PHYS-2010: General Physics I
Course Lecture Notes
Section V

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Edition 2.6

Abstract

These class notes are designed for use of the instructor and students of the course PHYS-2010: General Physics I taught by Dr. Donald Luttermoser at East Tennessee State University. These notes make reference to the *College Physics, 11th Edition* (2018) textbook by Serway and Vuille.

V. The Laws of Motion

A. Force.

1. We now have explored the concept of motion and defined its terms. We now can ask the question: *What causes a given motion*?

$\Longrightarrow$ This is answered in the field of

Classical Mechanics.

2. Assumptions of classical mechanics:

a) Objects are large with respect to the dimensions of the atom ($\approx 10^{-10}$ m) [if violated $\Longrightarrow$ use quantum mechanics].

b) Objects move at velocities much less than the speed of light ($c = 2.997925 \times 10^{8}$ m/s) [if violated $\Longrightarrow$ use special relativity].

3. Force represents the interaction of an object with its environment.

a) **Contact forces:** The act of pushing or pulling an object (sometimes called *mechanical force*).

b) **Field forces:** No physical contact necessary $\Longrightarrow$ force is transmitted via the "field" (*e.g.*, gravity).

B. Newton's Laws of Motion.

1. **Newton's 1st Law: Law of Inertia:** An object at rest remains at rest, and an object in motion continues in motion with a constant velocity, unless it is acted upon by an external force.

a) **Inertia** is the resistance that matter has to changes in motion.

b) The **mass** of an object measures that object's inertia.
$\Longrightarrow$ Mass is nothing more than a measure of matter's resistance to changes in motion.

2. **Newton's 2nd Law:** The acceleration (a) of an object is directly proportional to the resultant force (F) acting on it and inversely proportional to its mass (m). The direction of the acceleration is the same direction as the resulting force.

$$\boxed{\sum \vec{F} = m\,\vec{a}\ .} \tag{V-1}$$

a) **This is arguably the most important equation in physics and possibly all of science.**

b) The significance of the summation sign is that one needs to sum all of the forces acting on a mass to determine the acceleration of the mass, hence for N forces acting on a mass, the acceleration would be determined by

$$\vec{a} = \frac{\vec{F}_{\rm total}}{m} = \frac{1}{m}\sum_i^N \vec{F}_i = \frac{1}{m}\left(\vec{F}_1 + \vec{F}_2 + \cdots + \vec{F}_N\right)\ .$$

The summing of parts to make a whole as indicated in Eq. (V-1) is called the **principle of superposition**.

c) Also, since force is a vector, we must break the forces up into directional components for each force acting on a mass.

i) Again, the principle of superposition.

ii) As such, should we have motion in 3-dimensions in orthogonal coordinates, we would write the equation above as three separate equations:

$$\begin{aligned}
\vec{a}_x &= \frac{\vec{F}_{x\,\mathrm{total}}}{m} = \frac{1}{m}\sum_i^N \vec{F}_{x\,i} \\
\vec{a}_y &= \frac{\vec{F}_{y\,\mathrm{total}}}{m} = \frac{1}{m}\sum_i^N \vec{F}_{y\,i} \\
\vec{a}_z &= \frac{\vec{F}_{z\,\mathrm{total}}}{m} = \frac{1}{m}\sum_i^N \vec{F}_{z\,i} \ ,
\end{aligned}$$

and the total acceleration would be determined from vector arithmetic:

$$\vec{a} = \vec{a}_x + \vec{a}_y + \vec{a}_z \ .$$

d) Force is measured in **newtons** in the SI system:

$$\boxed{1\ \mathrm{N} \equiv 1\ \mathrm{kg{\cdot}m/s^2},} \qquad \text{(V-2)}$$

$\Longrightarrow$ or in the cgs system:

$$1\ \mathrm{dyne} \equiv 1\ \mathrm{g{\cdot}cm/s^2} = 10^{-5}\ \mathrm{N},$$

$\Longrightarrow$ or in the English system:

$$1\ \mathrm{lb} \equiv 1\ \mathrm{slug{\cdot}ft/s^2} = 4.448\ \mathrm{N}.$$

3. Newton's 3rd Law: If 2 bodies interact, the magnitude of the force exerted on body 1 by body 2 is equal to the magnitude of the force exerted on body 2 by body 1, and these forces are in opposite direction to each other.

a) Another way of saying this is "for every action, there is an opposite reaction."

b) We will see later in the course that Newton's 3rd law is nothing more than the *conservation of linear momentum.*

