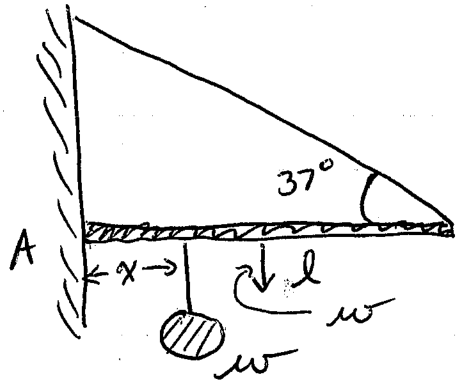
PHYS-2010
GENERAL PHYSICS I
DR. LUTTERMOSER'S CLASS
SOLUTION SET # 3
TO THE SUPPLEMENTARY HW PROBLEMS

1.

$l_{AB} = l = 4.0\ m$

$\mu_s = 0.50$ (rod on wall)

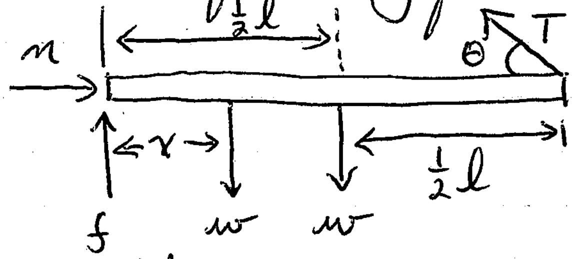
$\theta = 37^\circ$



w = weight of rod = weight of ball

When $x = x_{MIN}$, the rod is at the verge of slipping on the wall, so $f = (f_s)_{MAX} = \mu_s n = 0.50\, n$

We have the following forces in play:



Static equilibrium in x direction gives

$$\sum F_x = n - T\cos\theta = 0$$

$$n = T\cos\theta = T\cos 37^\circ = 0.80\,T$$

So

$$f = 0.50\,n = 0.50\,(0.80\,T) = 0.40\,T$$

Static equilibrium in y direction gives

$$\Sigma F_y = f + T\sin\theta - w - w = 0$$

$$0.40\,T + T\sin 37^\circ - 2w = 0$$

$$0.40\,T + 0.60\,T = 2w$$
$$T = 2w$$

Having now performed our 1st condition of static equilibrium, we can carry out the 2nd condition of equilibrium:

$$\Sigma \tau = 0.$$

Since we don't want the beam to rotate, we are free to place the rotation axis anyplace we wish. Let's choose the beam-wall interface, then the normal & the frictional force moment arms are zero and our torque equation becomes

$$\Sigma \tau = -w\underbrace{x}_{d_{ball}} - w\underbrace{\left(\tfrac{1}{2}l\right)}_{d_{beam}} + T\underbrace{l\sin\theta}_{d_T} = 0$$

where the ball and the beam torques are negative since these forces will cause clockwise rotation, and the tension force will cause CCW rotation. At $\Sigma\tau = 0$, $x = x_{MIN}$ so

$$w x_{MIN} = Tl\sin\theta - \tfrac{1}{2}lw$$

$$x_{MIN} = \frac{2w\, l \sin\theta - \frac{1}{2} l w}{w}$$

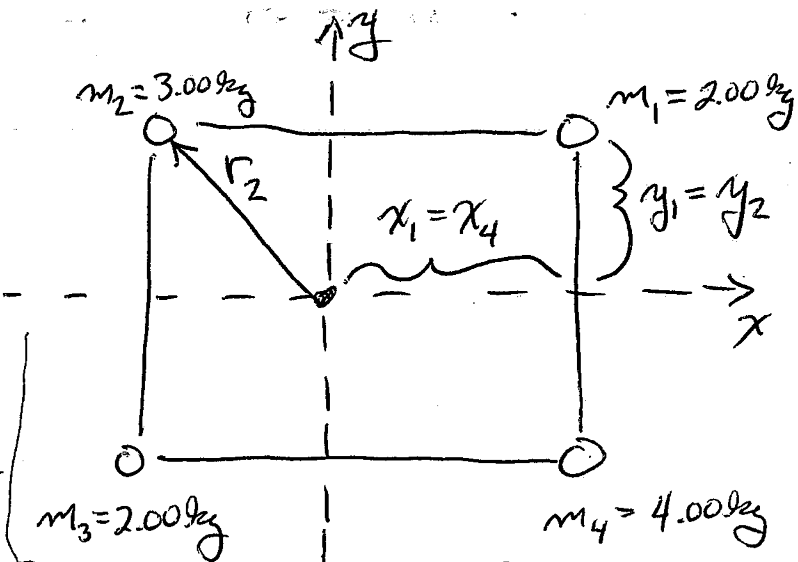
$$= \left(2\sin\theta - \tfrac{1}{2}\right) l$$

