PHYS-2010: General Physics I
Course Lecture Notes
Section II

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Abstract

These class notes are designed for use of the instructor and students of the course PHYS-2010: General Physics I taught by Dr. Donald Luttermoser at East Tennessee State University. These notes make reference to the *College Physics, 11th Edition* (2018) textbook by Serway and Vuille.

II. Mathematical Techniques

A. Dimensional Analysis.

1. Always make sure that all terms in an equation have the same dimensions (*i.e.*, units).

2. Then try to reduce a parameter in an equation to a combination of the three basic concepts: **length** [**L**], **mass** [**M**], and **time** [**T**].

3. For example, the acceleration of a body in a gravitational field is proportional to the mass of the primary body and inversely proportional to the square of the distance:

$$a = G\,\frac{M}{r^2}\ ,$$

where G is a constant. From this formula, find the dimensions of G.

$$[a] = \ \mathbf{L}\,\mathbf{T}^{-2} \qquad [M] = \ \mathbf{M} \qquad [r] = \ \mathbf{L}\ ,$$

where **L** represents *length*, **M** represents *mass*, and **T** represents *time*. Then

$$G = \frac{ar^2}{M} \quad \Longrightarrow \quad [G] = \frac{[a]\ [r]^2}{[M]} = \frac{\mathbf{L}\,\mathbf{T}^{-2}\,\mathbf{L}^2}{\mathbf{M}} = \mathbf{L}^3\,\mathbf{M}^{-1}\,\mathbf{T}^{-2}$$

or the dimensions of G in the *basic* (*i.e.*, *fundamental*) units in SI are $\mathrm{m}^3/\mathrm{kg}/\mathrm{s}^2$.

4. When a symbol or variable has *square brackets* around it, this means: *what are the dimensions (i.e., units) of this symbol or variable*?

B. Algebra Review.

1. Cross multiplication: $mx = ny \Longleftrightarrow \dfrac{x}{y} = \dfrac{n}{m}$.

2. Factoring: $y = mx + mb \Longleftrightarrow y = m(x+b)$.

3. Powers & Roots:

a) $a \times a \times a \times a \times a \times a \times a = a^7$
or
$a \times a \times \cdots (m - \text{times}) \cdots \times a = a^m$
"a" is raised to the "m^{th}" power.

b) $a^{1/m} = \sqrt[m]{a} \Longrightarrow$ "m^{th}" root of "a."

c) $a^0 \equiv 1$ (note that the "$\equiv$" symbol means "*defined to be*").

d) $a^{-m} = \dfrac{1}{a^m}$.

e) $(ab)^m = a^m b^m, \quad \left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m} = a^m b^{-m}$.

f) $a^m a^n = a^{m+n}, \quad \dfrac{a^m}{a^n} = a^{m-n}$.

g) $(a^m)^n = a^{mn}, \quad \sqrt[n]{a^m} = a^{m/n}$.

4. Exponentials and Logarithms:

$$\begin{array}{rcll} y & = & a^x & \text{(base ``}a\text{'' to power ``}x\text{'')} \\ x & = & \log_a y & \text{(the exponent of ``}a\text{'' that yields ``}y\text{'')} \end{array}$$

a) Product: $\log_a(xy) = \log_a x + \log_a y$.

b) Quotient: $\log_a\left(\dfrac{x}{y}\right) = \log_a x - \log_a y$.

c) Power: $\log_a(y^n) = n \log_a y$.

d) Two common bases:

i) Base 10 $\Rightarrow$ **common logarithms**:

$$\boxed{\log_a = \log_{10} \equiv \log}$$

$$x = \log y \quad \Longleftrightarrow \quad y = 10^x \ .$$

ii) Base $e = 2.71828...$ $\Rightarrow$ **natural logarithms**:

