \overline{p}\,\overline{(\sin^2\lambda)}$ since both p_i and $\sin^2\lambda_i$ are randomly uncorrelated.

e) We can also define the **secular parallax** caused by the solar motion through the LSR which occurs at about the rate 4.2 AU/yr:

$$H = \left(\frac{v_\odot}{4.74}\right) p \approx 4.2\, p. \qquad \text{(IX-21)}$$

14. In direct analogy to the mean parallaxes determined from the *upsilon* components, we may use the *tau* components — which are unaffected by solar motion — to find **statistical parallaxes**.

a) Assuming that our group of stars has random peculiar motions with respect to the LSR, we can write

$$\overline{|v_r(\star)|} = \overline{|v_\tau(\star)|} = \frac{4.74}{\overline{p}} \overline{|\tau(\text{obs})|}, \qquad \text{(IX-22)}$$

where the vertical bars denote absolute values.

b) The statistical parallax is thus

$$\boxed{\overline{p} = \frac{4.74}{\overline{|v_r(\star)|}} \overline{|\tau(\text{obs})|}} \qquad \text{(IX-23)}$$

and the $v_r(\star)$'s may be computed directly from Eq. (IX-8).

15. Table IX-1 summarizes this somewhat complicated discussion of parallaxes.

16. The *method of mean parallaxes* has been applied to determine the absolute magnitudes of A stars, RR Lyr stars, and Cepheid variables.

17. Note that in the *method of statistical parallaxes*, we must choose stars at approximately the same distance for both the radial and proper motions; otherwise, the proper motion sample will be biased towards nearer stars compared to the radial velocity sample.

Table IX–1: **Statistically-Determined Stellar Parallax**

Type of Parallax	Equation	Physical Basis
Annual	p	The Earth's solar orbit.
Mean	$\overline{p} = -\left(\frac{4.74}{v_\odot}\right)\left[\frac{\overline{u(\text{obs})\sin\lambda}}{\overline{\sin^2\lambda}}\right]$	The solar motion and upsilon components of proper motion
Secular	$H = -\left(\frac{v_\odot}{4.74}\right)\, p$	Reflected solar motion 90° from the solar apex
Statistical	$\overline{p} = \frac{4.74\overline{\lvert\tau(\text{obs})\rvert}}{\overline{v_r(\star)}}$	Radial speeds and tau components of proper motion

18. **Moving Clusters Method.** Stars that are members of star clusters will all exhibit the same peculiar motion with respect to the LSR as the cluster orbits the center of the Galaxy $\Longrightarrow$ the cluster members appear to *converge* or *diverge* from a single point on the celestial sphere.

   a) We can determine the distance to such a cluster based upon the cluster member's proper motion. This has been done for the

      i) Hyades cluster (about 200 members);

      ii) The Sirius group (which includes most of the bright stars of the Big Dipper);

      iii) The Coma group in Coma Berenices.

   b) If θ is the angle between radial velocity and space velocity of a cluster star, it can be shown that

$$p = \frac{4.74\mu}{v_r \tan\theta}. \qquad \text{(IX-24)}$$

   c) θ is determined from the direction that all the stars in the cluster appear to be heading on the celestial sphere. Hence by just measuring v_r for an individual star in a cluster, we get it's distance.

d) The distance to the Hyades is the foundation of the distance indicator latter than will be described later in the course. Change the distance to the Hyades, and everything in the Universe either gets closer or farther away.

19. Besides stellar motions, the structure of the Galaxy can be determined from **spiral tracers**. **OB associations** and *H II regions* are good examples of spiral tracers — these objects are always found in spiral arms since they are associated with stellar nurseries which form in spiral arms.

20. As spiral arms rotate around the center of the Galaxy, they act as snow plows and collect neutral hydrogen that exists in the disk of the Galaxy. H I in the ground state has **hyperfine structure** due to the spin of the electron in the ground state. The Galaxy has been found to have a spiral structure from spectral observations of the 21-cm hyperfine-structure line of H I (see Figure IX-2).

a) This 21-cm line results from an e^- (electron) spin-flip with respect to the spin of the proton.

b) In the lowest *orbital* electron energy-level, e^- & p (proton) can either be spinning in the same (*i.e.*, **parallel**) or opposite (*i.e.*, **antiparallel**) direction.

c) The antiparallel state is slightly lower energy than the parallel state.