C. Common Forces of Everyday Life.

1. The Force of Gravity — Weight and the Normal Force.

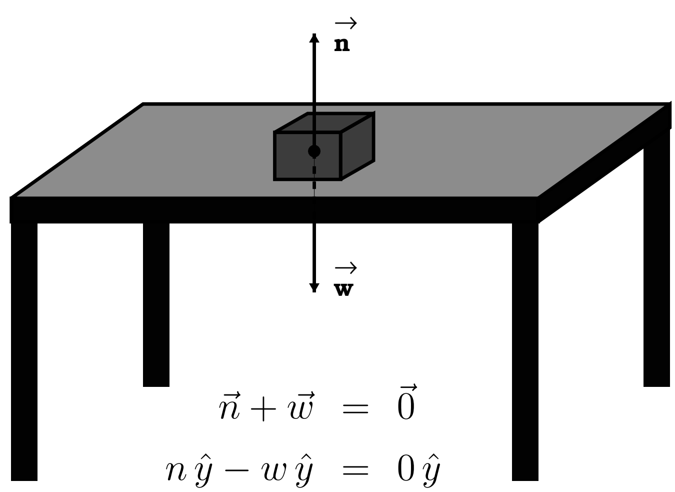
a) All objects with mass exert a force called **gravity**. It is one of the 4 forces of nature $\Longrightarrow$ a natural force.

b) The **weight** of an object is nothing more than a measure of the gravitational force on an object:

$$\boxed{\sum \vec{F}_g \equiv \vec{w} = m\,\vec{g}\ ,} \tag{V-3}$$

c) The acceleration $\vec{a}$ due to gravity is labeled $\vec{g}$ — $\vec{g}$ is referred to as the **surface gravity**. Each gravitating source has its own surface gravity. Most of the time in this course, $\vec{g}$ will refer to the surface gravity on Earth.

d) Assume we have an object <u>resting</u> on a desk. The weight of the object, or the gravitational force downward, is balanced by the force of the desk pushing upward (as there must be to keep the object from moving in response to gravity).



$$\begin{aligned} \vec{n} + \vec{w} &= \vec{0} \\ n\,\hat{y} - w\,\hat{y} &= 0\,\hat{y} \\ n - w &= 0 \\ n &= w \end{aligned}$$

i) This upward force is called the **normal force**, $\vec{n}$.

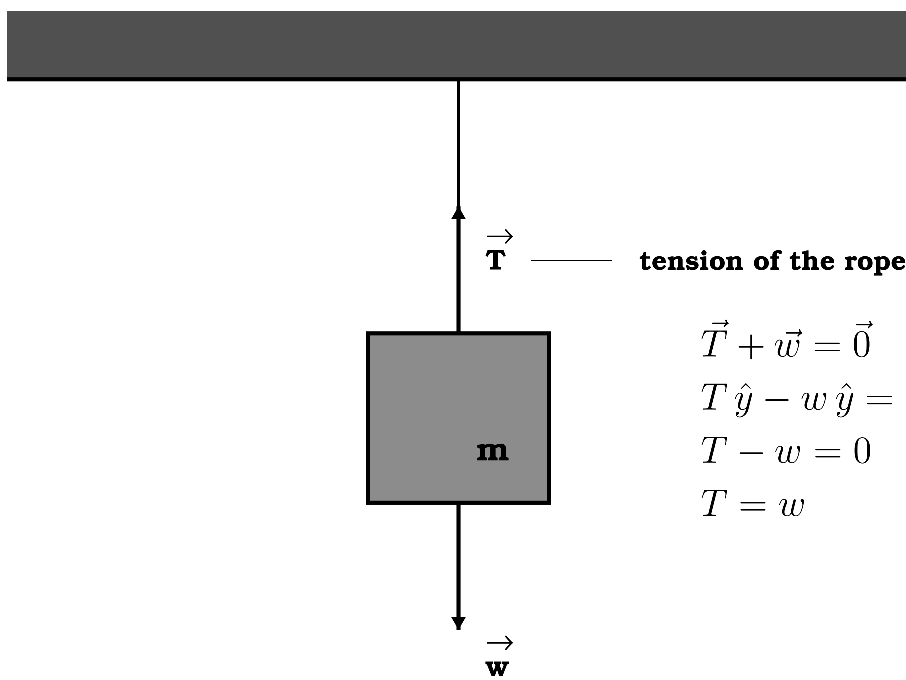
ii) “Normal” because this force is perpendicular to the surface of the desk.

iii) This normal force results from the electromagnetic forces between the atoms and molecules in the desk that make it rigid (*i.e.*, a solid).

