ASTR-3415-001: Astrophysics
Problem Set 1 (Due: 23 January 2003)

1. (40 pts total) Wien's Displacement Law.

   a) (24 pts) Derive Wien's Displacement Law from Planck's Law. Show **all** steps in your solution! (Hint: Note that at stellar temperatures, the peak flux of a blackbody occurs at either ultraviolet, optical, or near-infrared wavelengths. As such, $hc/\lambda \gg kT$, so $e^{hc/\lambda kT} \gg 1$. As such, you can simplify your derivative by ignoring the '1' in the denominator of the Planck function when deriving your solution. Note that even though this solution represents an approximation, it still gives the correct constant as compared to the exact solution derived numerically.)

   b) (16 pts) Calculate the wavelength (in Å) of peak light emission for the stars Spica (B1 V, $T_{\rm eff}$ = 25,400 K), Vega (A0 V, $T_{\rm eff}$ = 9600 K), the Sun (G2 V, $T_{\rm eff}$ = 5770 K), and UU Aur (N2 II, $T_{\rm eff}$ = 2500 K). In what band of the electromagnetic spectrum (*i.e.*, X-ray, ultraviolet, etc.) does each of these maximum emissions correspond?

2. (25 pts) Derive the Stefan-Boltzmann Law from Planck's Law. Show **all** steps in your solution! (Hint: You will need get the integral in the functional form of $\int_0^\infty \frac{x^3}{e^x-1}$ and use the relation:

$$\int_0^\infty \frac{x^{n-1}}{e^x-1}\,dx = \Gamma(n)\left(\frac{1}{1^n}+\frac{1}{2^n}+\frac{1}{3^n}+\cdots\right).$$

   Finally, note that the series

$$\left(\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+\cdots\right)=\frac{\pi^4}{90}$$

   and that $\Gamma(n) = (n-1)!$, where $\Gamma(n)$ is the Gamma Function. Unlike the solution to Wien's Displacement Law, where you take a derivative over λ, for the Stefan-Boltzmann Law you will have to integrate over ν to get the integral of the Planck function in the form mentioned above.)

3. (20 pts) Prove for an isotropic radiation field that $F_\nu = 0$.

4. (10 pts) Problem 3.2, Page 89 in the Carroll and Ostlie textbook.

5. (35 pts) Problem 3.8, Page 90 in the Carroll and Ostlie textbook.

6. (20 pts) Problems 5.10 and 5.11, Page 156 in the Carroll and Ostlie textbook.