$$= \left(2\sin 37^\circ - 0.5\right)(4.0\,m)$$

$$= (0.70)(4.0\,m) = \boxed{2.8\,m}$$

2.

Moment of Inertia



a) I about x-axis:

$$I_x = \sum_{i=1}^{4} m_i y_i^2$$

$$= m_1 y_1^2 + m_2 y_2^2 + m_3 y_3^2 + m_4 y_4^2$$

$$= (2.00\,kg)(3.00\,m)^2 + (3.00\,kg)(3.00\,m)^2 + (2.00\,kg)(-3.00\,m)^2 + (4.00\,kg)(-3.00\,m)^2$$

$$= (2.00\,kg)(9.00\,m^2) + (3.00\,kg)(9.00\,m^2) + (2.00\,kg)(9.00\,m^2) + (4.00\,kg)(9.00\,m^2)$$

$$= \boxed{99.0\ kg\,m^2}$$

b) I about the y-axis (now the lever arm is the x distance from the origin):

$$I_y = \sum_{i=1}^{4} m_i x_i^2 = m_1 x_1^2 + m_2 x_2^2 + m_3 x_3^2 + m_4 x_4^2$$

$$= (2.00\,kg)(2.00\,m)^2 + (3.00\,kg)(-2.00\,m)^2 + (2.00\,kg)(-2.00\,m)^2 + (4.00\,kg)(2.00\,m)^2$$

$$= \boxed{44.0\,kg\,m^2}$$

c) I ⊥ page through origin:

$$I_0 = \sum_{i=1}^{4} m_i r_i^2$$

Note that $r_i = \sqrt{x_i^2 + y_i^2}$ and that $x_1 = -x_2 = -x_3 = x_4$ and $y_1 = y_2 = -y_3 = -y_4$, so $r_1 = r_2 = r_3 = r_4 = r$ and

$$r = \sqrt{x_1^2 + y_1^2} = \sqrt{(2.00\,m)^2 + (3.00\,m)^2}$$
$$= \sqrt{(13.0\,m^2)} = 3.61\,m$$

Hence

$$I_0 = r^2 \sum_{i=1}^{4} m_i = r^2 (m_1 + m_2 + m_3 + m_4)$$

$$= (13.0\,m^2)(2.00\,kg + 3.00\,kg + 2.00\,kg + 4.00\,kg)$$

$$= \boxed{143\,kg\,m^2}$$

Torque:

$$\tau = I\alpha, \quad \alpha = 1.50\,rad/s^2$$

$$\tau_x = I_x \alpha = (99.0\,kg\,m^2)(1.50\,rad/s^2) = \boxed{149\,N\,m}$$

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$$\tau_y = I_y \alpha = (44.0\ \text{kg m}^2)(1.50\ \text{rad/s}^2) = \boxed{66.0\ \text{Nm}}$$

$$\tau_o = I_o \alpha = (143\ \text{kg m}^2)(1.50\ \text{rad/s}^2) = \boxed{215\ \text{Nm}}$$

3.

