PHYS-2010: General Physics I
Course Lecture Notes
Section XI

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Edition 2.6

Abstract

These class notes are designed for use of the instructor and students of the course PHYS-2010: General Physics I taught by Dr. Donald Luttermoser at East Tennessee State University. These notes make reference to the *College Physics, 11th Edition* (2018) textbook by Serway and Vuille.

XI. Solids and Fluids

A. States of Matter.

1. Matter exists in 3 different **states**:

a) A **solid** is a rigid body $\longrightarrow$ takes a lot of energy to change its shape. Solids can be classified into two types:

i) **Crystalline solids** have atoms that are structured in an *orderly* fashion.

ii) **Amorphous solids** have randomly arranged atoms.

b) A **liquid** is fluid in nature $\longrightarrow$ moderate energy required to change its shape.

c) A **gas** is also fluid in nature $\longrightarrow$ little energy required to change its shape.

2. If a gas gets hot enough, electrons circling the nucleus of the atoms in the gas are "ripped" away from the nucleus $\Longrightarrow$ the gas becomes *ionized* $\Longrightarrow$ ionized gas is called a **plasma**.

3. Matter consists of a distribution of particles (atoms and molecules).

a) **Atoms** consist of a nucleus surrounded by electrons (which are negatively [–] charged). The **nucleus** consists of protons (positive [+] charge) and neutrons (no [0] charge).

b) H, He, Li, Be, B, C, N, O, etc. are the elements of the **periodic table** = atoms. The number of protons in the nucleus defines each atom.

c) For elements heavier than H, typically the number of neutrons is equal to the number of protons. **Isotopes** of atoms contain different numbers of neutrons in the nucleus (*e.g.*, ^{12}C, ^{13}C, and ^{14}C are isotopes of carbon).

d) Neutral Atoms: # protons = # electrons.

i) If electrons are taken away from the atom such that the number of protons exceeds the number of electrons, the atom becomes a **positive ion** (*e.g.*, H^+ = H II = singly ionized hydrogen).

ii) If the number of electrons exceeds the number of protons in the nucleus, the atom becomes a **negative ion** (*e.g.*, H^- = negative hydrogen ion, a hydrogen atom with two electrons instead of one).

iii) The ionization stage of an atom can be labeled in a variety of different ways:

$\Longrightarrow$ Roman numerals: I = neutral atom (*e.g.*, He I, neutral helium), II = singly ionized (*e.g.*, Fe II), III = doubly ionized (*e.g.*, C III), etc.

$\Longrightarrow$ '+' exponents (positive ions): if no exponent appears, then we have a neutral atom (*e.g.*, He, neutral helium), '+' = singly ionized (*e.g.*, Fe^+), '3+' = triply ionized (*e.g.*, C^{3+}), etc.

$\Longrightarrow$ '−' exponents (negative ions): '−' = one extra electron (*e.g.*, H^-), '−−' = two extra electrons (*e.g.*, C^{--}), etc.

e) **Molecules** are a collection of atoms that are bound together by **molecular bonds**.

i) Salt: NaCl (1 sodium atom + 1 chlorine atom).

ii) Water: H_2O (2 hydrogen atoms + 1 oxygen atom).

iii) Methane: CH_4 (1 carbon atom + 4 hydrogen atoms).

f) Molecules can adhere to each other through **chemical bonds** making a structured lattice $\Longrightarrow$ solids.

B. The Deformation of Solids.

1. The amount of *shape change* (**deformation**) for a given amount of energy supplied to a solid is called the **elasticity** of the solid.

a) **Stress** is related to the force causing the deformation:

$$\text{stress} \equiv \frac{\text{force}}{\text{area}} = \frac{F}{A} \ .$$

b) **Strain** is a measure of the degree of deformation:

$$\text{strain} \equiv \frac{\text{change in shape}}{\text{initial shape}} \ .$$

c) For small stresses, strain is directly proportional to stress:

$$\text{elastic modulus} = \frac{\text{stress}}{\text{strain}} \ .$$

2. **Young's Modulus**: Elasticity in **Length**.

a) **Tensile stress** is the ratio of the magnitude of the external force applied F to the cross-sectional area of the object A.

b) Stress is the same thing as pressure P:

