PHYS-2010
GENERAL PHYSICS I SOLUTIONS
TO PROBLEM SET 4

1.

Since the cylinder is not leaking, the number of gas particles present remains constant, so n_2 (number of moles after compression) $= n_1$ (number of moles before compression).

Here we will use Form 1 of the ideal gas law: $PV = nRT$, and rewrite it as

$$n = \frac{PV}{RT}$$

SO:

$$n_2 = n_1 \quad \text{and } T_1 = 27^\circ C + 273 = 300\ K$$

$$\frac{P_2 V_2}{R T_2} = \frac{P_1 V_1}{R T_1}$$

$$\left(\frac{P_2 V_2}{P_1 V_1}\right) T_1 = T_2$$

$$T_2 = \left(\frac{0.800 \times 10^5\ Pa}{0.200 \times 10^5\ Pa}\right)\left(\frac{0.700\ m^3}{1.50\ m^3}\right)(300\ K)$$

$$= 560\ K - 273 = \boxed{287\ ^\circ C}$$

2.

a) $n = 1\,\text{mol}$ and $T = 300\,\text{K}$

Eq. (XIII-14) gives $\overline{KE} = \frac{3}{2} k_B T$ per particle

or

$$\overline{KE} = \frac{3}{2}\left(1.38\times10^{-23}\,\frac{J}{K}\right)(300\,K)/\text{particle}$$

$$= \boxed{6.21\times10^{-21}\ J/\text{particle}}$$

However, there are $n = 1\,\text{mol}$ or nN_A particles per mole, so we could also express this KE as

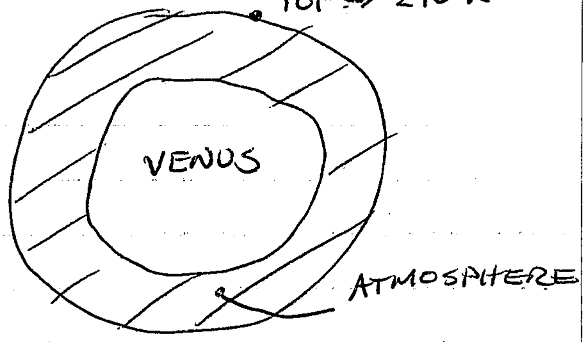
$$\overline{KE} = \left(6.21\times10^{-21}\,\frac{J}{\text{particle}}\right) n N_A$$

$$= \left(6.21\times10^{-21}\,\frac{J}{\text{particle}}\right)(1\,\text{mol})\left(6.02\times10^{23}\,\frac{\text{part.}}{\text{mol}}\right)$$

$$= \boxed{3740\ J}$$

total KE energy for all the particles making up this 1 mole of gas.

3.



$v_{esc} = 10.3\,\text{km/s}$

a) $v_{rms}(H_2) = ?$

$$v_{rms} = \sqrt{\frac{3k_B T}{m}}$$

$$k_B = 1.38\times10^{-23}\ J/K$$

$$m_{H_2} = 2m_H = 2(1.67\times10^{-27}\,kg) = 3.34\times10^{-27}\,kg$$

$$v_{rms}(H_2) = \sqrt{\frac{3(1.38\times10^{-23}\,J/K)(240K)}{3.34\times10^{-27}\,kg}}$$

$$(1\,J = 1\,kg\,m^2/s^2)$$

$$v_{rms}(H_2) = \sqrt{2.97\times10^{6}\,m^2/s^2} = 1.72\times10^{3}\,m/s$$

$$\boxed{v_{rms}(H_2) = 1.72\ km/s}$$

b) $v_{rms}(CO_2) = ?$

$$m(CO_2) = m_c + 2m_o = (12.01\,amu)(1.67\times10^{-27}\,\tfrac{kg}{amu}) + 2(16.00\,amu)(1.67\times10^{-27}\,kg/amu)$$

$$= 7.35\times10^{-26}\,kg$$

$$v_{rms}(CO_2) = \sqrt{\frac{3(1.38\times10^{-23}\,J/K)(240K)}{(7.35\times10^{-26}\,kg)}}$$

$$= 3.68\times10^{2}\,m/s = \boxed{0.368\ km/s}$$

c) $v_{esc} = 10.3\ km/s$, if $v_{rms} > \frac{v_{esc}}{6}$, then the gas will not be bound to the planet for an extended period of time.