Using the conservation of angular momentum:

$$L_{APHELION} = L_{PERIHELION}$$

$$I_{\underset{AP}{HALLEY}}\, \omega_{AP} = I_{\underset{PERI}{HALLEY}}\, \omega_{PERI}$$

where $I_{HALLEY} = m_{HALLEY} r^2 = m r^2$, so

$$m r_{AP}^2\, \omega_{AP} = m r_{PERI}^2\, \omega_{PERI}.$$

Note that $\omega = \frac{v}{r}$, so

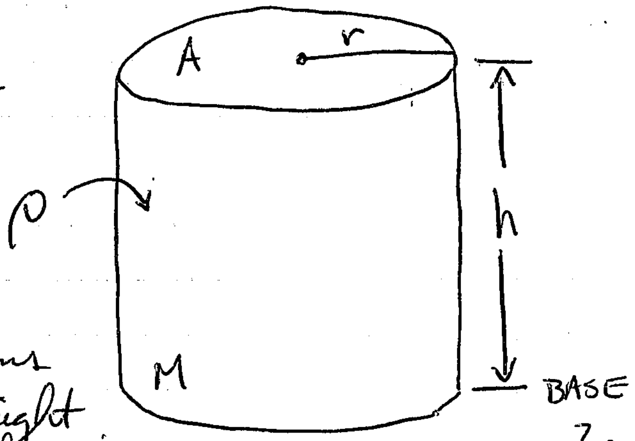
$$m r_a^2 \frac{v_a}{r_a} = m r_p^2 \frac{v_p}{r_p}$$

$$r_a v_a = r_p v_p$$

$$v_a = \left(\frac{r_p}{r_a}\right) v_p = \left(\frac{0.59\ \text{AU}}{35\ \text{AU}}\right)(54\ \text{km/s})$$

$$= \boxed{0.91\ \text{km/s}}$$

4.



$A \equiv$ surface area of cross-section

$\rho \equiv$ mass density

$M^* = \frac{w^*}{g}$ (Asterisk means mass and weight of $1.0\,m^3$ volume of material.)

$P_{MAX} = 1.7 \times 10^7$ Pa

M = total mass of cylinder

$w^* = 5.00 \times 10^4$ N and $V^* = 1.0\,m^3$

$$M^* = \frac{5.00 \times 10^4 \,N}{9.80\,m/s^2} = 5.10 \times 10^3\,kg$$

$$\rho = \frac{M}{V} = \frac{M^*}{V^*} = \frac{5.10 \times 10^3\,kg}{1.0\,m^3} = 5.10 \times 10^3\,kg/m^3$$

V = total volume of cylinder = Ah

Total weight of cylinder: $w = Mg = \rho V g = \rho A h g$

Pressure at base (ignoring air pressure) is:

$$P = \frac{F_g}{A} = \frac{w}{A} = \frac{\rho h A g}{A} = \rho h g$$

For max height: $P = P_{MAX} = \rho h g$ (note $Pa = N/m^2$)

$$h = \frac{P_{MAX}}{\rho g} = \frac{1.7 \times 10^7\,N/m^2}{(5.10 \times 10^3\,\frac{kg}{m^3})(9.80\,\frac{m}{s^2})} = \boxed{340\,m}$$

5.

The density of glucose solution is $\rho = 1.02\rho_{H_2O} = 1.02\times10^3\ kg/m^3$

The gauge pressure of the fluid at the level of the needle must be equal to the gauge pressure in the vein:

$$P_{gauge} = \rho g h = 1.33\times10^4\ Pa,$$

solving for h gives

$$h = \frac{P_{gauge}}{\rho g} = \frac{1.33\times10^4\ Pa}{(1.02\times10^3\ \frac{kg}{m^3})(9.80\ m/s^2)} = \boxed{1.33\ m}$$

6.

$w_0 = 300\ N$ and $w_{ess} = 200\ N$, $\rho_f = 0.700\,\rho_{H_2O} = 700\ kg/m^3$

Since the object is completely submerged:

$$V_o = V_f = V.$$

a)

Using Eq. (XI-17) gives

$$w_{ess} = w_o - \rho_f V g$$

or

$$V = \frac{w_o - w_{ess}}{\rho_f g} = \frac{300N - 100N}{(700\ \frac{kg}{m^3})(9.80\ \frac{m}{s^2})} = \boxed{1.46\times10^{-2}\ m^3}$$

b) Note that $\rho = \frac{m}{V} = \frac{w/g}{V} = \frac{w}{gV}$, so

$$\rho_0 = \frac{w_0}{gV} = \frac{300\,N}{(9.80\,\frac{m}{s^2})(1.46\times10^{-2}\,m^3)} = \boxed{2.10\times10^{3}\,kg/m^3}$$