$$\boxed{P \equiv \frac{F}{A} \ ,} \qquad \text{(XI-1)}$$

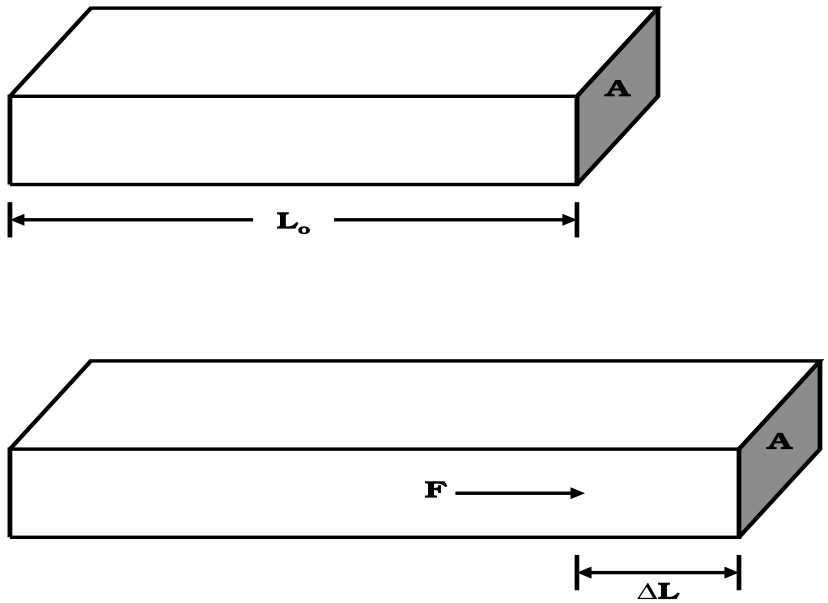
where pressure is measured in **pascals** (Pa):

$$\boxed{1 \text{ Pa} \equiv 1 \text{ N/m}^2 \ .}$$

c) **Young's Modulus**:

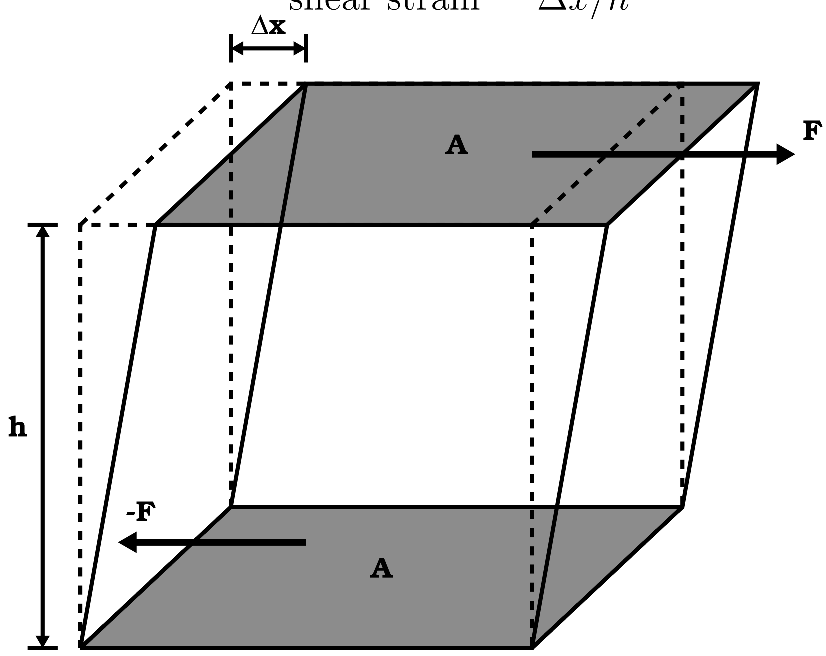
$$Y \equiv \frac{\text{tensile stress}}{\text{tensile strain}} = \frac{F/A}{\Delta L / L_\circ} = \frac{F\,L_\circ}{A\,\Delta L} \qquad \text{(XI-2)}$$

$\Longrightarrow$ force is $\perp$ to the cross-sectional area.



3. **Shear Modulus**: Elasticity in **Shape**.

$$S \equiv \frac{\text{shear stress}}{\text{shear strain}} = \frac{F/A}{\Delta x / h}\ , \qquad \text{(XI-3)}$$



where $[S] \equiv$ Pa $\Longrightarrow$ force F is tangential to the cross-sectional area A.

4. Bulk Modulus: Elasticity in **Volume**.

$$B \equiv \frac{\text{volume stress}}{\text{volume strain}} = -\frac{F/A}{\Delta V/V} = -\frac{\Delta P}{\Delta V/V} \ , \qquad \text{(XI-4)}$$

where $[B] \equiv$ Pa $\Longrightarrow$ force exerted in all directions (*i.e.*, squeezing).

a) The negative sign means that an increase in pressure (ΔP) results in a decrease in volume (ΔV).

b) $\dfrac{1}{B}$ is called the **compressibility** of the material.

5. Both solids and liquids have bulk moduli.

6. Only solids have shear and Young's moduli (liquids can flow).

Example XI–1. A steel wire of diameter 1 mm can support tension 0.2 kN. A cable to support tension 20 kN should have a diameter of what order of magnitude?