2. The Concept of Tension.

a) Above, we introduced the concept of the normal force (this is essentially an “equilibrium” type of force — see below).

b) If an object hangs from wires or ropes, another equilibrium force (*i.e.*, counteracting gravity) is the **tension** of the rope:



$$\vec{T} + \vec{w} = \vec{0}$$
$$T\,\hat{y} - w\,\hat{y} = 0\,\hat{y}$$
$$T - w = 0$$
$$T = w$$

3. Frictional Forces.

a) Bodies in motion often feel **frictional forces** (*i.e.*, from surfaces, air, etc.) which retard their motion $\Longrightarrow$ **frictional force is in the <u>opposite</u> direction of the direction of motion**.

b) If an object does not move on a surface ($\vec{a} = 0$), it may be experiencing a force of **static friction**, $\vec{f_s}$, with possible values of

$$\boxed{f_s \leq \mu_s \, n \ ,} \tag{V-4}$$

$$\Longrightarrow \mu_s \equiv \text{coefficient of static friction,}$$
$$n \equiv \text{magnitude of the normal force.}$$

c) Since the object doesn't move under applied force, $\vec{F}$,

$$\boxed{\vec{F} - \vec{f_s} = 0 \ .} \tag{V-5}$$

d) If one continues to increase the applied force until the object is just on the verge of slipping (*i.e.*, moving), we have reached the maximum of static friction

$$\boxed{f_{s,\,\max} = \mu_s \, n \ .} \tag{V-6}$$

e) Once the object is in motion, it experiences the force of **kinetic friction**, $\vec{f_k}$:

$$\boxed{f_k = \mu_k \, n \ ,} \tag{V-7}$$

$$\Longrightarrow \mu_k \equiv \text{coefficient of kinetic friction.}$$

f) Note that μ_k does not have to equal μ_s for the same surface. Usually, $\mu_k < \mu_s$ and ranges between 0.01 and 1.5.

D. Applications of Newton's Laws.

1. Bodies in Equilibrium ($\vec{a} = 0$).

a) Keywords that tell you an object is in **equilibrium** $\Longrightarrow$ <u>no acceleration</u>.

i) Body is at <u>rest</u> (not changing position).

ii) Body is static (not changing in time).

iii) Body is in steady state (no acceleration).

b) When objects are in equilibrium:

$$\boxed{\sum \vec{F} = 0} \qquad \text{(V-8)}$$

or in component format ($\sum$ means summation of all forces):

$$\begin{aligned} \sum F_x &= 0 \\ \sum F_y &= 0 \end{aligned}$$

2. Problem-Solving Strategy for Objects in Equilibrium.

a) Draw a diagram of the problem and label all forces acting upon the object (or objects if more than one).

b) Choose an appropriate coordinate system for each object and draw the coordinate(s) system on the diagram.

i) Choose the x and y axes such that the majority of the forces lie along one axis (either x or y).

ii) It is also a good idea to have the direction of motion be along one of the axes.

c) Write down all parameters that are given and desired. Get parameters into consistent units (preferably SI units).

d) Draw a vector diagram (called a "**free-body diagram**") for the isolated object under consideration, and label all forces acting upon it. Identify the + (positive) directions in the free-body diagram.

e) Resolve forces into x & y components (or some other convenient coordinate system).

f) Use equations of equilibrium:

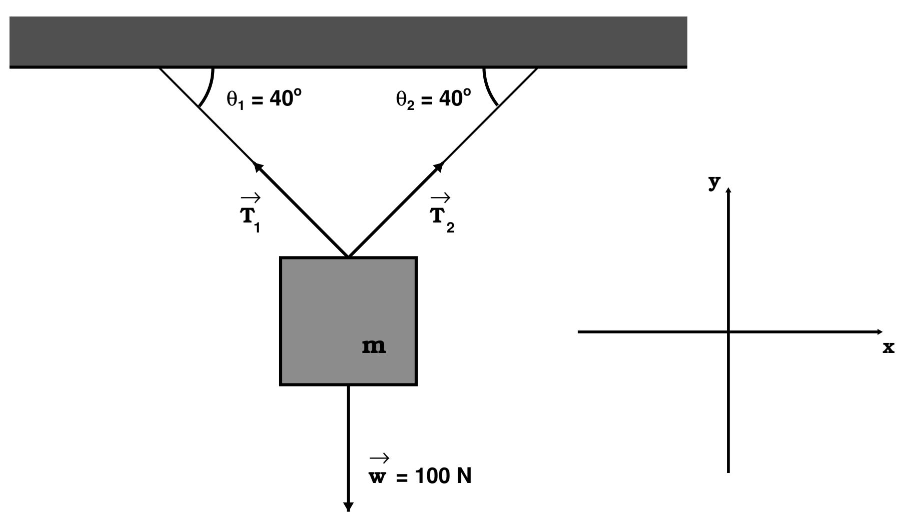
$$\sum F_x = 0 \quad \& \quad \sum F_y = 0 \ ,$$

keep track of the signs of the various force components.

g) Step (f) leads to a set of equations with several unknowns
$\rightarrow$ solve these simultaneous equations for these unknowns.

Example V–1. Find the tension in the two wires that support the 100-N object in the diagram below.