7.

a) The volume flow rate is Av and the mass flow rate (i.e, the mass flux) is

$$\rho A v = (1.0\,\tfrac{gm}{cm^3})(2.0\,cm^2)(40\,cm/s)$$
$$= \boxed{80\,gm/s}$$

b) From the equation of continuity, the speed in the capillaries is (assuming the blood is an ideal fluid)

$$v_{CAP}\,A_{CAP} = v_{AORTA}\,A_{AORTA}$$

or

$$v_{CAP} = \left(\frac{A_{AORTA}}{A_{CAP}}\right) v_{AORTA} = \left(\frac{2.0\,cm^2}{3.0\times10^{3}\,cm^2}\right)(40\,cm/s)$$
$$= 2.7\times10^{-2}\,cm/s = \boxed{0.27\,mm/s}$$

8.

From Poiseuille's law in Eq. (XI-29), the flow rate in the artery is (after converting to SI units)

$$Q = \frac{\Delta P \pi R^4}{8\eta L} = \frac{(400\,Pa)\pi(2.6\times10^{-3}\,m)^4}{8(2.7\times10^{-3}\,Ns/m^2)(8.4\times10^{-2}\,m)}$$

$$= 3.2\times10^{-5}\,m^3/s.$$

Thus, the flow speed is determined from $Q = vA$ or

$$v = \frac{Q}{A} = \frac{Q}{\pi r^2} = \frac{3.2\times10^{-5}\,m^3/s}{\pi(2.6\times10^{-3}\,m)^2}$$

$$= \boxed{1.5\,m/s}$$

9.

We are given $D = 2.0\,cm = 2.0\times10^{-2}\,m$, $v = 55\,\frac{cm}{s} = 0.55\,\frac{m}{s}$

$\rho = 1050\,kg/m^3$ and $\eta = 2.7\times10^{-3}\,Ns/m^2$.

We need to calculate the Reynold's number to see if the flow is turbulent (see Eq. XI-30):

$$R = \frac{\rho v D}{\eta} = \frac{(1050\,\frac{kg}{m^3})(0.55\,m/s)(2.0\times10^{-2}\,m)}{2.7\times10^{-3}\,Ns/m^2}$$

$$= 4.3\times10^3$$

Since $R > 3000$, the flow is $\boxed{\text{turbulent}}$!

10.

We have $\alpha = 130 \times 10^{-6}\ {}^\circ C^{-1}$. At $T_0 = 20.0^\circ C$, $r_0 = 2.20 cm$.
Need T to get $r = 2.21$ cm.

$$\Delta L = \alpha L_0 \Delta T \quad \text{or} \quad \Delta T = \frac{1}{\alpha}\frac{\Delta L}{L_0}$$

In reality, it is the circumference that is expanding, so $L = C = 2\pi r$ and

$$\frac{\Delta L}{L_0} = \frac{\Delta C}{C_0} = \frac{2\pi r - 2\pi r_0}{2\pi r_0} = \frac{2\pi(r - r_0)}{2\pi r_0}$$
$$= \frac{\Delta r}{r_0}$$

Hence, $$T - T_0 = \frac{1}{\alpha}\frac{\Delta r}{r_0}$$

or $$T = T_0 + \frac{1}{\alpha}\frac{\Delta r}{r_0} = 20.0^\circ C + \frac{1}{130\times 10^{-6}\ {}^\circ C^{-1}}\ \frac{2.21 cm - 2.20 cm}{2.20 cm}$$
$$= \boxed{55.0^\circ C}$$

11.

$m_{pellet} = m_l = 1.00 gm = 1.00 \times 10^{-3} kg$

$T_{pellet} = T_l = 200^\circ C$, $m_{water} = M_w = 500 gm = 0.500 kg$, $T_w = 20.0^\circ C$

$T_{eq} = T = 25.0^\circ C$. The total mass of the lead pellets is

$M_l = N m_l$ where N is the number of lead pellets.

Note that $c_l = 128 \frac{J}{kg\,^\circ C}$ and $c_w = 4186 \frac{J}{kg\,^\circ C}$.