Solution:

The cross-sectional area of a round cable/wire is $A = \pi R^2 = (\pi/4)D^2$, where R is the radius and D the diameter of the cable/wire. Call cable '1' the 1 mm diameter wire and cable '2' the one that is supporting the 20 kN tension. Then the cross-sectional area of cable 1 is $A_1 = (\pi/4)D_1^2 = \pi/4 \text{ mm}^2 = 0.8 \text{ mm}^2$. Let's assume that both the wire and the cable are both made of steel, hence both have the same Young's modulus and have the same lengths. Solving the Young's modulus formula for A gives

$$A = \frac{F L_\circ}{Y \Delta L} \ .$$

Now taking the ratio of the cross-sectional areas of these cables gives

$$\frac{A_2}{A_1} = \frac{F_2 L_\circ/(Y\,\Delta L)}{F_1 L_\circ/(Y\,\Delta L)} = \frac{F_2}{F_1}\ ,$$

or

$$A_2 = \frac{F_2}{F_1}\,A_1 = \left(\frac{20\ \text{kN}}{0.2\ \text{kN}}\right) 0.8\ \text{mm}^2 = 80\ \text{mm}^2.$$

Using our area formula from above gives

$$D_2 = \sqrt{\frac{4A_2}{\pi}} = \sqrt{\frac{320\ \text{mm}^2}{\pi}} = \boxed{10\ \text{mm}\ .}$$

C. Density and Pressure.

1. The **density** ρ of a substance of uniform composition is defined as the mass per unit volume:

$$\boxed{\rho \equiv \frac{m}{V}\ ,} \qquad \text{(XI-5)}$$

$[\rho] = \text{kg/m}^3$ in the SI system.

a) The density of a material changes as the pressure and temperature changes following the formula

$$\rho = \rho_\circ \left(\frac{273.15\,P}{P_\circ\,(T + 273.15)}\right)\ , \qquad \text{(XI-6)}$$

where $\rho_\circ$ is the density when the material is at pressure $P_\circ$ (the atmospheric pressure) at 0°C, and ρ is the density at pressure P and temperature T measured in °C (see §XII of the notes).

b) Often, the density of material is given in terms of its value as compared to the density of water ($\equiv$ 1000 kg/m^3). When the density is described in this manner, it is called the **specific gravity** of the material (*e.g.*, lead has a specific gravity of 11.4 since it has a density of 11,400 kg/m^3.

2. The **average pressure** P of a fluid is the ratio of the magnitude of the force exerted by the fluid to the surface area A that the fluid is pushing against:

$$\boxed{P \equiv \frac{F}{A}\ ,} \tag{XI-7}$$

$[P] = \mathrm{N/m^2} = \mathrm{Pa}$ (pascal).

a) Consider a column of fluid of height h in a cylinder of cross-sectional area A. The force on the fluid is just the gravitational force

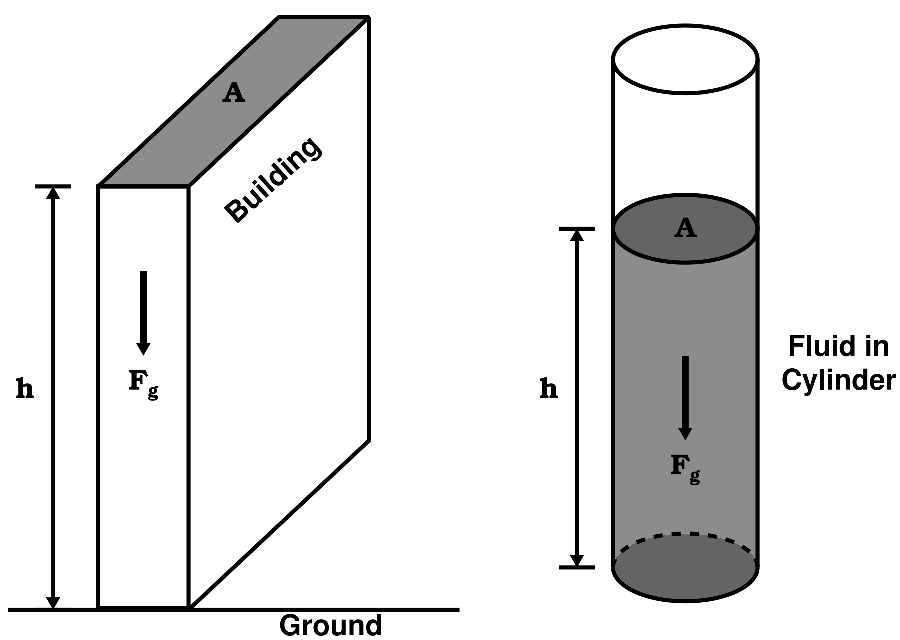
$$F_g = mg = (\rho V)g = \rho(Ah)g\ ,$$

where we have used Eq. (XI-5) and used the volume formula for a cylinder ($V = Ah$).

b) Using this force in Eq. (XI-7) we get

$$\boxed{P = \frac{F_g}{A} = \frac{\rho A h g}{A} = \rho g h\ .} \tag{XI-8}$$

c) This formula is valid for all states of matter (see the following figures).



Example XI–2. Water is to be pumped to the top of the Empire State Building, which is 1,200 ft high. What pressure is needed in the water line at the base of the building to raise the water to that height?

