PHYS-2010: General Physics I
Course Lecture Notes
Section VII

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Abstract

These class notes are designed for use of the instructor and students of the course PHYS-2010: General Physics I taught by Dr. Donald Luttermoser at East Tennessee State University. These notes make reference to the *College Physics, 11th Edition* (2018) textbook by Serway and Vuille.

VII. Linear Momentum and Collisions

A. Momentum and Impulse.

1. The **linear momentum** p of an object is equal to the product of the mass m and velocity v:

$$\boxed{\vec{p} \equiv m\,\vec{v}\ ,} \qquad \text{(VII-1)}$$

or in component form: $\vec{p} = p_x\,\hat{x} + p_y\,\hat{y} = mv_x\,\hat{x} + mv_y\,\hat{y}$.

2. Note that Newton's 2nd law can be written as

$$\boxed{\vec{F} = \frac{\text{change in momentum}}{\text{time interval}} = \frac{\Delta\vec{p}}{\Delta t}\ .} \qquad \text{(VII-2)}$$

The proof for this is

$$\begin{aligned} v_{\rm f} &= v_{\rm i} + at = v_{\rm i} + a\,\Delta t \\ a\,\Delta t &= v_{\rm f} - v_{\rm i} = \Delta v \\ a &= \frac{\Delta v}{\Delta t} \\ \Delta p = m\,\Delta v \ \text{ so } \ \Delta v &= \frac{\Delta p}{m} \\ a &= \frac{\Delta v}{\Delta t} = \frac{\Delta p/m}{\Delta t} = \frac{\Delta p}{m\,\Delta t} \\ \text{or} \quad \frac{\Delta p}{\Delta t} &= ma = F \ \ ! \end{aligned}$$

3. Rewriting Eq. (VII-2), we get

$$\boxed{\vec{F}\,\Delta t = \Delta\vec{p} = m\vec{v}_{\rm f} - m\vec{v}_{\rm i} = m(\vec{v}_{\rm f} - \vec{v}_{\rm i})\ .} \qquad \text{(VII-3)}$$

a) $\vec{F}\,\Delta t$ is called the **impulse** of the force, with $\vec{F}$ as the *average force* during a collision.

b) Some use the variable $\vec{I}$ for impulse (as your textbook does), though it is more common to use $\vec{F}\Delta t$.

c) Eq. (VII-3) says: the impulse of the force acting on an object equals the change in momentum of that object $\Longrightarrow$ **Impulse-Momentum theorem**.

i) The impulse is positive (+) if an object is being propelled (*i.e.*, velocity increasing) or if the velocity changes from the $-x$ direction to the $+x$ direction.

ii) The impulse is negative (−) if an object is being stopped (*i.e.*, velocity decreasing) or if the velocity changes from the $+x$ direction to the $-x$ direction.

Example VII–1. (a) Show that the kinetic energy of a particle of mass m is related to the magnitude of the momentum p of that particle by

$$\text{KE} = \frac{p^2}{2m} .$$

[*Note:* This expression is invalid for relativistic particles (those traveling at speeds near the speed of light).] (b) An object is moving so that its KE = 150 J and the absolute value of its momentum is 30.0 kg m/s. What is the mass of the object and at what velocity is it traveling?

Solution (a):

We have the defining equations for kinetic energy and momentum as

$$\text{KE} = \frac{1}{2}mv^2 , \quad p = mv .$$

Solving these 2 equations for v and squaring the momentum equation gives

$$v^2 = \frac{2\text{KE}}{m} , \quad v^2 = \frac{p^2}{m^2} .$$

Setting these two equations equal to each other and solving for KE gives

$$\begin{aligned} \frac{2\text{KE}}{m} &= \frac{p^2}{m^2} \\ \text{KE} &= \frac{mp^2}{2m^2} = \frac{p^2}{2m} \qquad \text{QED.} \end{aligned}$$

(Note that "QED" is used in mathematics when solving a proof. Think of it as meaning '*there, I have proved it!*')

Solution (b):