• **Make diagram and choose a coordinate system:**



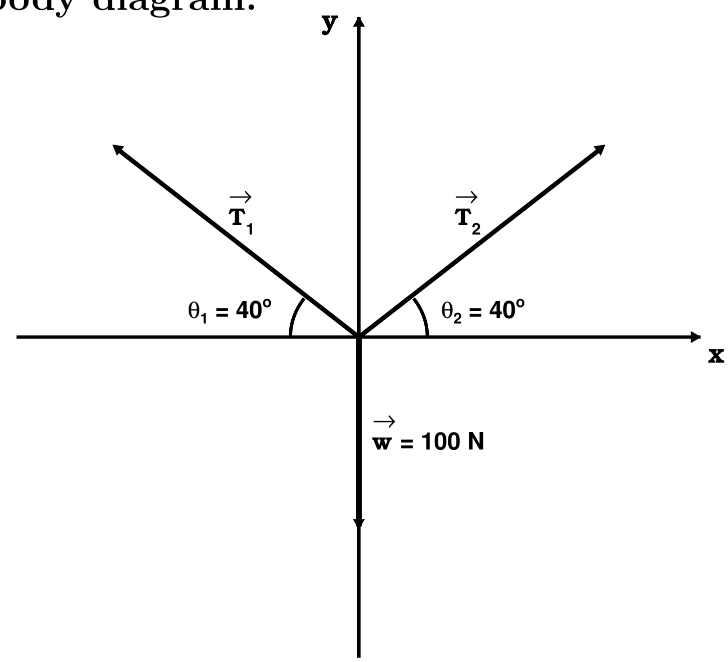
• **Parameters and check for consistent units:**

Given: $w = 100$ N, $\theta_1 = 40°$, $\theta_2 = 40°$.

Wanted: T_1 and T_2.

All units are consistent with the SI system.

- **Make free-body diagram:**



- **Resolve forces into components:**

$$\begin{aligned}\vec{T}_1 &= -T_1\cos\theta_1\,\hat{x} + T_1\sin\theta_1\,\hat{y} \;=\; -T_1\cos 40^\circ\,\hat{x} + T_1\sin 40^\circ\,\hat{y} \\ &= -0.7660\,T_1\;\hat{x} + 0.6428\,T_1\,\hat{y}\end{aligned}$$

$$\begin{aligned}\vec{T}_2 &= T_2\cos\theta_2\,\hat{x} + T_2\sin\theta_2\,\hat{y} \;=\; T_2\cos 40^\circ\,\hat{x} + T_2\sin 40^\circ\,\hat{y} \\ &= 0.7660\,T_2\;\hat{x} + 0.6428\,T_2\,\hat{y}\end{aligned}$$

$$\vec{w} = -(100\text{ N})\,\hat{y}$$

- **Use the equilibrium equations:**

$$\sum F_x = -0.7660\,T_1 + 0.7660\,T_2 = 0 \qquad \text{(Ex V.1-1)}$$
$$\sum F_y = 0.6428\,T_1 + 0.6428\,T_2 - 100\text{ N} = 0 \qquad \text{(Ex V.1-2)}$$

- **Simultaneously solve the equilibrium equations:**

Solve Eq. (Ex V.1-1) for T_2 and plug into Eq. (Ex V.1-2):

$$\text{Ex V.1-1:} \qquad T_2 = \frac{0.7660\,T_1}{0.7660} = 1.00\,T_1 = T_1 \qquad \text{(Ex V.1-3)}$$

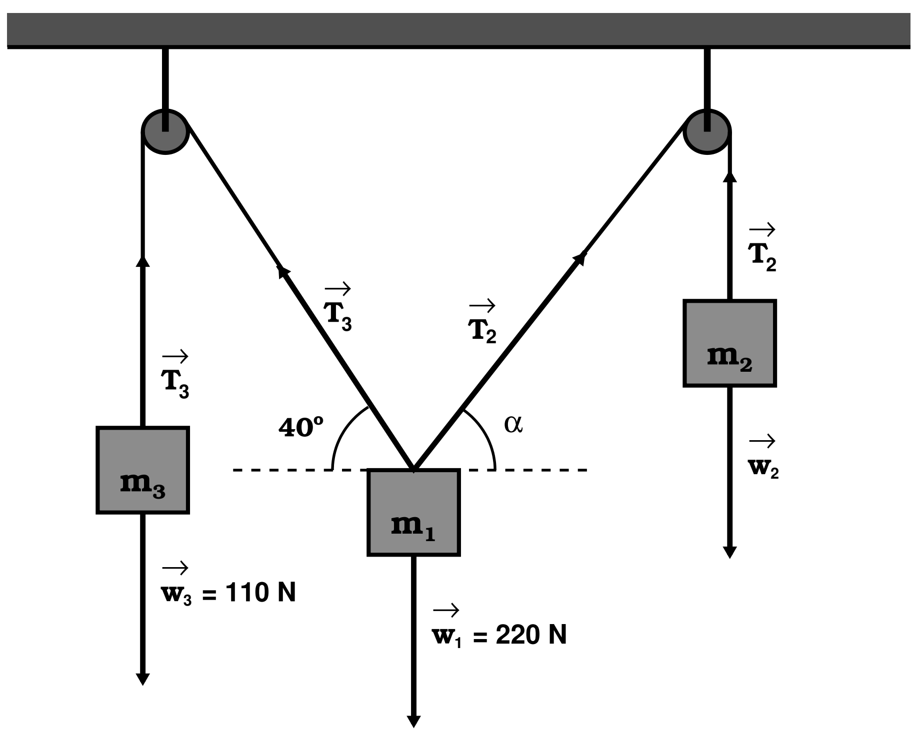
$$\text{Ex V.1-2:} \quad 0.6428\,T_1 + 0.6428\,(1.00\,T_1) = 100\ \text{N}$$
$$0.6428\,T_1 + 0.6428\,T_1 = 100\ \text{N}$$
$$1.2856\,T_1 = 100\ \text{N}$$