Using the conservation of heat energy, we get

$$Q_{LOSS} = Q_{GAIN}$$

$$Q_{LEAD} = Q_{WATER}$$

$$M_l c_l \Delta T_l = m_w c_w \Delta T_w$$

$$N m_l c_l (T_l - T) = m_w c_w (T - T_w)$$

$$N = \frac{m_w c_w (T - T_w)}{m_l c_l (T_l - T)}$$

$$N = \frac{(0.500\,kg)(4186\,J/kg\cdot°C)(25.0°C - 20.0°C)}{(1.00\times10^{-3}\,kg)(128\,J/kg\cdot°C)(200°C - 25.0°C)}$$

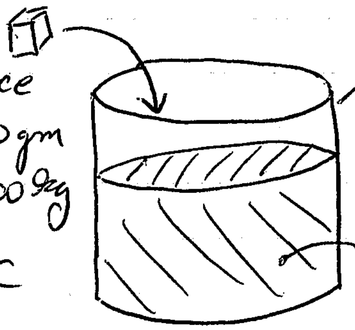
$$= \boxed{467}$$

12.

Ice

$m_I = 100\,gm$
$= 0.100\,kg$

$T_I = 0°C$



container is Al, $m_c = 100\,gm$
$= 0.100\,kg$

$T_c = 30°C$

$c_{Al} = 900\,\frac{J}{kg\,°C}$

water, $m_w = 180\,gm = 0.180\,kg$

$c_w = 4186\,\frac{J}{kg\,°C}$

$T_w = T_c = 30°C$ (at equilibrium)

a) How much ice remains after cooling container to $T_f = 0°C = T_I$?

$$\Delta Q_I = Q_{Al} + Q_w = m_c c_{Al}(T_c - T_I) + m_w c_w (T_w - T_I)$$

$$= (0.100\,kg)(900\,\tfrac{J}{kg\,°C})(30°C - 0°C)$$

$$+ (0.180\,kg)(4186\,\tfrac{J}{kg\,°C})(30°C - 0°C)$$

$$= 25{,}304\,J$$ (we'll take care of sig. digits later)

The amount of ice at 0°C which must melt to absorb this much heat is

$$m_I' = \frac{\Delta Q_I}{L_f} = \frac{25{,}304\,J}{3.33 \times 10^5\,J/kg} = 7.60 \times 10^{-2}\,kg = 76\,gm$$

The amount leftover is:

$$\Delta m_I = m_I - m_I' = 100\,gm - 76\,gm = \boxed{24\,gm}$$

b) Assume initial $m_I = 50\,gm$. Since 76 gm of ice is needed to get water to $T_f = 0°C$, all the ice will melt before $T = 0°C$ is reached.

$$\Delta Q_{SYS} = Q_{\substack{MELTING\\ICE}} + Q_{\substack{WATER\\FROM\ ICE}} + Q_{WATER} + Q_{CONTAINER} = 0$$

$$= m_I L_f + m_I c_w (T_f - T_I) + m_w c_w (T_f - T_w)$$

$$+ m_c c_{Al}(T_f - T_c) \doteq 0$$

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$$\Delta Q_{sys} = (0.050\,kg)(3.33\times10^5\ \tfrac{J}{kg})$$
$$+ (0.050\,kg)(4186\ J/kg\,^\circ C)(T_f - 0^\circ C)$$
$$+ (0.18\,kg)(4186\ J/kg\,^\circ C)(T_f - 30^\circ C)$$
$$+ (0.10\,kg)(900\ J/kg\,^\circ C)(T_f - 30^\circ C) = 0$$