We have the defining equations for kinetic energy and momentum as

$$\text{KE} = \frac{1}{2}mv^2 \ , \quad p = mv \ .$$

Solving these 2 equations for m gives

$$m = \frac{2\text{KE}}{v^2} \ , \quad m = \frac{p}{v} \ .$$

Setting these two equations equal, we can solve for v:

$$\begin{aligned} \frac{p}{v} &= \frac{2\text{KE}}{v^2} \\ \frac{v^2}{v} = v &= \frac{2\text{KE}}{p} = \frac{2(150 \text{ J})}{30.0 \text{ kg m/s}} \\ v &= \frac{300. \text{ kg m}^2/\text{s}^2}{30.0 \text{ kg m/s}} = \boxed{10.0 \text{ m/s} \ .} \end{aligned}$$

Plugging this value for v into the momentum 'm' equation above gives

$$m = \frac{p}{v} = \frac{30.0 \text{ kg m/s}}{10.0 \text{ m/s}} = \boxed{3.00 \text{ kg} \ .}$$

Example VII–2. A tennis player receives a shot with the ball (0.0600 kg) traveling horizontally at 50.0 m/s and returns the shot with the ball traveling horizontally at 40.0 m/s in the opposite direction. (a) What is the impulse delivered to the ball by the racquet? (b) What work does the racquet do on the ball?

Solution (a):

Assume the ball is initially traveling in the $-x$ direction away from the net. Then $v_{\rm i} = -50.0$ m/s and $v_{\rm f} = +40.0$ m/s. Using Eq. (VII-3), the impulse $F\,\Delta t$ is

$$\begin{aligned} F\,\Delta t &= \Delta p = m\,(v_f - v_i) \\ &= 0.0600\ \text{kg}\,[40.0\ \text{m/s} - (-50.0\ \text{m/s})] = 5.40\ \text{kg}\cdot\text{m/s} \\ &= \boxed{5.40\ \text{N}\cdot\text{s}\ ,} \end{aligned}$$

where the positive value indicates that the ball is now traveling in the $+x$ direction and that the impulse is supplied by the racquet.

Solution (b):

The work is just the change of kinetic energy as given by Eq. (VI-4) of the last section of the notes. As such,

$$\begin{aligned} W &= \Delta\ \text{KE} = \frac{1}{2}\,m\,(v_{\rm f}^2 - v_{\rm i}^2) \\ &= \frac{0.0600\ \text{kg}}{2}\,[(40.0\ \text{m/s})^2 - (-50.0\ \text{m/s})^2] \\ &= \boxed{-27.0\ \text{J}\ ,} \end{aligned}$$

where the negative sign means that the racquet is supplying work to the ball.

B. Conservation of Linear Momentum.

1. During a collision, assume no external forces (*e.g.*, gravity, friction, etc.) are present or that these external forces are small with respect to the force of the collision.

2. The impulse of 2 colliding particles, m_1 and m_2, are:

$$\textbf{a)} \quad \vec{\overline{F}}_1 \, \Delta t = m_1 \, \vec{v}_{1\mathrm{f}} - m_1 \, \vec{v}_{1\mathrm{i}} \qquad (\text{mass } m_1) \ . \tag{VII-4}$$

$$\textbf{b)} \quad \vec{\overline{F}}_2 \, \Delta t = m_2 \, \vec{v}_{2\mathrm{f}} - m_2 \, \vec{v}_{2\mathrm{i}} \qquad (\text{mass } m_2) \ . \tag{VII-5}$$

Where in these two equations $\overline{F}$ is the *average* force supplied during a collision.