$\Longrightarrow$ antiparallel $\longrightarrow$ parallel: absorption line at 21-cm.

$\Longrightarrow$ parallel $\longrightarrow$ antiparallel: emission line at 21-cm.

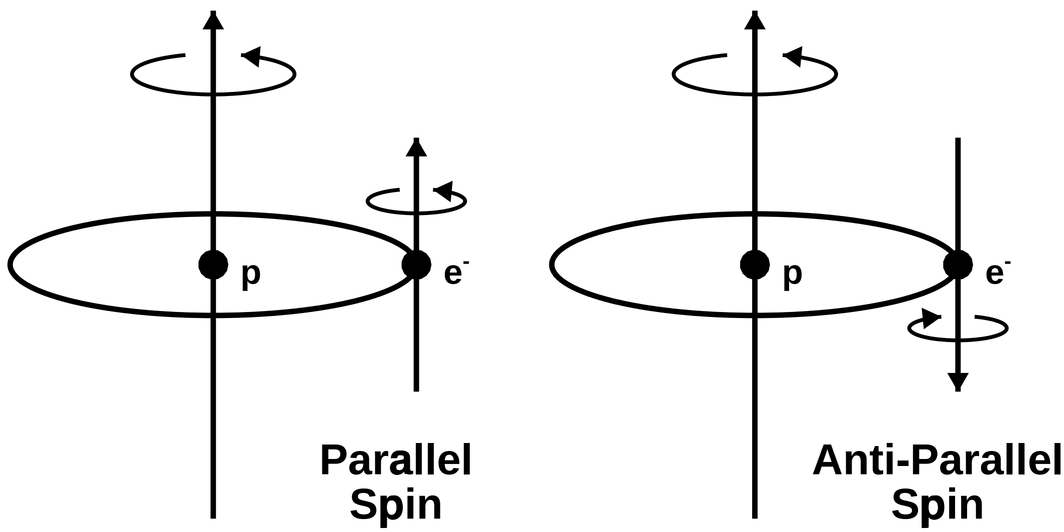

Figure IX–2: Hyperfine-Structure Transition of H I.

d) The exact frequency of this transition is 1420.406 MHz.

e) Since dust and gas are transparent at radio frequencies, Doppler-shifted 21-cm lines can propagate a long distance in the Galaxy with being destroyed $\Longrightarrow$ we can effectively map the whole Galaxy with this transition.

21. The structure of the Galaxy consists of two main components and shown in Figures 22.5 (page 916) and 22.6 (page 917) in the textbook. As well, data concerning these components is shown in Table 22.1 (page 918).

a) **Disk** contains:

i) Stars (majority Population I).

ii) Open star clusters and associations.

iii) Almost all of the ISM.

iv) Spiral arms (ISM located here).

v) We can subdivide the disk into 3 components:

- The **young thin disk** is closest to the galactic plane and contains the current star formation occurring in the Galaxy $\Longrightarrow$ the *extreme* Population I stars. It is about 100 pc thick.

- The **old thin disk** contain the younger Population I stars and is about 650 pc thick.

- The **thick disk** contain old Population I stars (the so-called *disk* population stars) and is about 2.8 kpc thick.

b) **Spherical component** contains:

i) **Halo** which contains:

- Stars (all Population II).

- Globular star clusters.

- Almost no ISM.

ii) **Nuclear bulge** which contains:

- Stars (mostly Population II).

- A few globular clusters.

- Perhaps a few million solar mass supermassive black hole at the center.

22. The disk of the Galaxy is 50 kpc (163,000 ly) in diameter and the thickness (in terms of *scale height*) for the three components is listed above. The Sun is in the disk (actually in the Orion spiral arm) at a distance of $R_\circ = 8.0 \pm 0.5$ kpc (26,000 ly) from the galactic center, where $R_\circ$ is called the **solar galactocentric distance**.

B. Stellar Populations.

1. There are two distinct populations of star that exist in the Milky Way:

a) **Population I stars**:

i) High metal abundance ($Z \approx 0.01$).

ii) Exist in the disk of the Galaxy.

iii) Have fairly circular orbits with low inclinations to the galactic plane.

iv) They are younger stars with $t_{\rm age} < 5 \times 10^9$ yrs.

b) **Population II stars**:

i) Low metal abundance ($Z \lesssim 0.001$).

ii) Exist primarily in the halo of the Galaxy.

iii) Have highly elliptical orbits with high inclinations to the galactic plane.

iv) They are older stars with $t_{\rm age} > 1.2 \times 10^{10}$ yrs.