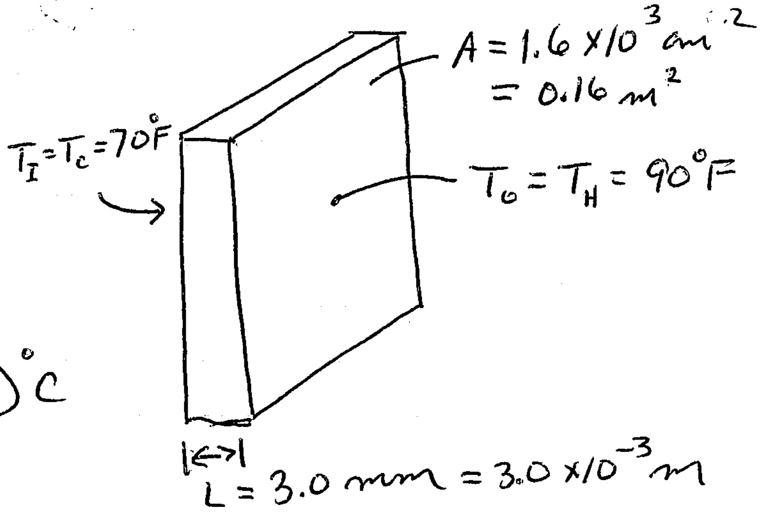
$$= 1.665\times10^4\,J + (209.3\ \tfrac{J}{^\circ C})T_f$$
$$+ (753.5\ J/^\circ C)T_f - 2.260\times10^4\,J$$
$$+ (90.0\ J/^\circ C)T_f - 2.700\times10^3\,J = 0$$

$$(1.053\times10^3\ \tfrac{J}{^\circ C})\,T_f = 8.65\times10^3\,J$$

$$\boxed{T_f = 8.2^\circ C}$$

13.

a) $\Delta T = T_H - T_c$
$= T_o - T_I$
$= 90^\circ F - 70^\circ F$
$= 20^\circ F$



$$\Delta T_c = \frac{5}{9}\Delta T_F = \frac{5}{9}(20)^\circ C$$
$$= 11.1^\circ C$$

$$P_{COND} = \frac{\Delta Q}{\Delta t} = \frac{kA\Delta T}{L} = \frac{(0.8\ \frac{J}{s\,m\,^\circ C})(0.16\,m^2)(11.1^\circ C)}{3.0\times10^{-3}\,m}$$

$$= 470\ J/s = \boxed{470\ W}$$ into house

b) Let $T_0 = T_c = 0°F$ and $T_I = T_H = 70°F$

$$\Delta T_C = \frac{5}{9}\Delta T_F = \frac{5}{9}(T_I - T_0) = \frac{5}{9}(70-0)°C$$
$$= 38.9°C$$

$$\mathcal{P}_{COND} = \frac{kA\Delta T}{L} = \frac{(0.8\frac{J}{s\,m\,°C})(0.16\,m^2)(38.9°C)}{\underline{3}\times 10^{-3}\,m}$$
$$= 1660\,\frac{J}{s} = \underline{1}660\,W = \boxed{\underline{2}000\,W}$$ out of house

14. Let STAR X = STAR 1 and STAR Y = STAR 2
Use Eq (XII-17) with $e=1$ and $A = 4\pi R^2$. Since both stars have the same radius, $A_1 = A_2$.

$$\frac{\mathcal{P}_{em-2}}{\mathcal{P}_{em-1}} = \frac{\sigma A_2 e T_2^4}{\sigma A_1 e T_1^4} = \left(\frac{T_2}{T_1}\right)^4$$
$$= \left(\frac{11,727\,K}{5,727\,K}\right)^4 = (2.05)^4 = \boxed{17.58}$$

15. The mass of the water in the heater is

$$m = \rho V = \underbrace{\left(10^3\,\frac{kg}{m^3}\right)}_{\rho}\underbrace{\left[(50.0\,gal)\left(\frac{3.786\,L}{1\,gal}\right)\left(\frac{1\,m^3}{10^3\,L}\right)\right]}_{V}$$

or

$$m = 189\,kg$$

The energy required to raise the temperature of the water from 20.0°C to 60.0°C is

$$Q = mc\Delta T = (189\ kg)\left(4186\ \frac{J}{kg\ ^\circ C}\right)(60.0^\circ C - 20.0^\circ C)$$
$$= 3.17 \times 10^7\ J$$

The heating element has a power of

$$P = 4800\ W = 4800\ \frac{J}{s}$$

and power is related to heat content by

$$P = \frac{Q}{t},$$

so the time required for the water heater to transfer this energy is

$$t = \frac{Q}{P} = \frac{3.17 \times 10^7\ J}{4800\ J/s} = 6600\ s$$

$$= 6600\ s \times \frac{1\ hr}{3600\ s} = \boxed{1.83\ hr}$$