Solution:

We just need to use Eq. (XI-8). Converting the height of the building to SI units, we get $h = 1{,}200 \times 0.3048$ m/ft $= 370$ m. Assuming we are using fresh water with density $\rho = 1000$ kg/m^3, we get a pressure of

$$\begin{aligned} P = \rho g h &= (10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(370 \text{ m}) = 3.6 \times 10^6 \text{ Pa} \\ &= \boxed{3.6 \text{ MPa}\ .} \end{aligned}$$

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3. If we wish to compare points between two different depths in a fluid, all we need to do is make use of Eq. (XI-8) and subtract the pressure at the these two different depths (either h or y are acceptable here to describe depth):

$$\Delta P = \rho\, g\, \Delta h = \rho\, g\, \Delta y\ . \tag{XI-9}$$

D. Pascal's Principle.

1. In words, **Pascal's Principle** states: *Pressure applied to an enclosed fluid is transmitted undiminished to every point of the fluid and to the walls of the containing vessel.*

2. Since we often measure pressure in fluids that are open to air at one end of a container, we can use Pascal's principle to write

$$\boxed{P = P_\circ + \rho g h\ ,} \tag{XI-10}$$

where $P_\circ$ is the atmospheric pressure at the Earth's surface:

$$\boxed{P_\circ = 1.01325 \times 10^5 \text{ Pa} \ .} \qquad \text{(XI-11)}$$

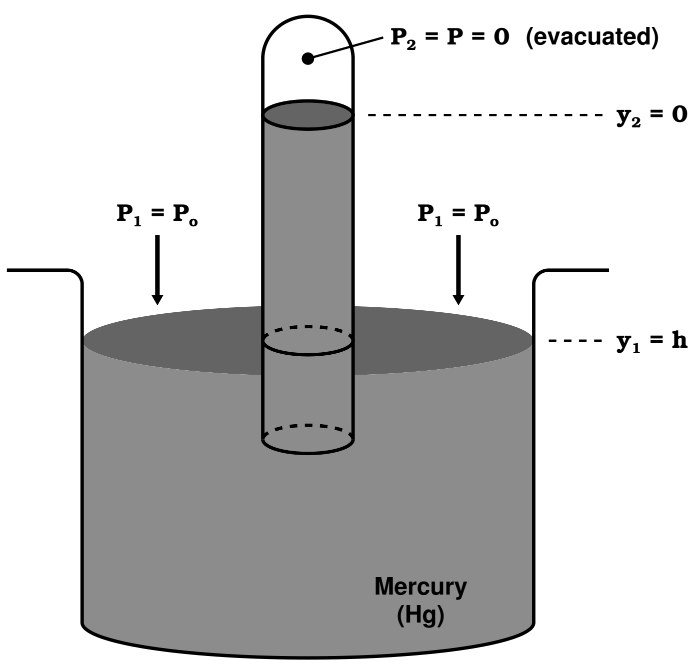
a) The atmospheric pressure in other units:

$$\begin{aligned} P_\circ &= 1.01325 \times 10^6 \text{ dynes/cm}^2 \quad \text{(cgs system)} \\ &= 14.7 \text{ lbs/in}^2 \quad \text{(English system)} \\ &= 1.00 \text{ atm} \quad \text{(atmosphere)} \\ &= 100 \text{ bars} \quad (1 \text{ bar } \approx 10^7 \text{ N/m}^2) \\ &= 760 \text{ mm-Hg} \quad \text{(mm of Mercury)} \end{aligned}$$

b) Note that Eq. (XI-9) is essentially a description of Pascal's principle as well.

c) Also note that the pressure as described by Eqs. (XI-9) and (XI-10) is not affected by the shape of the vessel.

3. A **barometer** is used to measure the atmospheric pressure and is based upon Pascal's Principle.



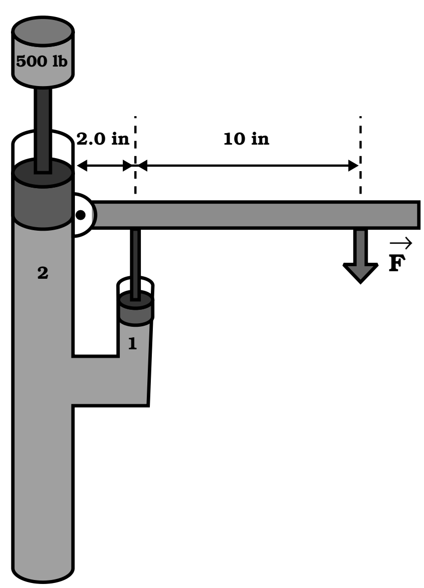
For the device shown in the figure above, Pascal's Principle (Eqs. (XI-9,10) gives the following relationship for a mercury barometer using the exact value of surface gravity for sea level at 45° north latitude:

$$
\begin{aligned}
P_2 - P_1 &= \rho g(y_2 - y_1) \\
P - P_\circ &= \rho g(0 - h) \\
-P_\circ &= -\rho g h \\
P_\circ &= \rho g h \\
&= (13.595 \times 10^3 \text{ kg/m}^3)(9.80665 \text{ m/s}^2)(0.7600 \text{ m}) \\
&= 1.01325 \times 10^5 \text{ Pa}
\end{aligned}
$$

$\Longrightarrow$ There's about 30,000 lbs of air pushing in on our bodies! Why don't we collapse?