$$\boxed{T_1 = 78\ \text{N}}$$

Now use this in Eq. (Ex V.1-3):

$$\boxed{T_2 = T_1 = 78\ \text{N}\ .}$$

Example V–2. Suppose we have a three mass and two pulley system as shown in this diagram:



Assume this system is in equilibrium. Determine the weight w_2 and the angle α in this diagram.

Solution:

- **The diagram, forces, and coordinate system:**

We have already been presented with a diagram with all of the forces indicated. Assume our Cartesian coordinate system has the x-axis going off to the right and that the y-axis goes upward. In addition, We will label the $40°$ angle as θ_3 in the equations below since it is associated with weight w_3.

We will assume that the cords connecting the masses do not stretch (hence change length). As a result, the tension coming off of the m_3 mass, $\vec{T}_3$, is the same as the tension of the cord coming off of mass m_1 connecting to mass m_3 as shown in the figure above. The same is true for the cord connecting masses m_1 with m_2.

- **Parameters and check for consistent units:**

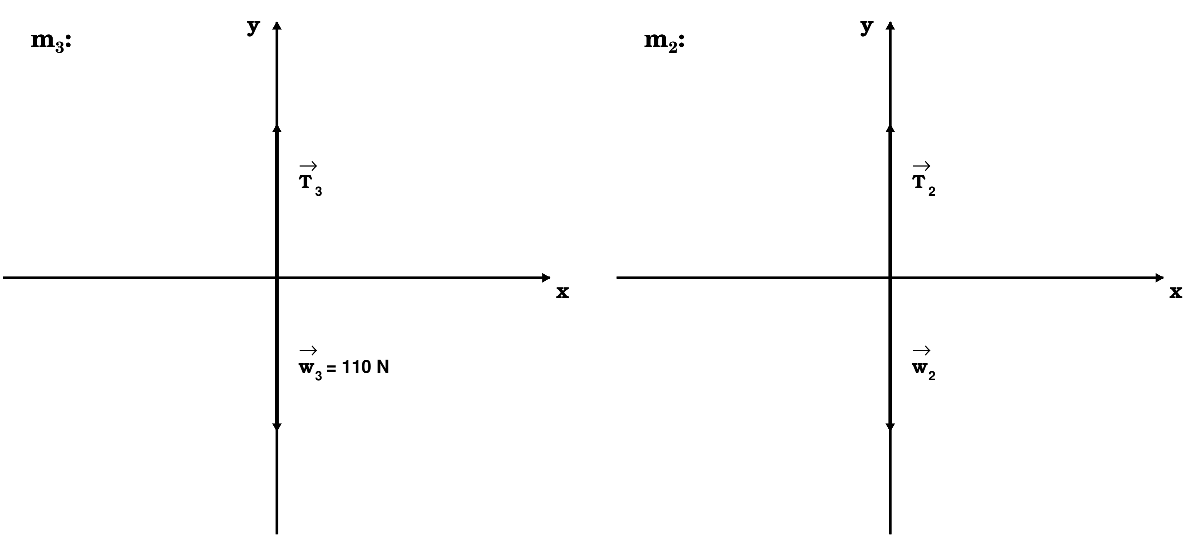
Given: $w_1 = 220$ N, $w_3 = 110$ N, $\theta_3 = 40°$.

Wanted: w_2 and α.