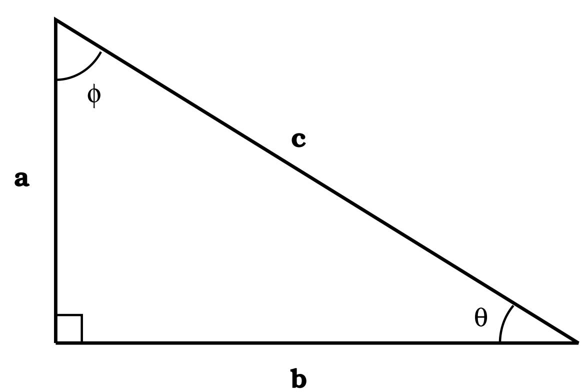
$$\boxed{\log_a = \log_e \equiv \ln}$$

$$x = \ln y \quad \Longleftrightarrow \quad y = e^x \ .$$

C. Basic Trigonometry.

1. Right-Angle Triangle Relationships:

a) Consider the triangle in the following figure – here a and b are called '**legs**' of this triangle and c is called the **hypotenuse**, which is the side opposite the right angle (indicated with a 'square box' in the figure below).



b) Note that the sum of the internal angles of a triangle is equal to 180°, as such, $\theta + \phi + 90° = 180°$. The angles θ and ϕ in the diagram above must add up to 90° since the third angle is the 'right' angle (*i.e.*, 90°). The angles θ and ϕ are said to be *complementary angles*. Also note that besides 'degrees,' we can also measure angles in units of 'radians,' where π radians $= 180°$.

c) Below we define the 3 primary trigonometry functions, *sine, cosine*, and *tangent*, and we show the Pythagorean Theorem in terms of the sides of a right-angle triangle (*i.e.*, $a^2+b^2=c^2$) and the angular version of this theorem (*i.e.*, $\sin^2\theta+\cos^2\theta=1$).

$$\begin{array}{ccc} \sin\theta = \dfrac{a}{c}, & \cos\theta = \dfrac{b}{c}, & \tan\theta = \dfrac{a}{b} = \dfrac{\sin\theta}{\cos\theta} \\ a^2+b^2=c^2 & \text{or} & \sin^2\theta+\cos^2\theta=1 \\ \theta+\phi & = 90° = & \dfrac{\pi}{2}\ \text{radians.} \end{array}$$

d) So we see that the **sine** of an angle (*i.e.*, using angle θ in the figure above) is equal to the ratio of the length of the *opposite side* (*i.e.*, side a) of the angle to the length of the *hypotenuse* (*i.e.*, side c); the **cosine** of the angle is defined to be the ratio of the length of the *adjacent side* (*i.e.*, side b) to the length of the *hypotenuse*, and the **tangent** of the angle is the length of the *opposite side* to the length of the *adjacent side*.

e) All trigonometric functions have an inverse function associated with the specific function. For example, for angle θ in the triangle on page II-3 we have:

$$\theta = \sin^{-1}\left(\frac{a}{c}\right) = \cos^{-1}\left(\frac{b}{c}\right) = \tan^{-1}\left(\frac{a}{b}\right) \ .$$

f) Two triangles are said to be *similar* if they have the same internal angles. When we have similar triangles, the ratio of corresponding sides in both triangles are always equal, *regardless* of the lengths of their sides. This fact is often useful when dealing with vectors in physics. We will make use of this when we represent vector addition and subtraction graphically.

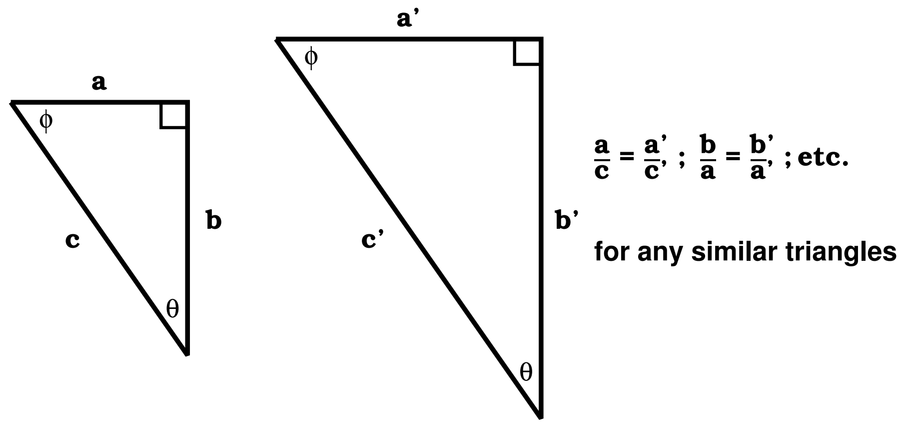
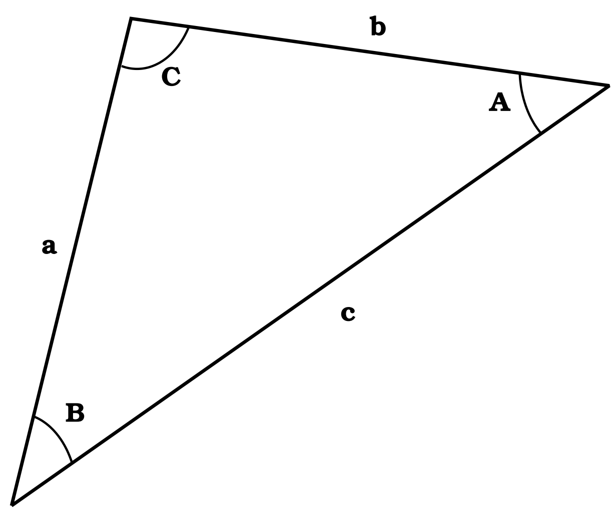


