# ASTR-3415, Astrophysics
## Solutions to Exam 2
## Spring 2003

1. (70 points total) Assume there is a spherical giant molecular cloud with a diameter of 20.0 ly, and is at a temperature of 50.0 K with a uniform particle density of $1.00 \times 10^4$ cm$^{-3}$. The mean molecular weight of the particles that make up this cloud is 0.770.

   (a) (15 points) What is the mass of this cloud in solar masses?

   **Solution (a):** Before we answer this specific question, we need to write down our given parameters, make some unit conversions, and calculate the mass density from the number density.

   $R = \frac{1}{2}D = 0.5 \cdot 20 \text{ ly} \cdot 9.4605 \times 10^{17} \text{cm/ly} \; = 9.46 \times 10^{18} \text{ cm}$

   $T = 50 \text{ K}, \qquad N = 1.00 \times 10^4 \text{ cm}^{-3}, \qquad \mu = 0.770$

   $\rho = \mu\, m_{\rm H}\, N = 0.770\,(1.67 \times 10^{-24} \text{ gm})\,(1.00 \times 10^4 \text{ cm}^{-3}) = 1.29 \times 10^{-20} \text{ gm/cm}^3$

   The mass can be determined from the above calculated mass density and volume with $\rho = M/V$, where the volume for a sphere is

   $$V = \frac{4}{3}\pi R^3 \; .$$

   As such, the mass of the cloud is

   $$\begin{aligned} M &= \rho V = \frac{4}{3}\pi\rho R^3 = \frac{4\pi}{3}(1.29 \times 10^{-20} \text{ gm/cm}^3)(9.46 \times 10^{18} \text{ cm})^3 \\ &= 4.57 \times 10^{37} \text{ gm} \left(\frac{1 M_\odot}{1.99 \times 10^{33} \text{ gm}}\right) \\ &= \boxed{2.30 \times 10^4 M_\odot \; .} \end{aligned}$$

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   (b) (20 points) Compare the total thermal energy in the cloud to the gravitational potential energy of the cloud.

   **Solution (b):** Using Eq. (V-2) of the notes, the thermal energy is

   $$\begin{aligned} E_{\rm th} &= P \cdot V = \frac{\rho k_{\rm B} T}{\mu m_{\rm H}} \cdot \frac{4}{3}\pi R^3 \\ &= \frac{(1.29 \times 10^{-20} \text{ gm/cm}^3)(1.3806 \times 10^{-16} \text{ erg/K})(50.0 \text{ K})}{(0.770)(1.67 \times 10^{-24} \text{ gm})} \times \\ &\qquad \frac{4\pi}{3}\left(9.46 \times 10^{18} \text{ cm}\right)^3 \\ &= \left(6.93 \times 10^{-11} \text{ erg/cm}^3\right)\left(3.55 \times 10^{57} \text{ cm}^3\right) \\ &= \boxed{2.46 \times 10^{47} \text{ erg} \; .} \end{aligned}$$

Meanwhile, the gravitational energy, given by Eq. (V-3) in the notes, is

$$
\begin{aligned}
E_g &= -16\pi^2 G\rho^2 R^5 \\
&= -16\pi^2 \left(6.673\times10^{-8}\ \text{dyne cm}^2/\text{gm}^2\right)\left(1.29\times10^{-20}\ \text{gm/cm}^3\right)^2 \times \\
&\qquad \left(9.46\times10^{18}\ \text{cm}\right)^5 \\
&= \boxed{-1.33\times10^{50}\ \text{erg}\ .}
\end{aligned}
$$

The absolute value of the gravitational energy is 541 times bigger than the thermal energy!

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(c) (5 points) Will this cloud expand, collapse, or remain stable? Base your answer on the solution to part (b).

**Solution (c):** Since $E_{\rm th} < |E_g|$, the thermal pressure is insufficient to support the weight of the cloud, hence it $\boxed{\text{collapses .}}$

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(d) (20 points) Calculate both the Jeans' length (in light years) and Jeans' mass (in solar masses) of this cloud.

**Solution (d):** Using Eq. (V-4) from the notes, the Jeans' radius is

$$
\begin{aligned}
R_{\rm J} &= \left(\frac{k_{\rm B}}{12\pi G\mu m_{\rm H}}\right)^{1/2}\left(\frac{T}{\rho}\right)^{1/2} = \left(\frac{k_{\rm B}T}{12\pi G\mu m_{\rm H}\rho}\right)^{1/2} \\
&= \left(\frac{(1.3806\times10^{-16}\ \text{erg/K})(50.0\ \text{K})}{12\pi(6.673\times10^{-8}\ \text{dyne cm}^2/\text{gm}^2)(0.770)(1.67\times10^{-24}\ \text{gm})}\right)^{1/2} \times \\
&\qquad \left(\frac{1}{1.29\times10^{-20}\ \text{gm/cm}^3}\right)^{1/2} \\
&= \left(4.62\times10^{7}\ \text{gm}^{1/2}\text{cm}^{-1/2}\right)\left(8.80\times10^{9}\ \text{cm}^{3/2}\text{gm}^{-1/2}\right) \\
&= 4.07\times10^{17}\ \text{cm}\ \times\left(\frac{1.00\ \text{ly}}{9.4605\times10^{17}\ \text{cm}}\right) \\
&= \boxed{0.430\ \text{ly}\ .}
\end{aligned}
$$