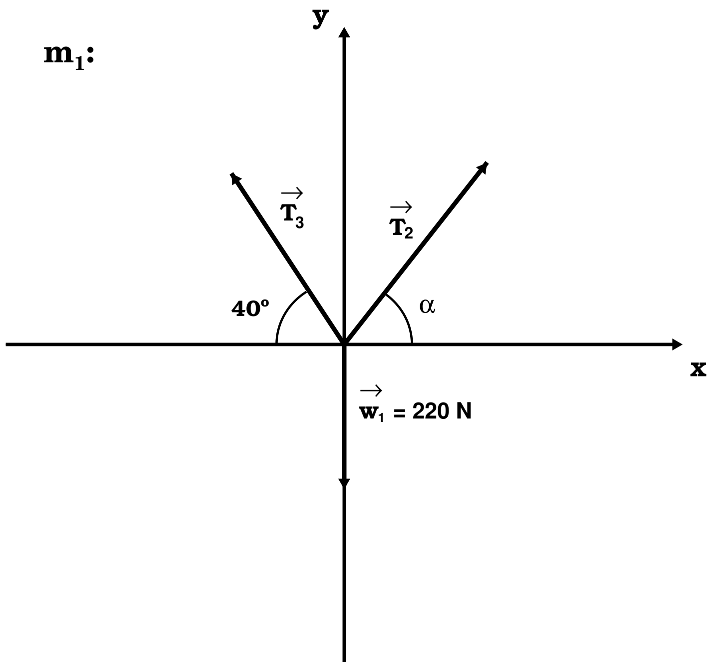
All units are consistent with the SI system.

- **Make free-body diagrams for all masses:**

For masses m_2 and m_3, we have the following two free-body diagrams:



Now we draw the free-body diagram for mass m_1:



• Resolve forces into components and plug into the force equilibrium equations:

Mass 3 (there are no forces in the x direction):

$$\begin{aligned} \sum F_{3y} &= T_{3y} - w_{3y} = T_3 - w_3 = T_3 - 110 \text{ N} \ = 0 \\ T_3 &= 110 \text{ N} \qquad \text{(Ex V.2-1)} \end{aligned}$$

Mass 2 (there are no forces in the x direction):

$$\begin{aligned} \sum F_{2y} &= T_{2y} - w_{2y} = T_2 - w_2 = 0 \\ T_2 &= w_2 \qquad \text{(Ex V.2-2)} \end{aligned}$$

Mass 1:

$$\begin{aligned} \sum F_{1x} &= -T_{3x} + T_{2x} \ = \ -T_3 \cos\theta_3 + T_2 \cos\alpha = 0 \\ &= -T_3 \cos 40^\circ + T_2 \cos\alpha \ = \ 0 \\ &= -(110 \text{ N})\,(0.7660) + T_2 \cos\alpha \ = \ 0 \\ &= -84.265 \text{ N} \ + T_2 \cos\alpha \ = \ 0 \\ \cos\alpha &= (84.265 \text{ N})/T_2 \qquad \text{(Ex V.2-3)} \end{aligned}$$

$$\begin{aligned}
\sum F_{1y} &= T_{3y} + T_{2y} - w_{1y} \;=\; T_3 \sin\theta_3 + T_2 \sin\alpha - w_1 = 0 \\
&= T_3 \sin 40^\circ + T_2 \sin\alpha - 220\text{ N} \;=\; 0 \\
&= (110\text{ N})\,(0.6428) + T_2 \sin\alpha - 220\text{ N} \;=\; 0 \\
&= 70.707\text{ N} \;+ T_2 \sin\alpha - 220\text{ N} \;=\; 0 \\
&= T_2 \sin\alpha - 149.293\text{ N} \;=\; 0 \\
T_2 \sin\alpha &= 149.293\text{ N} \\
\sin\alpha &= (149.293\text{ N})/T_2 \qquad\qquad \text{(Ex V.2-4)}
\end{aligned}$$

- **Simultaneously solve the equilibrium equations:**

If we first divide Eq. (Ex V.2-4) by Eq. (Ex V.2-3), we can determine the angle α:

$$\begin{aligned}
\frac{\sin\alpha}{\cos\alpha} &= \frac{(149.293\text{ N})/T_2}{(84.265\text{ N})/T_2} \\
\tan\alpha &= \frac{149.293\text{ N}}{84.265\text{ N}} \;=\; 1.7717 \\
\alpha &= \tan^{-1}(1.7717) = 6\underline{0}.5585^\circ
\end{aligned}$$

$$\boxed{\alpha = 61^\circ}$$

Now use this in Eq. (Ex V.2-4) to determine T_2 and finally use that value in Eq. (Ex V.2-2) to determine w_2:

$$\begin{aligned}
\sin\alpha &= (149.293\text{ N})/T_2 \\
T_2 &= (149.293\text{ N})/\sin\alpha \;=\; (149.293\text{ N})/\sin 6\underline{0}.5585^\circ \\
&= 1\underline{7}1.432\text{ N}
\end{aligned}$$

$$\boxed{T_2 = 170\text{ N}\ .}$$

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3. Bodies in Non-equilibrium ($\vec{a} \neq 0$).

a) If a body "feels" a net force that produces an acceleration, it is no longer in equilibrium.