Figure IX–3: Color-color diagram of Population I main sequence (V) and supergiant (I) stars.

2. Metalicity has important effect on the colors of stars, which becomes evident on either an H-R diagram or a **color-color** (or **two-color**) **diagram**.

a) Figure IX-3 shows a color-color diagram of both main sequence and supergiant stars. As can be seen, it is possible to distinguish stars of different spectral and luminosity classes from one another, assuming that adequate allowance can be made for interstellar reddening (see below).

b) It is fairly standard to plot $(U-B)$ versus $(B-V)$ in color-color diagrams. The large dip in the main sequence near $(B-V)=0$ results from the strong Balmer discontinuity (*i.e.*, continuum) near 3646 Å in the A and B stars.

c) The red part of the curve $(B-V) > 0.8$ falls away from a representative blackbody curve due to the large increase

Figure IX–4: The effects of metalicity and interstellar reddening on a color-color plot.

of **line blanketing** for late-type stars.

d) Population II stars appear bluer than Population I stars on both an H-R diagram and a color-color diagram due to their lower metal abundances (see Figure IX-4).

e) Meanwhile, interstellar reddening causes a star to appear redder than it normally would due blue light being scattered more effectively than red light by interstellar dust (see Figure IX-4).

3. Besides representing metalicity with the "Z" index (*i.e.*, mass fraction of metals to all particles), metalicity also is defined by a star's Fe abundance with respect to the Sun following:

$$[\mathrm{Fe/H}] \equiv \log_{10}\left(\frac{n(\mathrm{Fe})}{n(\mathrm{H})}\right) - \log_{10}\left(\frac{n(\mathrm{Fe})}{n(\mathrm{H})}\right)_{\odot} \qquad \text{(IX-25)}$$

as given by Eq. (II-10) in these course notes.

a) The Sun's Fe abundance is about $\log_{10}\left(\frac{n(\mathrm{Fe})}{n(\mathrm{H})}\right)_{\odot} = 10^{-5}$.

b) If a star has solar metalicity, then [Fe/H] = 0.0.

c) Population I stars range from $-0.5 <$ [Fe/H] $< +0.5$.

d) Population II stars have [Fe/H] < -0.8.

e) The few stars that have $-0.8 <$ [Fe/H] < -0.5 are typically called old disk population stars, though they are typically grouped with the Population I stars.

C. Star Clusters and the ISM.

1. Galactic (Open) Star Clusters.

a) Composed of young stars ($t_{\mathrm{age}} \lesssim 5 \times 10^9$ years) $\Longrightarrow$ many are near giant molecular clouds — proves that stars form in groups.

b) Most open clusters contain *extreme* Population I stars $\Longrightarrow$ [Fe/H] > 0.0.

c) Primarily found in spiral arms.

d) Contain from 100 to 1000 stars that are somewhat *randomly* distributed.

e) Small clusters with tens of stars are called **associations** (*i.e.*, OB associations & T Tau associations).

f) Famous examples include the Hyades, Pleiades, Praesepe, and h & χ Persei.

2. **Globular Star Clusters.**

   a) Composed of old stars ($t_{\rm age} \approx 12 - 15 \times 10^9$ years) $\Longrightarrow$ oldest stars of the Galaxy! The age of these clusters tell us the age of the Milky Way.

   b) Contain old Population II stars $\Longrightarrow -2.2 < [\mathrm{Fe/H}] < -0.8$.

   c) Primarily found in the galactic halo though some are in the nuclear bulge.

   d) Contain from 10^5 to 10^6 stars that are fairly *spherically* distributed.

   e) Some famous examples include M13 and ω Cen.

3. When determining the distance to objects in the Galaxy, we need to take proper account of **interstellar absorption**, which diminishes the total brightness of the object, and **interstellar reddening**, which affects the colors of these objects. We need to understand the structure of the **interstellar medium** (ISM) in the Milky Way Galaxy.

   a) Figure IX-5 gives the *3 phase* structure of the ISM: gas: 98-99% by weight; dust: 1-2% by weight.

   b) Intensity of light gets diminished as it traverses the ISM via:

$$I = I_\circ \, e^{-\tau} \,, \qquad \text{(IX-26)}$$

   where I is the emergent intensity, $I_\circ$ the incident intensity, and τ the optical depth of the ISM cloud.