Figure II–1: Similar triangles and their mathematical relationships.

2. **Generic triangle relationships:**

Usually, the mathematics of right-angle triangles is sufficient to add and subtract vector quantities. Occasionally, the more general triangle relations summarized here may be useful.



a) Law of sines:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \ .$$

b) Law of cosines:

$$
\begin{aligned}
a^2 &= b^2 + c^2 - 2bc\cos A \ , \\
b^2 &= a^2 + c^2 - 2ac\cos B \ , \\
c^2 &= a^2 + b^2 - 2ab\cos C \ .
\end{aligned}
$$

c) The equations above correspond to the generic triangle shown in the above figure. There are other useful trigonometric identities that are sometimes useful in solving physics problems. In the equations below, angles α and β are two angles in a given generic triangle shown above.

3. Additional useful trigonometric identities:

a) Angle-sum and angle-difference relations:

$$
\begin{aligned}
\sin(\alpha+\beta) &= \sin\alpha\,\cos\beta + \cos\alpha\,\sin\beta \\
\sin(\alpha-\beta) &= \sin\alpha\,\cos\beta - \cos\alpha\,\sin\beta \\
\cos(\alpha+\beta) &= \cos\alpha\,\cos\beta - \sin\alpha\,\sin\beta \\
\cos(\alpha-\beta) &= \cos\alpha\,\cos\beta + \sin\alpha\,\sin\beta \\
\tan(\alpha+\beta) &= \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\,\tan\beta} \\
\tan(\alpha-\beta) &= \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\,\tan\beta}
\end{aligned}
$$

b) Double-angle relations:

$$
\begin{aligned}
\sin 2\alpha &= 2\sin\alpha\,\cos\alpha = \frac{2\tan\alpha}{1+\tan^2\alpha} \\
\cos 2\alpha &= \cos^2\alpha - \sin^2\alpha = 2\cos^2\alpha - 1 = 1 - 2\sin^2\alpha \\
&= \frac{1-\tan^2\alpha}{1+\tan^2\alpha} \\
\tan 2\alpha &= \frac{2\tan\alpha}{1-\tan^2\alpha}
\end{aligned}
$$

D. Scalars and Vectors.

1. A **scalar** has magnitude but <u>no</u> directional information (*e.g.*, 'v' is a scalar).

a) 4 kg and 600 K are scalars.

b) 420 km/s is a scalar (*i.e.*, speed).

2. A **vector** has <u>both</u> magnitude and directional information (*e.g.*, '$\vec{v}$' is a vector).

a) 420 km/s to the NW (northwest) is a vector (*i.e.*, velocity).

b) 420 km/s NW is <u>not</u> equal to 420 km/s SE (southeast)!

c) Note that in these course notes I will always represent a vector with an arrow over the variable letter (*e.g.*, $\vec{A}$), whereas your textbook indicates a vector with a boldface letter (*e.g.*, $\boldsymbol{A}$).

d) Arithmetic for scalars and vectors are handled <u>differently</u> with respect to each other. We will describe vector arithmetic in §IV of these notes.