b) We must now use Newton's 2nd Law to solve the problem.

$$\boxed{\sum \vec{F} = m\,\vec{a}} \tag{V-9}$$

or in component format:

$$\begin{aligned} \sum F_x &= m\,a_x \\ \sum F_y &= m\,a_y \end{aligned}$$

4. Problem-Solving Strategy using Newton's 2nd Law (Objects in Non-equilibrium).

a) Use the same procedure as described for equilibrium conditions ...

b) <u>Except</u>, do not use the equilibrium condition that $\sum \vec{F} = 0$, instead use

$$\boxed{\sum \vec{F} = m\,\vec{a}} \tag{V-10}$$

$\Longrightarrow$ Newton's 2nd Law.

c) <u>And</u>, set the coordinate system such that $\vec{a}$ is in the positive direction for each object for a <u>single</u> axis (hence zero along the other axis).

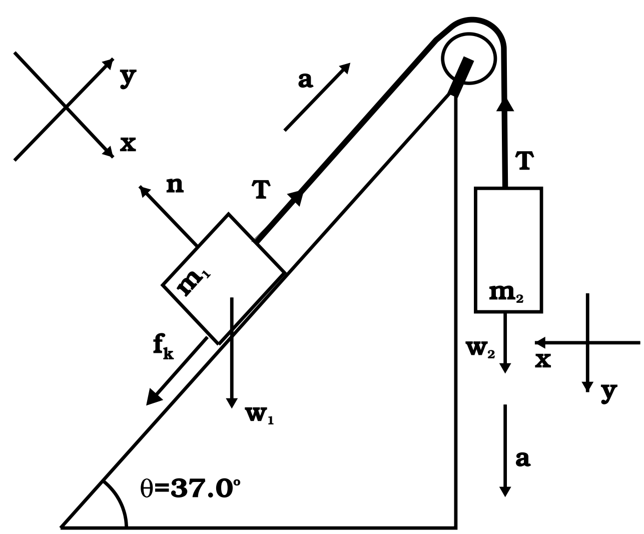
Example V–3. Consider two masses, $m_1 = 7.00$ kg and $m_2 = 12.0$ kg, connected by a rigid cord as shown in the diagram below. Find the tension on the cord and the normal force on the 7.00-kg object in addition to the acceleration experienced by each of the two objects. Note that the coefficient of kinetic friction between the

7.00-kg object and the plane is 0.250.

Solution:

- **Make diagram:**

  Since $m_2 > m_1$, mass m_1 will slide up the inclined plane as m_2 falls through the gravitational field. This will set the direction of motion which is needed to define the frictional force vector. This also helps us define our coordinate systems since we want the acceleration to point towards a positive direction of either the x or y axes. All of the force vectors of this problem are drawn on this diagram. In this problem, we are told that the cord that connects m_1 and m_2 is rigid, hence it does not stretch during the experiment. As such, the tension of the cord off of mass 1 will be the same as the tension of the rope off of mass 2 which we will label as T for both masses. In addition, a rigid cord will result in both masses experiencing the same acceleration.



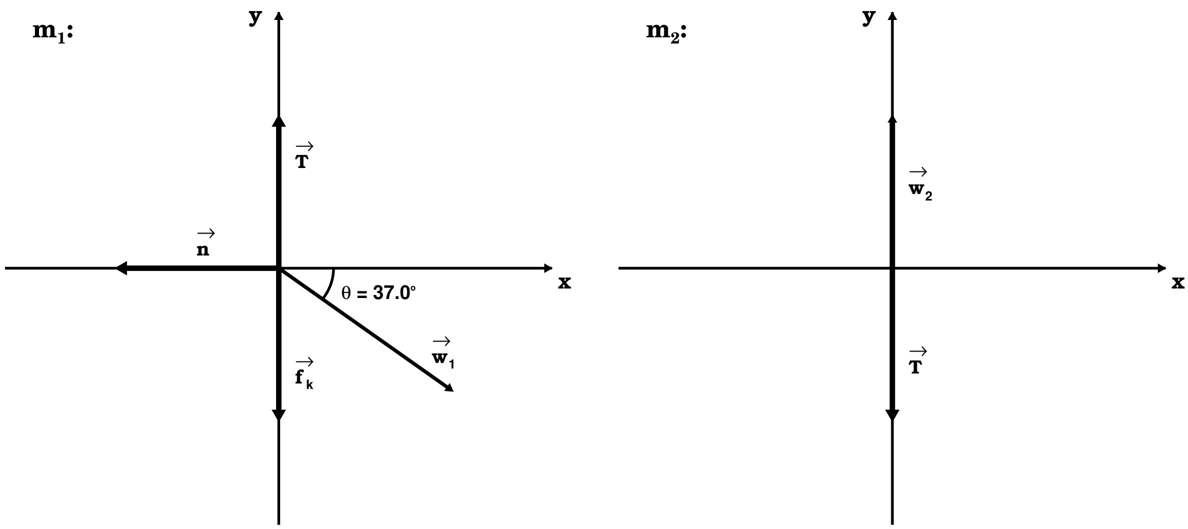
- **List parameters and check for consistent units:**

  Given: $m_1 = 7.00$ kg, $m_2 = 12.0$ kg, $\theta = 37.0°$, and $\mu_k = 0.250$.