Figure IX–5: The 3 phase structure of the ISM.

c) In terms of magnitude change, Eq. (IX-26) becomes

$$\Delta m = -2.5 \log \left(\frac{I}{I_\circ} \right) = -2.5 (\log_{10} e) \ln e^{\tau} = 1.086\, \tau \ . \tag{IX-27}$$

Note that $\log_{10} e = 0.434$ and multiplying this by 2.5 gives 1.086. Hence, a star will get fainter by $1.086\,\tau$ magnitudes as its light passes through an ISM cloud.

d) From §I of the notes, the optical depth can be expressed as

$$\tau = \int_0^x \kappa_\lambda(x')\, n(x')\, dx' \tag{IX-28}$$

$$= \kappa_\lambda(x) \int_0^x n(x')\, dx' \tag{IX-29}$$

$$= \kappa_\lambda(x)\, N(x) \ , \tag{IX-30}$$

where we are making the assumption that the absorption coefficient κ_λ (cm^2/particle) is relatively constant in the ISM cloud (this is typically the case in the ISM), $n(x)$ (particle/cm^3) is the particle density of the opacity

Figure IX–6: Plane parallel model of galactic interstellar absorption.

source, and $N(x)$ (particle/cm^2) is the column density of the cloud.

e) Since most of the ISM is in the plane of the Galaxy, we use a plane parallel model (see Figure IX-6) to map out optical depths within the disk as a function of galactic latitude b (note that $b = 0$ is defined to be the galactic plane and that the galactic longitude ℓ is measured with respect to the galactic center designated by the radio source Sagittarius A). Then

$$\Delta m(b) = 1.086\,\tau\ \csc b\ . \qquad \text{(IX-31)}$$

f) Light also gets reddened as it travels through the ISM. This arises from scattering off of dust, molecules, and ices. The spectrum of the interstellar absorption/scattering, and the causes of this absorption/scattering, is shown in Figure (IX-7).

Figure IX–7: Optical depth as a function of wavenumber for interstellar absorption/scattering.

g) We can define a **color excess** $E(U - B) = (U - B) - (U - B)_\circ$ or $E(B - V) = (B - V) - (B - V)_\circ$ which results from interstellar reddening. It has been shown through the observations or O, B, and A stars, that

$$\frac{E_{U-B}}{E_{B-V}} = 0.72 \qquad \text{(IX-32)}$$

in the disk of the Galaxy.

h) Using the absorption profile of the ISM shown in Figure (IX-7), we can deduce an interstellar absorption term A as a function the color excess in various various filters along the spectrum. Typically, one works with the visual filter, hence we can write a distance modulus formula (see Eq. II-2) that takes interstellar absorption into account:

$$V - M_V = 5 \log \left(\frac{r}{10 \text{ pc}} \right) + A_V \ . \qquad \text{(IX-33)}$$

i) Observations of stars on our side of the Milky Way have shown that

$$R_V = \frac{A_V}{E_{B-V}} \ , \qquad \text{(IX-34)}$$

where $R_V = 3.2 \pm 0.2$, a fairly constant value. (Note that there are one or two peculiar regions in the Milky Way where $R_V \gtrsim 4$.)

ii) The same equations can be used for other filters too $\Longrightarrow$ B or U would replace V in the equations above and R would have different values for different filters.

iii) As is shown here, interstellar absorption can have a major impact in determining the distances to interstellar and extragalactic objects, Fortunately reddening information can give a good handle on the amount of absorption that has taken place.

i) The ISM has other regions of interest, including planetary nebulae and supernova remnants (both which have already been discussed). A large number of astronomers specialize in the study and modeling of the ISM due to its importance in so many areas in astrophysics.

D. Galactic Rotation

1. By studying the motion of stars in the Galaxy, we can deduce the rotation rate of the Galaxy as a whole. Unfortunately due to the heavy obscuration in the disk of the Galaxy from the ISM, stellar motions are hard to deduce except for stars in the solar neighborhood.

2. However, with use the H I 21-cm line, we have been able to deduce a rotation curve ($v_{\rm orb}(R) = \Theta(R)$, where R is the distance from the center of the Galaxy) of the Milky Way. Before describing the results of this work, we need to develop the mathematics to describe this rotation.