3. Newton's 3rd law states:

$$\vec{\overline{F}}_1 = -\vec{\overline{F}}_2 \ , \tag{VII-6}$$

or

$$\vec{\overline{F}}_1 \, \Delta t = -\vec{\overline{F}}_2 \, \Delta t \tag{VII-7}$$

$$m_1 \, \vec{v}_{1\mathrm{f}} - m_1 \, \vec{v}_{1\mathrm{i}} = -(m_2 \, \vec{v}_{2\mathrm{f}} - m_2 \, \vec{v}_{2\mathrm{i}}) \ , \tag{VII-8}$$

and finally

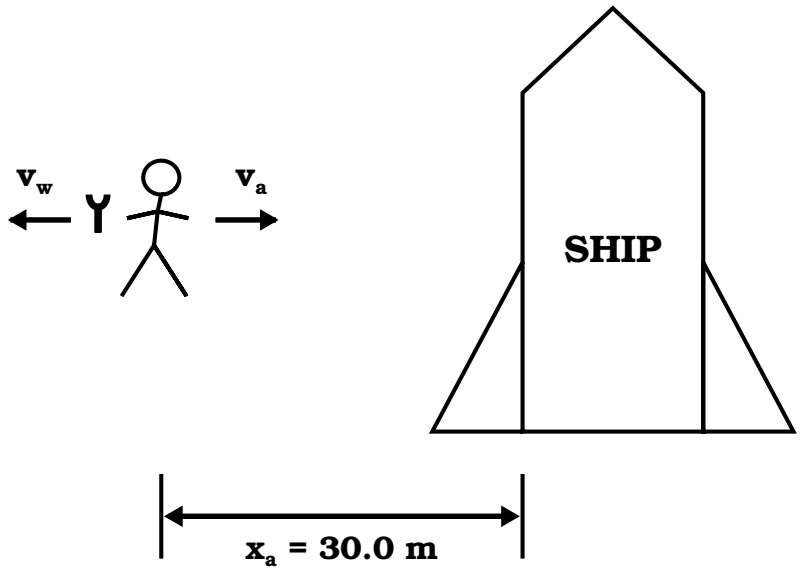
$$\boxed{m_1 \, \vec{v}_{1\mathrm{i}} + m_2 \, \vec{v}_{2\mathrm{i}} = m_1 \, \vec{v}_{1\mathrm{f}} + m_2 \, \vec{v}_{2\mathrm{f}}} \tag{VII-9}$$

$\Longrightarrow$ **conservation of linear momentum**.

a) When no external forces are acting on a system (or these forces are small with respect to the impulse force), the total momentum before a collision is equal to the total momentum after a collision.

b) The conservation of linear momentum (Eq. VII-9) is nothing more than Newton's 3rd law of motion!

Example VII–3. An 80.0-kg astronaut is working on the engines of her spaceship, which is drifting through space with a constant velocity. The astronaut, wishing to get a better view of the Universe, pushes against the ship and later finds herself 30.0 m behind the ship and moving so slowly that she can be considered at rest with respect to the ship. Without a thruster, the only way to return to the ship is to throw a 0.500-kg wrench with a speed of 20.0 m/s in the opposite direction from the ship. How long will it take to get back to the ship (in minutes) once the wrench has been thrown?



Solution:

$$v_{\rm wf} = -20.0 \text{ m/s} \qquad v_{\rm af} = ?$$
$$m_{\rm w} = 0.500 \text{ kg} \qquad m_{\rm a} = 80.0 \text{ kg}$$

$$\text{wrench+astronaut initial mom.} = \text{wrench+astronaut final mom.}$$
$$p_{\rm wi} + p_{\rm ai} = p_{\rm wf} + p_{\rm af} \ .$$

Initially, both the astronaut and wrench are at rest so

$$p_{\rm ai} = 0 \ , \ \text{ since } \ v_{\rm ai} = 0$$
$$p_{\rm wi} = 0 \ , \ \text{ since } \ v_{\rm wi} = 0 \ .$$

Plugging these values into our momentum equation above and solving for the final astronaut velocity, we get

$$\begin{aligned} 0 &= m_{\rm w}\, v_{\rm wf} + m_{\rm a}\, v_{\rm af} \\ v_{\rm af} &= -\frac{m_{\rm w}}{m_{\rm a}}\, v_{\rm wf} = -\frac{0.500\ {\rm kg}}{80.0\ {\rm kg}}\,(-20.0\ {\rm m/s}) \\ &= 0.125\ {\rm m/s}\ . \end{aligned}$$

