ASTR-3415, Astrophysics
Solutions to Exam 3
Spring 2003

1. (70 points total) We observe the H I line (rest wavelength of 21.10611 cm) at 22.58173 cm in an Sb galaxy that has an apparent magnitude of 13.23 in the blue filter once intergalactic extinction has been subtracted. This 21-cm line has a width that gives a maximum velocity of 406 km/s for the rotation of this galaxy.

   (a) (10 points) What is the absolute magnitude of this galaxy in the blue filter? What is the name of the relationship that you used to figure this out?

Solution (a): This solution can be obtained with the

Tulley-Fisher Relation,

as given by Eq. (23.4) in the textbook. For an Sb galaxy, a maximum rotation velocity $V_{\rm max}$ of 406 km/s gives an absolute blue magnitude of

$$\begin{aligned} M_B &= -10.2 \log V_{\rm max} + 2.71 \\ &= -10.2 \log(406) + 2.72 = -26.6 + 2.71 \\ &= \boxed{-23.9\ .} \end{aligned}$$

(b) (10 points) What is the distance to this galaxy in Mpc? What is the name of the formula that you used to find this distance?

Solution (b): We use the **distance modulus formula** to determine this distance which is

$$\begin{aligned} B - M_B &= 5 \log\left(\frac{d}{10\ \rm pc}\right) \\ \frac{d}{10\ \rm pc} &= 10^{0.2\,(B-M_B)} = 10^{0.2\,[13.23-(-23.9)]} = 10^{7.43} = 2.69 \times 10^{7} \\ d &= 2.69 \times 10^{8}\ {\rm pc}\ \times \frac{1\ \rm Mpc}{10^{6}\ \rm pc} \\ &= \boxed{269\ \rm Mpc\ .} \end{aligned}$$

(c) (10 points) What is the radial velocity of this galaxy in km/s? What is the name of the formula that you used to figure this out?

Solution (c): Here we use the **Doppler effect formula** to determine the radial velocity. The redshift of this galaxy is

$$
\begin{aligned}
z &= \frac{\Delta\lambda}{\lambda_\circ} = \frac{22.58173 \text{ cm } - 21.10611 \text{ cm}}{21.10611 \text{ cm}} = \frac{1.47562 \text{ cm}}{21.10611 \text{ cm}} \\
&= 0.0699144 \; .
\end{aligned}
$$

Since this is fairly close to unity, we must use the relativistic form of the Doppler effect to determine the radial velocity as given by Eq. (X-18) of the notes:

$$
\begin{aligned}
\frac{v_r}{c} &= \frac{(z+1)^2 - 1}{(z+1)^2 + 1} \\
&= \frac{(1.0699144)^2 - 1}{(1.0699144)^2 + 1} = 0.0674759 \\
v_r &= (0.0674759)\,(2.997925 \times 10^5 \text{ km/s}) \\
&= \boxed{2.02288 \times 10^4 \text{ km/s} \; .}
\end{aligned}
$$

(d) (15 points) From the information above, what is the current Hubble constant? Name the relationship that you used to figure this out. Assuming that the WMAP value of Hubble's constant is the accurate value, what is the percent error of the value of your Hubble's constant?

Solution (d): We simply use **Hubble's law** to figure out the current Hubble constant $H_\circ$:

$$
\begin{aligned}
H_\circ &= \frac{v_r}{d} = \frac{2.02288 \times 10^4 \text{ km/s}}{269 \text{ Mpc}} \\
&= \boxed{75.2 \text{ km/s/Mpc} \; .}
\end{aligned}
$$

The WMAP value for Hubble's constant is given on page X-14 of the notes as 71$\pm$4 km/s/Mpc. The percent error of our value here to the WMAP value is

$$
\begin{aligned}
\%\text{–error} &= \frac{75.2 \text{ km/s/Mpc } - 71 \text{ km/s/Mpc}}{71 \text{ km/s/Mpc}} \times 100\% \\
&= \boxed{5.9\% \; .}
\end{aligned}
$$

(e) (10 points) What must be the maximum age (in years) of the Universe for the Hubble constant determined above? Is this consistent with the age of the globular star clusters in the Milky Way Galaxy?