3. A great deal of effort has gone into the derivation of the galactocentric distance $R_\circ$, the Oort constants A and B which describe the differential galactic rotation, the motion of the Sun with respect to the LSR, and the circular velocity of the LSR about the galactic center, $\Theta_\circ$. In 1966, the IAU adopted the following values: $R_\circ = 10$ kpc (1 kpc = 1000 pc), $A = 15$ km/s/kpc, $B =$ -10 km/s/kpc, $\Theta_\circ = 250$ km/s. In recent years, new measurements have reset some of these constants.

a) Observations of O & B stars by Balona & Feast (1974) and by Crampton *et al.* (1976) give $R_\circ = 9$ and 8 kpc, respectively.

b) Harris (1976) found $R_\circ = 8.5$ kpc from the distribution of globular clusters.

c) Oort & Plaut (1975) deduced $R_\circ = 8.7$ kpc from the distribution of RR Lyr stars.

d) $\Theta_\circ$ can only be deduced from the Oort constants A and B (see below). Using values of A determined from optical radial velocities and values of B determined from the velocity *ellipsoid*, Lynden-Bell & Lin (1977) can only say that 230 km/s $< \Theta_\circ <$ 340 km/s ($R_\circ = 10$ kpc), or 180 km/s $< \Theta_\circ <$ 270 km/s ($R_\circ = 8$ kpc).

Figure IX–8: Geometric relationship among quantities entering the differential rotation formula.

4. Gunn, Knapp, & Tremaine (1979, *Astronomical Journal*, **84**, 1181) improved on these values by making use of the H I 21-cm line. Their results were as follows:

a) Sun's distance from galactic center: $R_\circ = 8.5$ kpc; and $A\,R_\circ = 110$ km/s.

b) Local circular velocity: $\Theta_\circ = 220$ km/s.

c) H I mass in the Milky Way: $M_{\rm H\ I} = 2 \times 10^9 M_\odot$.

d) Luminosity of Milky Way: $L_{\rm MW} = 1.7 \times 10^{10} L_\odot$.

5. Figure IX-8 the relationship between the Sun's motion and a star's about the galactic center. Through this diagram and the assumption that stellar orbits are circular about the center of the Galaxy, we can empirically describe the stellar velocity about the center of the Galaxy $\Longrightarrow$ the **rotation curve** of the Milky Way.

a) The linear rotation velocity of a star about the center of the Galaxy is

$$\Theta = \omega\, R\ , \qquad \text{(IX-35)}$$

where ω is the angular velocity of the star. Note that $\omega_\circ = \omega(R_\circ)$ is the angular velocity of the Sun.

b) From Figure IX-8, the corresponding radial velocity of the star (positive sign implies recession) is

$$v_r = \Theta\, \cos\alpha - \Theta_\circ\, \sin\ell\ . \qquad \text{(IX-36)}$$

where ℓ is the galactic longitude and α is the angle between the star's radial velocity vector and its galactic rotation velocity vector.

c) From the law of sines,

$$\frac{\sin\ell}{R} = \frac{\sin(90^\circ + \alpha)}{R_\circ} = \frac{\cos\alpha}{R_\circ}\ . \qquad \text{(IX-37)}$$

so

$$\begin{aligned} v_r &= \left(\frac{\Theta R_\circ}{R}\right) \sin\ell - \Theta_\circ\, \sin\ell \qquad &\text{(IX-38)} \\ &= (\omega - \omega_\circ)\, R_\circ\, \sin\ell\ . &\text{(IX-39)} \end{aligned}$$

d) From Figure IX-8, we see that the corresponding tangential velocity is then

$$v_t = \Theta\, \sin\alpha - \Theta_\circ\, \cos\ell\ , \qquad \text{(IX-40)}$$

where v_t is measured positive in the direction of increasing ℓ.

e) With the help of Figure IX-9, we see that

$$R\, \sin\alpha = R_\circ\, \cos\ell - d\ , \qquad \text{(IX-41)}$$

so that

$$\begin{aligned} v_t &= \left(\frac{\Theta}{R}\right) (R_\circ\, \cos\ell - d) - \Theta_\circ\, \cos\ell \qquad &\text{(IX-42)} \\ &= (\omega - \omega_\circ)\, R_\circ\, \cos\ell - \omega\, d\ . &\text{(IX-43)} \end{aligned}$$

Figure IX–9: Geometric construction used in the derivation of Equation (IX-41).