However, we want to know how long it will take to get to the ship, we just use this velocity in a 1-D equation of motion. Since the velocity of the astronaut will be constant once the wrench is thrown, $a = 0$, so $v_{\rm af} = x_{\rm a}/t$, and

$$t = \frac{x_{\rm a}}{v_{\rm af}} = \frac{30.0\ {\rm m}}{0.125\ {\rm m/s}} = 240\ {\rm s}\ \times \frac{1\ {\rm min}}{60\ {\rm s}} = \boxed{4.00\ {\rm min}\ .}$$

C. Collisions in One Dimension.

1. An **inelastic collision** is one in which momentum is conserved but kinetic energy is not. Hence for this type of collision, we can only use Eq. (VII-9) to try and solve the problem. (Of course, we can always use Newton's 2nd law of motion in addition to this equation if we need more equations.)

a) Some of the kinetic energy goes into deformation (and heat) of the surfaces in contact.

b) If the two objects stick together during a collision, it is called a **perfectly inelastic collision**. For such a collision, Eq. (VII-9), the conservation of momentum, becomes

$$\boxed{m_1\,\vec{v}_{1\rm i} + m_2\,\vec{v}_{2\rm i} = (m_1 + m_2)\ \vec{v}_{\rm f}\ .} \qquad \text{(VII-10)}$$

2. An **elastic collision** is one where <u>both</u> momentum (CM) and kinetic energy (CE) are conserved (*e.g.*, billiard balls, air molecules, etc.). To solve such a collision problem, we use

$$\boxed{\begin{array}{lrcll} \textbf{CM:} & m_1\,\vec{v}_{1\mathrm{i}} + m_2\,\vec{v}_{2\mathrm{i}} & = & m_1\,\vec{v}_{1\mathrm{f}} + m_2\,\vec{v}_{2\mathrm{f}} & (a) \\ \textbf{CE:} & \frac{1}{2}\,m_1\,v_{1\mathrm{i}}^2 + \frac{1}{2}\,m_2\,v_{2\mathrm{i}}^2 & = & \frac{1}{2}\,m_1\,v_{1\mathrm{f}}^2 + \frac{1}{2}\,m_2\,v_{2\mathrm{f}}^2 & (b) \end{array}} \quad . \tag{VII-11}$$

a) Here we are assuming the potential energy (PE) remains constant during the collision.

b) Note that we can rewrite Eq. (VII-11b) as

$$m_1\,v_{1\mathrm{i}}^2 - m_1\,v_{1\mathrm{f}}^2 = m_2\,v_{2\mathrm{f}}^2 - m_2\,v_{2\mathrm{i}}^2 \ .$$

c) Factoring gives

$$\begin{aligned} m_1\,\left(v_{1\mathrm{i}}^2 - v_{1\mathrm{f}}^2\right) &= m_2\,\left(v_{2\mathrm{f}}^2 - v_{2\mathrm{i}}^2\right) \\ m_1\,(v_{1\mathrm{i}} - v_{1\mathrm{f}})(v_{1\mathrm{i}} + v_{1\mathrm{f}}) &= m_2\,(v_{2\mathrm{f}} - v_{2\mathrm{i}})(v_{2\mathrm{f}} + v_{2\mathrm{i}}) \ . \end{aligned} \tag{VII-12}$$

d) We can rewrite Eq. (VII-11a) as

$$\begin{aligned} m_1\,v_{1\mathrm{i}} - m_1\,v_{1\mathrm{f}} &= m_2\,v_{2\mathrm{f}} - m_2\,v_{2\mathrm{i}} \\ m_1\,(v_{1\mathrm{i}} - v_{1\mathrm{f}}) &= m_2\,(v_{2\mathrm{f}} - v_{2\mathrm{i}}) \ . \end{aligned} \tag{VII-13}$$

e) Now, divide Eq. (VII-12) by Eq. (VII-13):