  Wanted: a, T, and n.

All units are consistent with the SI system.

- **Make free-body diagrams for both masses:**



- **Resolve forces into components and plug into Newton's 2nd Law:**

Mass 1:

$$\begin{aligned}\sum F_{1x} &= w_{1x} - n = m_1 g \cos\theta - n = 0 \\ \sum F_{1y} &= T - f_k - w_{1y} = T - \mu_k n - m_1 g \sin\theta = m_1 a\ .\end{aligned}$$

Mass 2 (there are no forces in the x direction):

$$\sum F_{2y} = w_{2y} - T = m_2 g - T = m_2 a\ .$$

- **Simultaneously solve the force equations:**

We will first use the equation for F_{1x} to solve for n which will be needed in the F_{1y} equation and requested in the question:

$$\begin{aligned} n &= w_{1x} = m_1 g \cos\theta \\ &= (7.00\ \text{kg})(9.80\ \text{m/s}^2)\cos 37.0^\circ \\ &= \boxed{54.8\ \text{N}\ .}\end{aligned}$$

We will now use this equation in the equation for F_{1y} and solve for the acceleration 'a':

$$\begin{aligned} m_1 a &= T - \mu_k n - m_1 g \sin\theta \\ m_1 a &= T - \mu_k m_1 g \cos\theta - m_1 g \sin\theta \\ a &= \frac{T}{m_1} - g\left(\mu_k \cos\theta + \sin\theta\right) \ . \end{aligned}$$

Now solve the equation of F_{2y} for 'a':

$$\begin{aligned} m_2 a &= w_2 - T \\ m_2 a &= m_2 g - T \\ a &= g - \frac{T}{m_2} \ . \end{aligned}$$

Subtract the acceleration equation of mass 2 from the acceleration equation for mass 1 and solve for the tension T:

$$\begin{aligned} a - a &= \frac{T}{m_1} - g\left(\mu_k \cos\theta + \sin\theta\right) - \left(g - \frac{T}{m_2}\right) \\ 0 &= \left(\frac{1}{m_1} + \frac{1}{m_2}\right) T - g\ \left(\mu_k \cos\theta + \sin\theta + 1\right) \\ \left(\frac{m_1 + m_2}{m_1 m_2}\right) T &= g\ \left(\mu_k \cos\theta + \sin\theta + 1\right) \\ T &= g\left(\frac{m_1 m_2}{m_1 + m_2}\right)\left(\mu_k \cos\theta + \sin\theta + 1\right) \\ T &= (9.80\ \mathrm{m/s^2}) \cdot \left(\frac{(7.00\ \mathrm{kg})(12.0\ \mathrm{kg})}{(7.00\ \mathrm{kg}\ + 12.0\ \mathrm{kg})}\right) \cdot \\ &\quad\ (0.250 \cos 37.0^\circ + \sin 37.0^\circ + 1) \\ T &= (9.80\ \mathrm{m/s^2})\,(4.42\ \mathrm{kg})\,(1.80) = \boxed{78.1\ \mathrm{N}\ .} \end{aligned}$$

Finally, using this tension in the acceleration equation of mass 2, we can solve for the acceleration of both masses:

$$a = g - \frac{T}{m_2} = 9.80\ \mathrm{m/s^2} - \frac{78.1\ \mathrm{N}}{12.0\ \mathrm{kg}} = \boxed{3.30\ \mathrm{m/s^2}\ .}$$

E. Kinematics and Dynamics Summary.

1. The word **mechanics** in physics means the *study of motion*.

- **a)** The study of motion without regards to its causes is called **kinematics**.
- **b)** The study of the causes of motion is called **dynamics**.

2. As we have seen, objects that move in **uniform motion** (*i.e.*, at a **constant velocity**) have the following characteristics:

- **a)** Since $\Delta v = 0$ for uniform motion, objects with such motion are **not accelerating** (remember $a = \Delta v / \Delta t$).
- **b)** From Newton's 2nd law (see Eq. V-1), zero acceleration means that the total force in all directions (*i.e.*, the vector sum of all forces acting on the object) equals zero.
- **c)** As such, if an object is moving in uniform motion and there is an applied force causing this motion, there must be a *retarding* force (such as friction) acting in the opposite direction.
- **d)** Objects (and fluids) that are moving in uniform motion are said to be in **static equilibrium**.