4. When taking pressure measurements, P is called the **absolute pressure** and the difference $P - P_\circ$ is called the **gauge pressure**.

Example XI–3. Piston '1' in the figure below has a diameter of 0.25 in; piston '2' has a diameter of 1.5 in. In the absence of friction, determine the force F on the pump handle necessary to support the 500 lb weight.



Solution:

Normally, we would convert the English units to SI, but since we will be taking ratios, there is no need to do this. The larger (left-side) piston is labeled as piston '1;' whereas the smaller (right-side) piston is labeled as piston '2,' as indicated in the problem. Remember that force is a product of pressure times area, where here, the area is the cross-sectional area of each piston. The downward force acting on piston 2 will be designated as F_2. This force is the weight of the mass that sits upon the larger piston plate: $F_g = w = 500$ lb. This force will produce a pressure P_2 in the fluid of the larger piston.

The pressure in the smaller piston is P_1 which will produce an upward directed force F_1 on that piston's plate (appearing just above the number '1' in the diagram). Pascal's law tells us that the pressures between the two connected pistons (*i.e.*, 1 and 2) must be in equilibrium, hence $P_1 = P_2$. The cross-sectional area of each piston is $A = \pi R^2 = \frac{\pi}{4} D^2$, hence the force that the small piston produces from the 500 lb mass via Eq. (XI-7) is

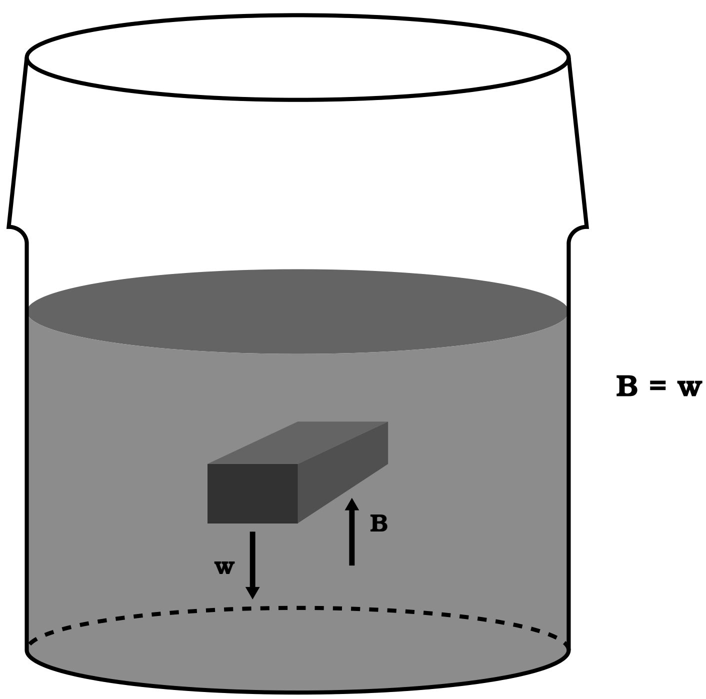
$$\begin{aligned}
P_1 &= P_2 \\
\frac{F_1}{A_1} &= \frac{F_2}{A_2} = \frac{w}{A_2} \\
F_1 &= \frac{A_1}{A_2}\, w = \frac{(\pi/4) D_1^2}{(\pi/4) D_2^2}\, w = \left(\frac{D_1}{D_2}\right)^2 w \\
&= \left(\frac{0.25 \text{ in}}{1.5 \text{ in}}\right)^2 500 \text{ lb} \; = 14 \text{ lb} .
\end{aligned}$$

Now to figure the downward force required on the handle (F) to produce an equilibrium condition, we must use the second condition of equilibrium concerning torques: $\sum \tau = 0$. For this torque problem, F_1 is pointing upward at a lever arm of $d_1 = 2.0$ inches from the pivot point which will produce a positive torque, since this force wants to push the rotation counter-clockwise. The applied force F is pointing downward (as shown in the diagram) with a lever arm of $d = 2.0$ inch $+$ 10 inch $= 12$ inches, which produces a negative torque due to the implied clockwise rotation. These are the only forces acting on this "balance." As such, the second condition of equilibrium gives

$$\begin{aligned}\sum \tau &= 0 \\ +F_1 d_1 - F d &= 0 \\ F_1 d_1 &= F d \\ F &= \frac{d_1}{d} F_1 = \frac{2.0 \text{ in}}{12 \text{ in}} 14 \text{ lb} \\ &= \boxed{2.3 \text{ lb}} .\end{aligned}$$

E. Buoyancy.

1. **Archimedes' Principle**: Any body completely or partially submerged in a fluid is buoyed up by a force equal to the weight of the fluid displaced by the body.