6. The Galaxy does not rotate like a solid disk — there is **differential rotation**. We will now derive mathematical relationships that will allow us to make observations to construct a rotation curve $\Theta(R)$ following the technique first describe by Oort.

a) Oort's work was based upon observations of stellar motions. Since we can only get this type of information from stars in the solar neighborhood, the distance to the stars d can be assumed small with respect to the galactic center ($d \ll R \approx R_\circ$).

b) Consider first Eq. (IX-39) for fixed ℓ. $R_\circ$ is fixed, and the only term $(\omega - \omega_\circ)$ can depend on distance. We can estimate the variation of $(\omega - \omega_\circ)$ by use of a Taylor expansion

$$(\omega - \omega_\circ) \approx \left(\frac{d\omega}{dR}\right)_{R_\circ} (R - R_\circ) \ . \qquad \text{(IX-44)}$$

c) Now

$$\frac{d\omega}{dR} = \frac{d}{dR}\left(\frac{\Theta}{R}\right) = \frac{1}{R}\frac{d\Theta}{dR} - \frac{\Theta}{R^2} \qquad \text{(IX-45)}$$

so that

$$\left(\frac{d\omega}{dR}\right)_{R_\circ} = \frac{1}{R_\circ}\left(\frac{d\Theta}{dR}\right)_{R_\circ} - \frac{\Theta_\circ}{R_\circ^2} \; . \qquad \text{(IX-46)}$$

Therefore to first order

$$v_r = \left[\left(\frac{d\Theta}{dR}\right)_{R_\circ} - \frac{\Theta_\circ}{R_\circ}\right] (R - R_\circ) \sin\ell \; . \qquad \text{(IX-47)}$$

d) Since $d \ll R_\circ$, the projection of R onto $R_\circ$ is almost equal to R, so that, to good approximation,

$$R_\circ - R \approx d \cos\ell \; , \qquad \text{(IX-48)}$$

which, when substituted into Eq. (IX-47), yields

$$v_r \approx \left[\frac{\Theta_\circ}{R_\circ} - \left(\frac{d\Theta}{dR}\right)_{R_\circ}\right] d \sin\ell \cos\ell \; . \qquad \text{(IX-49)}$$

e) Then, using the trigonometric identity $\sin\ell \cos\ell = \frac{1}{2}\sin 2\ell$, and defining

$$A \equiv \frac{1}{2}\left[\frac{\Theta_\circ}{R_\circ} - \left(\frac{d\Theta}{dR}\right)_{R_\circ}\right] \; , \qquad \text{(IX-50)}$$

which is called the *Oort's constant* A, we obtain, to first order in d,

$$v_r = A\, d \sin 2\ell \; . \qquad \text{(IX-51)}$$

i) Hence, because of differential galactic rotation, the radial velocities of stars in the galactic disk will show a *double sine-wave* variation with galactic longitude (which has been observed), with an amplitude that increases linearly with distance.

ii) Eq. (IX-51) is valid for disk stars with $d \ll R_\circ$.

iii) Hence, if we can observe v_r and estimate d for a large sample of stars, we can estimate A and hence place constraints on the local values of $\omega_\circ$ and $(d\omega/dR)_{R_\circ}$.

iv) In most work, v_r is expressed in km/s and d in kpc, hence A is often given in units of km/s/kpc.

f) We can also do a Taylor expansion of the tangential velocity equation (*e.g.*, Eq. IX-43). We will express the second term in this equation as $\omega d = \omega_\circ d + (\omega - \omega_\circ)d$. Now since $(\omega - \omega_\circ)$ is already of the first order in d, retaining only the first-order terms, we can replace ωd with $\omega_\circ d$. Equation (IX-43) thus becomes

$$v_t \approx \left[\left(\frac{d\Theta}{dR}\right)_{R_\circ} - \frac{\Theta_\circ}{R_\circ}\right] (R - R_\circ) \cos\ell - \omega_\circ d \qquad \text{(IX-52)}$$

$$v_t \approx \left[\left(\frac{\Theta_\circ}{R_\circ} - \frac{d\Theta}{dR}\right)_{R_\circ}\right] d \cos^2\ell - \left(\frac{\Theta_\circ}{R_\circ}\right) d \; . \qquad \text{(IX-53)}$$