For H_2, is $v_{rms} > \frac{1}{6}(10.3\,\frac{km}{s}) = 1.72\ km/s$? Since $v_{rms}(H_2) \approx v_{esc}/6$, H_2 will escape, but just barely. Meanwhile $v_{rms}(CO_2) < \frac{1}{6}v_{esc}$, so CO_2 will remain for an extended period of time.

4.

(a) The initial absolute pressure in the tire is

$$P_1 = P_0 + (P_1)_{GAUGE}, \text{ where } P_0 = P_{atm} = 1.00 \text{ atm}$$

$$= 1.00 \text{ atm} + 1.80 \text{ atm} = 2.80 \text{ atm}$$

The final pressure is thus $P_2 = 1.00 \text{ atm} + 2.20 \text{ atm}$
$= 3.20 \text{ atm}$

Since volume is constant use the Charles and Gay-Lussac Law: $\frac{P_1}{T_1} = \frac{P_2}{T_2}$, solving for T_2 gives

$$T_2 = T_1\left(\frac{P_2}{P_1}\right) = (300\text{ K})\left(\frac{3.20 \text{ atm}}{2.80 \text{ atm}}\right) = \boxed{343\text{ K}}$$

b) When the quantity of gas varies while T and V are constant, the ideal gas law gives $\frac{V}{T} = \frac{nR}{P}$ = constant, so

$$\frac{n_2 R}{P_2} = \frac{n_3 R}{P_3} \quad \text{or} \quad \frac{n_3}{n_2} = \frac{P_3}{P_2}$$

so

$$\frac{n_3}{n_2} = \frac{2.80 \text{ atm}}{3.20 \text{ atm}} = 0.875$$

At the end, we have 87.5% of the original mass of air remaining, or

$\boxed{\text{12.5\% of the original mass}}$ was released.

5.

a) Work done by changing volume is $W = P\Delta V$

$$P = 0.800\,atm \times 1.013\times10^{5}\,\frac{Pa}{atm} = 8.104\times10^{4}\,Pa$$

$$\Delta V = V_f - V_i = 2.00\,L - 9.00\,L = -7.00\,L \times 10^{-3}\,\frac{m^3}{L}$$

$$= -7.00\times10^{-3}\,m^3$$

$$W = -P\Delta V = -(8.104\times10^{4}\,Pa)(-7.00\times10^{-3}\,m^3)$$

$$= +5.67\times10^{2}\,N\cdot m = \boxed{+567\,J}$$

b) Change in internal energy: $\Delta U = Q + W$

$Q = -400\,J$ (negative since it flows out)

$$\Delta U = -400\,J + (567\,J) = \boxed{+167\,J}$$

6.

a) Engine's efficiency (e)?

heat at coldest temp

$$e = \frac{W}{Q_h} = \frac{\text{work done during cycle}}{\text{heat at hottest temp}} = \frac{Q_h - Q_c}{Q_h}$$

$$= \frac{1700\,J - 1200\,J}{1700\,J} = \frac{500\,J}{1700\,J} = \boxed{0.29}$$

29% efficiency

b) Work done in each cycle?

$$W = Q_h - Q_c = 1700\,J - 1200\,J = \boxed{500\,J}$$

c) Power output if each cycle lasts 0.300 s?

$$P = \frac{W}{\Delta t} = \frac{500\,J}{0.300\,s} = 1.67\times 10^3\,W = 1.67\,kW$$

7.

$$T_\odot = 5700\,K, \quad T_\oplus = 290\,K, \quad Q = 1000\,J$$

$$\Delta S_\odot = \frac{-Q}{T_\odot}$$ (entropy is negative since Sun is losing it)

↑ SUN

$$\Delta S_\oplus = \frac{Q}{T_\oplus}$$ (positive since Earth is gaining it)

↑ EARTH

$$\Delta S_{TOTAL} = \Delta S_\odot + \Delta S_\oplus = -\frac{Q}{T_\odot} + \frac{Q}{T_\oplus} = Q\left(\frac{1}{T_\oplus} - \frac{1}{T_\odot}\right)$$

$$= (1000\,J)\left(\frac{1}{290\,K} - \frac{1}{5700\,K}\right) = \boxed{3.27\,J/K}$$

8. Let probability $\equiv P$

a) Only 1 ace of spades per 52 cards: $\boxed{P = \frac{1}{52}}$

b) Only 4 aces out of 52 cards: $P = \frac{4}{52} = \boxed{\frac{1}{13}}$

c) 13 spades out of 52 cards: $P = \frac{13}{52} = \boxed{\frac{1}{4}}$