$$\begin{aligned} \frac{m_1\,(v_{1\mathrm{i}} - v_{1\mathrm{f}})(v_{1\mathrm{i}} + v_{1\mathrm{f}})}{m_1\,(v_{1\mathrm{i}} - v_{1\mathrm{f}})} &= \frac{m_2\,(v_{2\mathrm{f}} - v_{2\mathrm{i}})(v_{2\mathrm{f}} + v_{2\mathrm{i}})}{m_2\,(v_{2\mathrm{f}} - v_{2\mathrm{i}})} \\ v_{1\mathrm{i}} + v_{1\mathrm{f}} &= v_{2\mathrm{f}} + v_{2\mathrm{i}} \ , \end{aligned}$$

and finally,

$$\boxed{v_{1\mathrm{i}} - v_{2\mathrm{i}} = -(v_{1\mathrm{f}} - v_{2\mathrm{f}}) \ .} \tag{VII-14}$$

f) We can use Eqs. (VII-11a, 11b, and 14) to solve *head-on* elastic collisions.

3. Problem-Solving Strategy for One-Dimensional Collisions:

a) Set up a coordinate axis and define your velocities with respect to this axis. It is convenient to make your axis coincide with one of the initial velocities direction. Determine which quantities are given (and make sure they all have units that are consistent) and which need to be determined.

b) Make a sketch the situation and draw all velocity vectors and display the given information for both before and after the collision.

c) Write expressions for the momentum of each object *before* and *after* the collision. Remember to include the appropriate signs for the velocity vectors.

d) Now write expressions for the *total* momentum of the system of the objects *before* and *after* the collision and equate the two momenta sum.

e) If the collision is *inelastic*, the kinetic energy of the system is <u>not</u> conserved. Proceed to solve the momentum equations for the unknown quantities.

f) If the collision is *elastic*, the kinetic energy of the system <u>is</u> conserved, so you can equate the total kinetic energies before and after the collision. Proceed to solve the system of equations simultaneously for the unknown quantities.

Example VII–4. A 12.0-g bullet is fired horizontally into a 100-g wooden block that is initially at rest on a frictionless horizontal surface and connected to a spring having a spring constant of 150 N/m. The bullet becomes embedded in the block. If the bullet-block system compresses the spring by a maximum of 80.0 cm, what

was the speed of the bullet at impact with the block?

Solution:

Since we have two bodies sticking together, the collision will be *perfectly inelastic*, hence Eq. (VII-10) will be the equation we will use for CM. In order to solve this problem, we need an additional equation. Even though kinetic energy is not conserved, the total mechanical energy is conserved assuming we ignore energy loss due to deformation and heat caused by the friction between the bullet and wood. As such, we will use the conservation of mechanical energy (*i.e.*, Eq. VI-9) in conjunction with the conservation of linear momentum for this perfectly inelastic collision problem. Since we are not changing height in the gravitational field, $\mathrm{PE}_{\text{g-i}} = \mathrm{PE}_{\text{g-f}}$ in the CE equation. However, the potential energy of the spring is changing, hence we will need the potential energy equation of a spring:

$$\mathrm{PE}_s = \frac{1}{2}kx^2 \ ,$$

where $k = 150$ N/m is the spring constant.

There are two steps that we have to worry about here. It is easier to work in the reverse order that the events take place. Part (a) concerns the events just after impact of the bullet and block. For this part we will call the '*initial*' epoch just after the bullet embeds in the wood block and the '*final*' epoch when the bullet+block have compressed the spring to its maximum extent. Part (b) concerns the epoch just '*before*' the bullet/block collision and the epoch just '*after*' the collision.