g) Using the trigonometric identity $\cos^2\ell = \frac{1}{2}(1 + \cos 2\ell)$, we get

$$v_t = \frac{1}{2}\left[\left(\frac{\Theta_\circ}{R_\circ} - \frac{d\Theta}{dR}\right)_{R_\circ}\right] d \cos 2\ell - \frac{1}{2}\left[\left(\frac{\Theta_\circ}{R_\circ} + \frac{d\Theta}{dR}\right)_{R_\circ}\right] d \; . \qquad \text{(IX-54)}$$

h) We can now define *Oort's constant B* as

$$B \equiv -\frac{1}{2}\left[\left(\frac{\Theta_\circ}{R_\circ} + \frac{d\Theta}{dR}\right)_{R_\circ}\right] , \qquad \text{(IX-55)}$$

such that the tangential velocity equation becomes

$$v_t = d\,(A\,\cos 2\ell + B)\ . \qquad \text{(IX-56)}$$

i) In terms of proper motion (see Eq. IX-5), we get

$$\mu_\ell = \frac{A\,\cos 2\ell + B}{4.74}\ . \qquad \text{(IX-57)}$$

Note, however, that in this equation, both A and B must be in units of km/s/pc instead of the typically posted km/s/kpc.

j) It follows from Eqs. (IX-50) and (IX-55) that the angular rotation rate of the Sun and its derivative can be deduced from Oort's A and B constants:

$$\omega_\circ = \frac{\Theta_\circ}{R_\circ} = A - B \qquad \text{(IX-58)}$$

$$\left(\frac{d\Theta}{dR}\right)_{R_\circ} = -(A + B)\ . \qquad \text{(IX-59)}$$

k) The Oort's constant A may be found by one of the following methods:

i) From the amplitude of variation with galactic longitude of the radial velocity of objects of known distances using Eq. (IX-51).

ii) From amplitude of longitude dependence of proper motions using Eq. (IX-57).

iii) From the definition

$$A = \frac{1}{2}\left[\frac{\Theta_\circ}{R_\circ} - \left(\frac{d\Theta}{dR}\right)_{R_\circ}\right] = -\frac{1}{2} R_\circ \left(\frac{\omega}{dR}\right)_{R_\circ}, \qquad \text{(IX-60)}$$

using an empirically determined rotation curve.

Table IX–2: **H I 21-cm Results of Galactic Constants.**

Constant		Value
$R_\circ$	=	8.5 ± 0.5 kpc
$\Theta_\circ$	=	220 ± 10 km/s
A	=	13 ± 2 km/s/kpc
B	=	-13 ± 2 km/s/kpc

l) The Oort's constant B:

i) The only *direct* way to measure the Oort's constant B is through measurements of proper motion of stars using Eq. (IX-57) along with an already determined A value.

ii) An *indirect* method can be used to determine B where the velocity dispersions of the peculiar velocity of stars (σ_r – radial direction and σ_t – tangential direction) through the equations

$$\frac{-B}{A-B} = \frac{\sigma_t^2}{\sigma_r^2} \qquad \text{(IX-61)}$$

$$\frac{-B}{A} = \frac{1}{(\sigma_r^2/\sigma_t^2) - 1} \,. \qquad \text{(IX-62)}$$

7. The above described methods gives the relevant information for the Sun. The overall rotation curve of the Galaxy is determined from H I 21-cm observations and is shown in Figure IX-10. The results from 21-cm observations are listed in Table IX-2.

8. The rotation curve for $R > R_\circ$ is a bit more difficult to obtain. We won't get into the details of determining the outer rotation curve here, but note that it has been deduced to a distance out to $R = 25$ kpc — and it remains fairly flat out to this distance!

Figure IX–10: The rotation curve for the Milky Way Galaxy based from 21-cm observations (*dotted line*). The smooth curve is based on a dynamical model constrained to fit the observed data.