2. Mathematically (note that the "f" subscript means "fluid"):

$$\begin{aligned} B &= \Delta P \cdot A = (\rho_{\rm f} g h)(A) \\ &= \rho_{\rm f} g (hA) = \rho_{\rm f} g V_{\rm f} \\ &= M_{\rm f} g = w_{\rm f} \end{aligned} \qquad \text{(XI-12)}$$

3. Sink or Float?

a) The weight of a submerged object is

$$w_{\rm o} = m_{\rm o} g = \rho_{\rm o} V_{\rm o} g \ ,$$

where the "o" subscript represents "object."

b) From Archimedes' principle, for an object to float,

$$B = w_{\rm o} \quad \Longrightarrow \quad w_{\rm f} = w_{\rm o} \ ,$$

so

$$\rho_{\rm f} g V_{\rm f} = \rho_{\rm o} g V_{\rm o} \qquad \text{(XI-13)}$$

or

$$\boxed{\frac{\rho_{\rm o}}{\rho_{\rm f}} = \frac{V_{\rm f}}{V_{\rm o}} \ ,} \qquad \text{(XI-14)}$$

c) What if $B \neq w_{\rm o}$?

$$B - w_{\rm o} = \rho_{\rm f} g V_{\rm f} - \rho_{\rm o} g V_{\rm o} \neq 0 \ . \qquad \text{(XI-15)}$$

i) If an object is completely submerged, $V_{\rm f} = V_{\rm o}$ and Eq. (XI-15) becomes

$$B - w_{\rm o} = (\rho_{\rm f} - \rho_{\rm o})\, V_{\rm o} g \ . \qquad \text{(XI-16)}$$

ii) If $\rho_{\rm f} > \rho_{\rm o}$, $B - w_{\rm o} > 0 \Longrightarrow$ the object <u>floats</u>!

iii) If $\rho_{\rm f} < \rho_{\rm o}$, $B - w_{\rm o} < 0 \Longrightarrow$ the object <u>sinks</u>!

4. From Eq. (XI-15), we see that there are 2 competing forces involved for submerged objects. From this, we can define an *effective weight* of such an object as

$$w_{\rm eff} = w_{\rm o} - B = F_{\rm g} - F_{\rm b} = mg - \rho_{\rm f} V_{\rm f} g \ . \qquad \text{(XI-17)}$$

Since $V_{\rm o} = V_{\rm f}$ and $V_{\rm o} = m/\rho_{\rm o}$, we can rewrite this equation as

$$\begin{aligned} w_{\rm eff} &= mg - \rho_{\rm f}\left(\frac{m}{\rho_{\rm o}}\right) g = mg\left(1 - \frac{\rho_{\rm f}}{\rho_{\rm o}}\right) \\ &= w_{\rm air}\left(1 - \frac{\rho_{\rm f}}{\rho_{\rm o}}\right) \ . \qquad \text{(XI-18)} \end{aligned}$$

5. But what exactly do we mean by *effective weight*? Since weights are usually measured via its effect on an equilibrium force (*i.e.*, the normal force as the object rests on a scale, or the tension if hanging from a scale), the effective weight is nothing more than the tension on a cord that suspends the mass from a scale (see the figures associated with Example 9.5 in the textbook). As such, we also could write Eq. (XI-17) as

$$T = w_{\rm o} - B \ . \qquad \text{(XI-19)}$$

Note that if we were to take the object out of the water, we would get $T = w_{\rm o}$ (as we saw in §V of the notes) since the buoyancy force would be zero then.

Example XI–4. An object weighing 300 N in air is immersed in water after being tied to a string connected to a balance. The scale now reads 265 N. Immersed in oil, the object appears to weigh 275 N. Find (a) the density of the object and (b) the density of the oil.

Solution (a):

This is simple enough problem that we don't need to draw a figure. This is nothing more than an effective weight type of problem. Note that we have $w_{\rm air} = 300$ N, $w_{\rm water} = 265$ N, and $w_{\rm oil} = 275$ N, with $w_{\rm water}$ and $w_{\rm oil}$ representing our effective weights. We know that the density of water is $\rho_{\rm water} = 1000$ kg/m^3. Using Eq. (XI-18), we can solve for $\rho_{\rm o}$, the density of the object:

$$\begin{aligned}
w_{\rm water} &= w_{\rm air}\left(1 - \frac{\rho_{\rm water}}{\rho_{\rm o}}\right) \\
\frac{w_{\rm water}}{w_{\rm air}} &= 1 - \frac{\rho_{\rm water}}{\rho_{\rm o}} \\
\frac{\rho_{\rm water}}{\rho_{\rm o}} &= 1 - \frac{w_{\rm water}}{w_{\rm air}} \\
\frac{\rho_{\rm water}}{\rho_{\rm o}} &= 1 - \frac{265 \text{ N}}{300 \text{ N}} = 1 - 0.883 = 0.117 \\
\frac{\rho_{\rm o}}{\rho_{\rm water}} &= \frac{1}{0.117} = 8.57 \\
\rho_{\rm o} &= 8.57\rho_{\rm water} = 8.57\,(1000 \text{ kg/m}^3) \\
&= \boxed{8570 \text{ kg/m}^3 .}
\end{aligned}$$

Solution (b):