The input parameters are

$$\begin{aligned} m = 12.0 \text{ g } = 0.0120 \text{ kg} &: \text{ mass of the bullet} \\ M = 100 \text{ g } = 0.100 \text{ kg} &: \text{ mass of the block} \\ m + M = 0.112 \text{ kg} &: \text{ combined mass of bullet+block} \end{aligned}$$

$v_{\text{bullet}} = ?$: velocity of bullet prior to impact

$v_{\text{block}} = 0$ (rest) : velocity of block prior to impact

$V_{\text{i}} = ?$: velocity of bullet+block at just after impact

$V_{\text{f}} = 0$: velocity of bullet+block at max spring compression

$x_{\text{i}} = 0$: position of bullet+block just after impact

$x_{\text{f}} = 80.0$ cm $= 0.800$ m : position of bullet+block at max spring compression

Part (a): The conservation of mechanical energy is now applied where the initial epoch is the point just after the collision has taken place and the final epoch occurs at maximum spring compression.

$$\begin{aligned}
\left[\text{KE}_{\text{(bullet+block)}} + \text{PE}_s\right]_{\text{i}} &= \left[\text{KE}_{\text{(bullet+block)}} + \text{PE}_s\right]_{\text{f}} \\
\frac{1}{2}(m+M)V_{\text{i}}^2 + \frac{1}{2}kx_{\text{i}}^2 &= \frac{1}{2}(m+M)V_{\text{f}}^2 + \frac{1}{2}kx_{\text{f}}^2 \\
\frac{1}{2}(m+M)V_{\text{i}}^2 + 0 &= 0 + \frac{1}{2}kx_{\text{f}}^2 \\
V_{\text{i}}^2 &= \frac{kx_{\text{f}}^2}{(m+M)} \\
V_{\text{i}} &= \sqrt{\frac{kx_{\text{f}}^2}{(m+M)}} \\
V_{\text{i}} &= \sqrt{\frac{(150\text{ N/m})(0.800\text{ m})^2}{0.112\text{ kg}}} = 29.3\text{ m/s}
\end{aligned}$$

Part (b): Let's now relabel $V_{\text{i}} = V$ which will represent the velocity of the bullet+block just after the collision. The conservation of linear momentum can now be used to determine the initial velocity of the bullet (here the subscripts 'b' mean before the collision and 'a' after the collision).

$$p_{\text{b}} = p_{\text{a}}$$

$$
\begin{aligned}
p_{\text{b-bullet}} + p_{\text{b-block}} &= p_{\text{a}} \\
m v_{\text{bullet}} + M v_{\text{block}} &= (m+M)\,V \\
m v_{\text{bullet}} + 0 &= (m+M)\,V \\
m v_{\text{bullet}} &= (m+M)\,V \\
v_{\text{bullet}} &= \frac{m+M}{m}\,V \\
v_{\text{bullet}} &= \left(\frac{0.112\text{ kg}}{0.0120\text{ kg}}\right)(29.3\text{ m/s}) = \boxed{273\text{ m/s .}}
\end{aligned}
$$

D. Glancing (Two-Dimensional) Collisions.

1. Colliding masses rebound at some angle relative to the line of motion.

2. Conservation of momentum still applies, but is applied for each component of the motion:

$$\sum p_{\text{ix}} = \sum p_{\text{fx}} \qquad \text{(VII-15)}$$

$$\sum p_{\text{iy}} = \sum p_{\text{fy}}\ . \qquad \text{(VII-16)}$$

3. Problem-Solving Strategy for Two-Dimensional Collisions:

a) Define a 2-D coordinate system (usually Cartesian) and identify the masses and velocities. It is convenient to make your x-axis coincide with one of the initial velocities direction. Determine which quantities are given (and make sure they all have units that are consistent) and which need to be determined.