Now, using Eq. (V-5) from the notes, the Jean's mass is

$$
M_{\rm J} = \frac{4\pi}{3}\left(\frac{k_{\rm B}}{12\pi G\mu m_{\rm H}}\right)^{3/2}\frac{T^{3/2}}{\rho^{1/2}} = \left(\frac{k_{\rm B}T}{12\pi G\mu m_{\rm H}}\right)^{3/2}\cdot\frac{1}{\rho^{1/2}}
$$

$$= \frac{4\pi}{3}\left(\frac{(1.3806\times10^{-16}\ \mathrm{erg/K})(50.0\ \mathrm{K})}{12\pi(6.673\times10^{-8}\ \mathrm{dyne\ cm^2/gm^2})(0.770)(1.67\times10^{-24}\ \mathrm{gm})}\right)^{3/2}\times \left(\frac{1}{1.29\times10^{-20}\ \mathrm{gm/cm^3}}\right)^{1/2}$$

$$= \frac{4\pi}{3}\left(9.86\times10^{22}\ \mathrm{gm^{3/2}cm^{-3/2}}\right)\left(8.80\times10^{9}\ \mathrm{cm^{3/2}gm^{-1/2}}\right)$$

$$= 3.64\times10^{33}\ \mathrm{gm}\ \times\left(\frac{1.00\,M_\odot}{1.99\times10^{33}\ \mathrm{gm}}\right)$$

$$= \boxed{1.83\,M_\odot\ .}$$

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(e) (10 points) Are these 'Jeans' values consistent with your *cloud expansion/collapse/equilibrium* statement above in part (c)? (Present logical reasons, just don't answer yes or no!)

**Solution (e):** Note that $R = 10$ ly and $R_{\mathrm{J}} = 0.430$ ly, hence $R > R_{\mathrm{J}} \Longrightarrow$ the cloud has to collapse due to the Jeans' stability criterion (see page V-2 of the notes).

Also, $M = 2.30\times10^4 M_\odot$ and $M_{\mathrm{J}} = 1.83 M_\odot$, hence $M > M_{\mathrm{J}}$ (actually $M \gg M_{\mathrm{J}}$) $\Longrightarrow$ the cloud has to collapse due to the Jeans' stability criterion (see page V-3 of the notes).

This is consistent with the $E_{\mathrm{th}} < |E_g|$ (*i.e.*, thermal pressure vs. weight of the cloud) condition found in part (c).

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2. (30 pts) Answer the following questions about stellar evolution and its relation to interior structure. A paragraph for each part (containing anywhere between 3 to 5 sentences) is sufficient, but make sure you supply enough details in your answers.

   (a) (15 points) Why does a 0.8 $M_\odot$ star become a red giant whereas a 0.2 $M_\odot$ does not?

**Solution (a):** Stars ascend the red giant branch when H-shell burning starts. This occurs after an inert He core starts to collapse once the main sequence H-core burning ends. Energy dumped into the envelope from the H-shell burning 'pumps' up the star causing it to expand. As the surface expands, the energy flux per unit area drops and the surface layers cool — the star becomes a red giant. This occurs in a 0.8 $M_\odot$ star.

A 0.2 $M_\odot$ star on the main sequence is completely convective, whereas only the outer envelope is convective in the 0.8 $M_\odot$ star. The He produced in the core during the main sequence is spread throughout the entire star and fresh hydrogen from the outer layers is brought into the core. Due to this, no inert He core ever forms, and as a result, no H-shell burning ever begins. Hence no ascent is made up the red giant branch. Instead, the pure He star will collapse once the H burning ends to degenerate state forming a He-rich white dwarf star.

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   (b) (15 points) What is the core helium flash? Does it occur in both of the two stars mentioned in part (a)? Why or why not?

**Solution (b):** As the inert He-core collapses in the 0.8 $M_\odot$ star, a time is reached when the gas becomes electron degenerate (which occurs before He-fusion ignition). At this point, increases in temperature will not increase pressure as is the case with ideal gas. As material from the H-burning shell above continuously rains down onto the degenerate core, the temperature continues to increase until the temperature gets high enough for He ignition. He fusion then increases the temperature more, which increase the triple-alpha reaction rates, which increase the temperature more $\Longrightarrow$ a *thermonuclear runaway* occurs. This causes a sharp increase in the luminosity of the outer layers of the star $\Longrightarrow$ a **core helium flash**! This continues until the temperature gets high enough to lift degeneracy. Once the degeneracy is lifted, the ideal gas equation of state is re-established and the temperature-pressure regulator once again operates, which results in a reaction rate/temperature equilibrium.

The 0.2 $M_\odot$ never forms an inert He core with a hydrogen-rich layer above it. Instead, the conclusion of the H-burning main sequence stage results in the He 'star' collapsing to a degenerate state. However, since the gravitational energy is so weak in these stars, temperatures never approach the triple-alpha ignition values, hence no runaway occurs and these types of stars never experience a He flash.