Now we use the same technique as in (a), but use the oil weight for the effective weight:

$$\begin{aligned}
w_{\rm oil} &= w_{\rm air}\left(1 - \frac{\rho_{\rm oil}}{\rho_{\rm o}}\right) \\
\frac{w_{\rm oil}}{w_{\rm air}} &= 1 - \frac{\rho_{\rm oil}}{\rho_{\rm o}} \\
\frac{\rho_{\rm oil}}{\rho_{\rm o}} &= 1 - \frac{w_{\rm oil}}{w_{\rm air}} \\
\frac{\rho_{\rm oil}}{\rho_{\rm o}} &= 1 - \frac{275 \text{ N}}{300 \text{ N}} = 1 - 0.917 = 0.0833 \\
\rho_{\rm oil} &= 0.0833\rho_{\rm o} = 0.0833\,(8570 \text{ kg/m}^3) \\
&= \boxed{714 \text{ kg/m}^3 .}
\end{aligned}$$

F. Fluid Dynamics — Fluids in Motion.

1. We will often make the assumption that a fluid is "ideal" — a fluid is **ideal** when:

a) The fluid is **non-viscous** $\Longrightarrow$ no internal friction.

b) The fluid is **incompressible** $\Longrightarrow$ density is constant.

c) The fluid is **steady** $\Longrightarrow$ no fluid acceleration.

d) The fluid is **non-turbulent** $\Longrightarrow$ zero angular velocity in fluid.

2. Equation of Continuity.

a) The **continuity** condition requires the **mass flux** of a fluid through a pipe is constant:

$$\text{Mass Flux} = \rho A v = \text{ constant} \quad [\text{kg/s}],$$

or

$$\boxed{\rho_1 A_1 v_1 = \rho_2 A_2 v_2 \ .} \qquad \text{(XI-20)}$$

b) If the fluid is ideal: $\rho_1 = \rho_2$, so

$$\boxed{A_1 v_1 = A_2 v_2 \ .} \qquad \text{(XI-21)}$$

i) $A \equiv$ cross-sectional area of pipe at two different points (1) and (2).

ii) $v \equiv$ velocity of fluid at two different points (1) and (2).

iii) $Av \equiv$ **flow rate**.

iv) Eq. (XI-21) is known as the equation of continuity $\Longrightarrow$ the smaller you make the cross-sectional area of a pipe, the faster the fluid flow.

c) Instead of the word 'trajectory,' the path that a fluid follows is called a **streamline**.

i) Smooth streamline flow (the streamlines don't cross one another) is referred as **laminar flow**.

ii) Fluid flow where streamlines cross one another (hence an angular velocity is present) is referred as **turbulent flow**.

3. Bernoulli's Equation: Fluid flow with change in height.

a) In fluids, the work done by a fluid is given by

$$W = P_1 \cdot \Delta V - P_2 \cdot \Delta V - g\rho\Delta V(h_2 - h_1) \ . \qquad \text{(XI-22)}$$

b) According to the work-energy theorem,

$$\Delta\text{KE} = W \ , \qquad \text{(XI-23)}$$

where

$$\text{KE} = \frac{1}{2}mv^2 = \frac{1}{2}(\rho\Delta V)v^2 \ . \qquad \text{(XI-24)}$$

c) Combining Eqs. (XI-22,23,24), we get

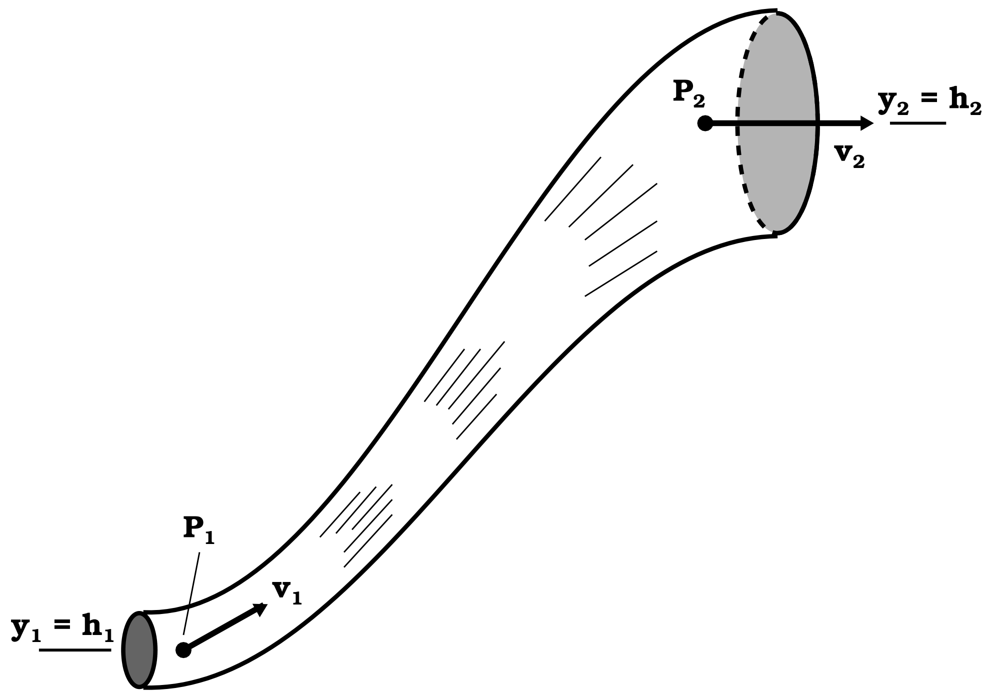
$$\frac{1}{2}\rho\Delta V v_2^2 - \frac{1}{2}\rho\Delta V v_1^2 = P_1 \cdot \Delta V - P_2 \cdot \Delta V - g\rho\Delta V(h_2 - h_1) \ ,$$