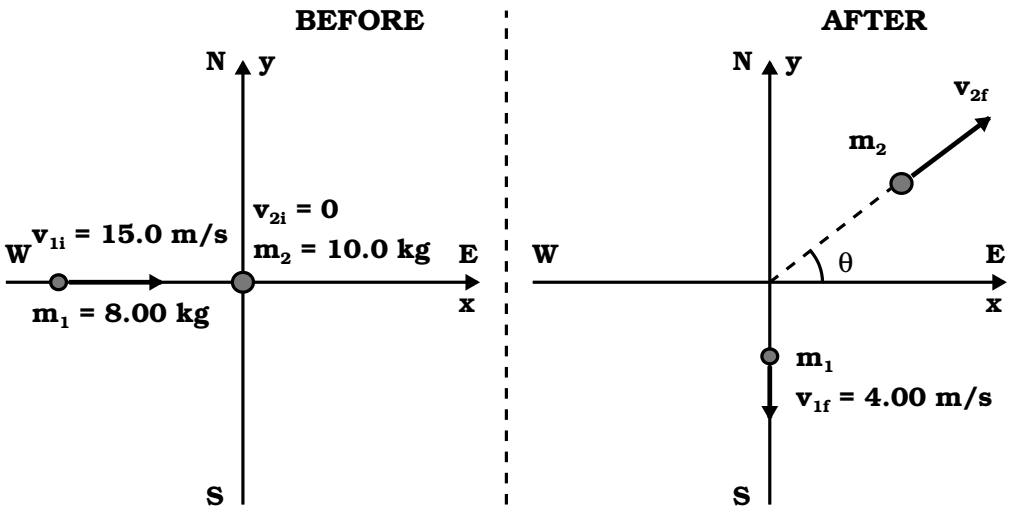
b) Make sketches of the situations for both *before* and *after* the collision with respect to the coordinate system you have defined.

c) Write the equations for the *total* momentum *before* and *after* the collision for both coordinates (*i.e.*, p_x and p_y). This will produce two sets of equations since the momentum is a vector (see Eqs. VII-11a, VII-15,16).
(Note that if one object is moving and the second is stationary, with both objects having the same mass, the angle between the two outgoing objects is 90°.)

d) If the collision is *inelastic*, the kinetic energy of the system is <u>not</u> conserved. Proceed to solve the momentum equations simultaneously for the unknown quantities. If the collision is perfectly inelastic, the final velocities of the two objects are equal since they stick together.

e) If the collision is *elastic*, the kinetic energy of the system <u>is</u> conserved, so you can equate the total kinetic energies before and after the collision. Proceed to solve the system of equations (momentum and kinetic energy) simultaneously for the unknown quantities.

Example VII–5. An 8.00-kg object moving east at 15.0 m/s on a frictionless horizontal surface collides with a 10.0-kg object that is initially at rest. After the collision, the 8.00-kg object moves south at 4.00 m/s. (a) What is the velocity of the 10.0-kg object after the collision? (b) What percentage of the initial kinetic energy is lost in the collision?



Solution (a):

Since we are told that kinetic energy will be lost during the collision, the collision is not an elastic collision, hence we cannot use the conservation of kinetic energy equation here. We are given sufficient information, however, to solve the problem using the conservation of linear momentum (in 2-D). Since the first object has changed direction and the second object was initially at rest, the final velocity of the second object will have both an x and y component in order to conserve momentum in both axes as shown in the diagram above. We are given the following parameters:

BEFORE	**AFTER**
$\vec{v}_{1\mathrm{i}} = (15.0 \text{ m/s})\ \hat{x}$	$\vec{v}_{1\mathrm{f}} = (-4.00 \text{ m/s})\ \hat{y}$
$\vec{v}_{2\mathrm{i}} = 0$	$\vec{v}_{2\mathrm{f}} = ?$
	$\theta = ?$
$m_1 = 8.00 \text{ kg}$	
$m_2 = 10.0 \text{ kg}$	

We have defined the eastern direction along the positive x axis and the northern direction along the positive y axis. Angle θ is

the direction of motion of the second object after the collision with respect to the eastern direction.