9. Various aspects of the Galaxy's mass can be deduced from the rotation curve.

a) The total mass of the *visible* Galaxy is $M_{\rm MW} = 1.5 \times 10^{11} M_\odot$.

b) As one gets farther out from the galactic center, there should come a position where most of the Galaxy's mass is inside the orbit of a star. When that happens, the rotation curve of the Galaxy should resemble Kepler's 3rd law of planetary motion ($P^2 \propto R^3$) and the equation for circular orbital velocity:

$$\Theta = \sqrt{\frac{GM_{MW}}{R}} \; . \qquad \text{(IX-63)}$$

c) The fact that the rotation curve is *non-Keplerian* in its outer regions implies that there is **a lot** of mass in the outer reaches of the Galaxy $\Longrightarrow$ a **massive halo** exists surrounding the disk of the Galaxy.

d) The motion of the Magellanic Clouds and the relative motions of M31 and the Milky Way indicate that the total mass of the Galaxy is $M_{\rm MW} = 1.0 \times 10^{12} M_{\odot}$! $\Longrightarrow$ Much of the mass (90% of it) is not visible — **dark matter**. This dark matter is often called the *missing mass* problem of the Galaxy.

e) Many candidates have been given for this missing mass:

i) A preponderance of red dwarfs and brown dwarfs in the halo.

ii) Many stellar black holes are seeded in the Galaxy.

iii) Neutrinos outnumber photons in the Universe by a factor of a million! However, the recent mass estimates for these lepton ($\sim$25 eV/c^2) from neutrino oscillation experiments show that neutrinos have insufficient mass to account for the missing mass.

iv) Exotic particles: massive magnetic monopoles, axions, and weakly interactive massive particles (WIMPs).

E. The Spiral Structure of the Milky Way

1. **The Density Wave Theory**: A disturbance in the Galaxy's gravitational field propagates around the Galaxy in a spiral structure — like a compression wave.

a) Dust and gas get compressed in this wave which triggers star formation $\Longrightarrow$ O & B stars form which outline this spiral wave in our and other galaxies.

b) The density wave/giant molecular cloud interaction is very efficient in creating stars.

c) These waves (2 main ones) are thought to have arisen from a gravitational instability in the Milky Way's gravitational potential well through interactions with the Milky Way's satellite galaxies, the Large and Small Magellanic Clouds.

d) This theory predicts only 2 spiral arms per galaxy (and sometimes a linear bar that goes through the nucleus of the galaxy connecting the inner portion of the arms).

e) Branches and spurs that are seen in our and other galaxies result from supernova explosions and OB association ionization fronts $\Longrightarrow$ this is known as **self-sustaining star formation**.

2. **The Differential Rotation Theory**:

   a) Supernova shocks make stars and the differential rotation of the Galaxy causes the spiral structure.

   b) This does not work as a viable theory since differential rotation would rip the spiral structure apart after just a few revolutions

3. Even though the Sun is currently close to a spiral arm (*e.g.*, the Orion arm), there is not a lot of ISM in the Sun's vicinity — the Sun is in a large, somewhat empty, bubble called the **Local Bubble** which is about 300 ly across.

   a) This was only realized within the past 15 years through observations with the EUVE (Extreme Ultraviolet Explorer) satellite — hydrogen is very opaque at the wavelengths in which EUVE is sensitive, we did not expect to see very far with this telescope, but we did, implying that there is not a lot of ISM in our vicinity.

b) It has been suggested that there may have been a nearby supernova within the last million years that formed this hole.

c) A pulsar called **Geminga** with peculiar properties was discover a few decades ago. It has a large proper motion across the sky meaning that it is relatively nearby (probably less than 300 ly). Its pulsation time indicates that it is about 300,000 yrs old. The supernova that produced Geminga may be the cause of the Local Bubble. When this supernova went off, it would have had an apparent magnitude of $-13 \Longrightarrow$ brighter than the full Moon!

F. The Galactic Nucleus.

1. The central nuclear region of the Galaxy is completely invisible at visible wavelengths $\Longrightarrow$ dust and gas obscuration.

2. We can see it in radio waves however! The center of the galaxy is the brightest radio source in the sky $\Longrightarrow$ called **Sagittarius A**.

3. There is evidence for a small 10 AU in diameter region at the center that has a mass of 1-4 million (10^6) solar masses! X-rays and gamma rays have been detected from this region as well.

4. Something massive and energetic is at the galactic center $\Longrightarrow$ perhaps a million solar mass **supermassive black hole**!

5. Velocity measurements of stars in the nuclear bulge and the distribution of light in the *COBE* far-infrared observations suggest that the Milky Way has a bar going through the center of the Galaxy! Prior to this, the Milky Way was though to be an Sbc spiral galaxy (see §10 of the notes). As a result of this, it now has the classification of SBb.