Solution (e): Use Eq. (XI-2) from the notes where $h = 0.752$ from our solution above,

$$t_{\rm H} = 9.78 \times 10^9\, h^{-1} \text{ yr} = \frac{9.78 \times 10^9}{0.752} \text{ yr}$$
$$= \boxed{1.30 \times 10^{10} \text{ years} ,}$$

or about 13.0 Gyr. Since the age of the globular star clusters in the Milky Way ranges from 12 to 13 Gyr old, this value is consistent with this value.

(f) (15 points) Based on the data for this galaxy, what is the current mass density of the Universe if $\Omega_\Lambda = 0$ and the deceleration parameter is 0.250? From this data, what is the matter density parameter equal to? Is such a Universe closed, flat, or open? Why do you say this?

Solution (f): To solve this, we use Eq. (XI-48) from the notes (since $\Lambda = 0$ from $\Omega_\Lambda = 0$) in conjunction with Eq. (XI-1) with $h = 0.752$, and solve for $\rho_\circ$, giving

$$\rho_\circ = \frac{3q_\circ H_\circ^2}{4\pi G} = 2.51 \times 10^{-36} \frac{q_\circ\, h^2 \text{ s}^2}{G}$$
$$= 2.51 \times 10^{-36} \frac{(0.25)\,(0.752)^2 \text{ s}}{6.673 \times 10^{-8} \text{ dyne cm}^2/\text{gm}^2}$$
$$= \boxed{5.32 \times 10^{-30} \text{ gm/cm}^3 .}$$

To determine the density parameter, we must determine the critical density for our Hubble's constant using Eq. (XI-54):

$$\rho_c = 1.88 \times 10^{-29}\, h^2 \text{ gm/cm}^3 = 1.06 \times 10^{-29} \text{ gm/cm}^2 .$$

The matter density parameter is then

$$\Omega_m = \Omega_\circ = \frac{\rho_\circ}{\rho_c} = \frac{5.32 \times 10^{-30} \text{ gm/cm}^3}{1.06 \times 10^{-29} \text{ gm/cm}^3}$$
$$= \boxed{0.502 .}$$

Since $\Lambda = 0$ and $\Omega_\circ < 1$, the Universe is open with a negative curvature and hyperbolic shape.

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2. (30 pts) These questions deal with stellar populations:

(a) (10 points) What are the differences between Population I, Population II, <u>and</u> Population III stars? Give all characteristics of each. Where do we find each type of star in the Milky Way?

Solution (a): Population I stars have the following characteristics: (a) high metal abundance ($Z \approx 0.01$); (b) exist in the disk of the Galaxy; (c) have fairly circular orbits with low inclinations to the galactic plane; and (d) are younger stars with $t_{\rm age} < 5 \times 10^9$ yrs

Population II stars have the following characteristics: (a) low metal abundance ($0 < Z \lesssim 0.001$); (b) exist primarily in the halo of the Galaxy; (c) have highly elliptical orbits with high inclinations to the galactic plane; and (d) are older stars with $t_{\rm age} > 1.2 \times 10^{10}$ yrs.

Population III stars were the first stars to be born out of the Big Bang. They have the following characteristics: (a) <u>no</u> metal abundance ($Z = 0$); (b) no longer exist in the Galaxy; and (c) would have ages comparable to the age of the Universe if they still existed.

(b) (10 points) Draw the two-color diagram for Population I main sequence stars and compare it to blackbody data. Why do the two deviate and why do main sequence stars show a big dip near A0 spectral class?

Solution (b): We need to redraw Figure IX-3 from page IX-15 in the notes:

The primary reason that the two deviate is due to line blanketing in the real stellar spectra. The large dip in the main sequence near $(B - V) = 0$ (*i.e.*, spectral class A0) results from the strong Balmer discontinuity (*i.e.*, continuum) near 3646 Å.

(c) (10 points) How and why does interstellar reddening affect stars plotted on the two-color diagram? Describe in words and indicate on a two-color diagram.

Solution (c): Interstellar reddening causes a star to appear redder than it normally would due blue light being scattered more effectively than red light by interstellar dust (due to Mie scattering) as shown in the diagram below (see Figure IX-4 in the notes).