Conservation of linear momentum in the x direction:

$$\begin{aligned} m_1\, v_{1\mathrm{ix}} + m_2\, v_{2\mathrm{ix}} &= m_1\, v_{1\mathrm{fx}} + m_2\, v_{2\mathrm{fx}} \\ m_1\, v_{1\mathrm{ix}} + 0 &= 0 + m_2\, v_{2\mathrm{fx}} \\ v_{2\mathrm{fx}} &= \frac{m_1}{m_2}\, v_{1\mathrm{ix}} \\ &= \left(\frac{8.00\ \mathrm{kg}}{10.0\ \mathrm{kg}}\right)(15.0\ \mathrm{m/s}) \\ &= 12.0\ \mathrm{m/s}\ . \end{aligned}$$

Conservation of linear momentum in the y direction:

$$\begin{aligned} m_1\, v_{1\mathrm{iy}} + m_2\, v_{2\mathrm{iy}} &= m_1\, v_{1\mathrm{fy}} + m_2\, v_{2\mathrm{fy}} \\ 0 + 0 &= m_1\, v_{1\mathrm{fy}} + m_2\, v_{2\mathrm{fy}} \\ -m_2\, v_{2\mathrm{fy}} &= m_1\, v_{1\mathrm{fy}} \\ v_{2\mathrm{fy}} &= -\frac{m_1}{m_2}\, v_{1\mathrm{fy}} \\ &= -\left(\frac{8.00\ \mathrm{kg}}{10.0\ \mathrm{kg}}\right)(-4.00\ \mathrm{m/s}) \\ &= 3.20\ \mathrm{m/s}\ . \end{aligned}$$

The magnitude of the final velocity of object 2 is then found from the Pythagorean theorem:

$$\begin{aligned} v_{2\mathrm{f}} &= \sqrt{v_{2\mathrm{fx}}^2 + v_{2\mathrm{fy}}^2} \\ &= \sqrt{(12.0\ \mathrm{m/s})^2 + (3.20\ \mathrm{m/s})^2} = 12.4\ \mathrm{m/s}\ . \end{aligned}$$

The angle θ can now easily be found with

$$\begin{aligned} \tan\theta &= \frac{v_{2\mathrm{fy}}}{v_{2\mathrm{fx}}} \\ \theta &= \tan^{-1}\left(\frac{v_{2\mathrm{fy}}}{v_{2\mathrm{fx}}}\right) = \tan^{-1}\left(\frac{3.20\ \mathrm{m/s}}{12.0\ \mathrm{m/s}}\right) \\ &= 14.9^\circ\ . \end{aligned}$$

Thus,

$$\boxed{v_{2\mathrm{f}} = 12.4 \text{ m/s at } 14.9^\circ \text{ N of E} .}$$

Solution (b):

The percentage of kinetic energy lost is given by

$$\frac{\mathrm{KE}_{\mathrm{lost}}}{\mathrm{KE}_{\mathrm{i}}} = \frac{\mathrm{KE}_{\mathrm{i}} - \mathrm{KE}_{\mathrm{f}}}{\mathrm{KE}_{\mathrm{i}}} = 1 - \frac{\mathrm{KE}_{\mathrm{f}}}{\mathrm{KE}_{\mathrm{i}}} .$$

The initial kinetic energy is just the kinetic energy of object 1 since object 2 is initially at rest, hence

$$\mathrm{KE}_{\mathrm{i}} = \frac{1}{2} m_1 v_{1\mathrm{i}}^2 = \frac{1}{2} (8.00 \text{ kg}) (15.0 \text{ m/s})^2 = 900 \text{ J} .$$

The final kinetic energy is the sum of the kinetic energies of object 1 and object 2, hence

$$\begin{aligned} \mathrm{KE}_{\mathrm{f}} &= \frac{1}{2} m_1 v_{1\mathrm{f}}^2 + \frac{1}{2} m_2 v_{2\mathrm{f}}^2 \\ &= \frac{1}{2} (8.00 \text{ kg}) (-4.00 \text{ m/s})^2 + \frac{1}{2} (10.00 \text{ kg}) (12.4 \text{ m/s})^2 \\ &= 64.0 \text{ J} + 770 \text{ J} = 834 \text{ J} . \end{aligned}$$

Hence the percentage lost is

$$\frac{\mathrm{KE}_{\mathrm{lost}}}{\mathrm{KE}_{\mathrm{i}}} = 1 - \frac{834 \text{ J}}{900 \text{ J}} = 1 - 0.928 = 0.072 ,$$

or $\boxed{7.2\%}$ of the original kinetic energy is lost in the collision.