or

$$\boxed{P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2 \ ,} \qquad \text{(XI-25)}$$

or

$$\boxed{P + \frac{1}{2}\rho v^2 + \rho g h = \text{ constant.}} \qquad \text{(XI-26)}$$



d) Bernoulli's equation is nothing more than the conservation of energy for a fluid.

e) This equation says that swiftly moving fluids exert less pressure than do slowly moving fluids.

Example XI–5. An airplane is cruising at altitude 10 km. The pressure outside the craft is 0.287 atm; within the passenger compartment the pressure is 1.00 atm and the temperature is 20°C. The density of the air is 1.20 kg/m^3 at 20°C and 1.00 atm of pressure. A small leak occurs in the window seals in the passenger compartment. Model the air as an ideal fluid to find the speed of the stream of air flowing through the leak.

Solution:

This is simple enough problem that we don't need to draw a figure. Ideal fluids are incompressible ($\rho_1 = \rho_2 = \rho = 1.20$ kg/m^3) which is surely unrealistic in our case here, but it does allow for an estimate of the speed of the leaking gas. Use Bernoulli's equation

(Eq. XI-25), define point '1' as the outside of the aircraft and point '2' as the inside, and set $y_1 = y_2 = 0$ with $v_2 \approx 0$ inside the aircraft. Then solve for v_1 (the speed of the leaking gas outside the aircraft):

$$\begin{aligned}
P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 &= P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2 \\
P_1 + \frac{1}{2}\rho v_1^2 + 0 &= P_2 + 0 + 0 \\
\frac{1}{2}\rho v_1^2 &= P_2 - P_1 \\
v_1^2 &= \frac{2\,(P_2 - P_1)}{\rho} \\
v_1 &= \sqrt{\frac{2\,(P_2 - P_1)}{\rho}}
\end{aligned}$$

$$\begin{aligned}
v_1 &= \sqrt{\frac{2(1.00 \text{ atm} - 0.287 \text{ atm})\left(\frac{1.013\times10^5 \text{ N/m}^2}{1 \text{ atm}}\right)}{1.20 \text{ kg/m}^3}} \\
&= \sqrt{\frac{1.44\times10^5 \text{ N/m}^2}{1.20 \text{ kg/m}^3}} = \sqrt{1.20\times10^5 \text{ m}^2/\text{s}^2} \\
&= \boxed{347 \text{ m/s}} .
\end{aligned}$$

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4. **Viscosity and Poiseuille's Law.**

   a) Seldomly fluids travel in an ideal fashion. Most fluids have internal resistance. This *internal friction* is called **viscosity**.

   i) Assume we have a fluid flowing through a pipe of cross-sectional area A ($= \pi R^2$, where R is the radius of the pipe) over a length L between to points labeled '1' and '2.'

ii) Using Eq. (XI-21) can write this ***volume* flow rate** Q through a small slice of the pipe of length Δx as

$$Q = A\,v = A\,\frac{\Delta x}{\Delta t} = \frac{\Delta V}{\Delta t}\ . \tag{XI-27}$$

iii) If the fluid is not ideal, then internal friction of the fluid will set up an internal frictional force called the **viscous force**, $F_{\rm visc}$. In fluid mechanics, the size of this force is measured by **coefficient of viscosity** (often just called the 'viscosity'), η (the Greek letter 'eta').

iv) Viscosity is measured with the unit of **poise**, where

$$\boxed{\begin{aligned} 1\ \text{poise} &= 1\ \text{dyne}\cdot\text{s/cm}^2 \\ &= 0.1\ \text{N}\cdot\text{s/m}^2 = 0.1\ \text{Pa}\cdot\text{s}. \end{aligned}} \tag{XI-28}$$

b) The resistive force set up by this internal friction can be determined from a change in pressure along a streamline:

$$P_1 - P_2 = \frac{8Q\eta L}{\pi R^4}\ . \tag{XI-29}$$

Rewriting this gives the volume flow rate for the fluid:

$$Q = \frac{\Delta V}{\Delta t} = \frac{\pi R^4\,(P_1 - P_2)}{8\eta L}\ , \tag{XI-30}$$

which is called **Poiseuille's law**. The derivation of this law from first principles requires high-level integral calculus, as such, we will just state this law here without proof.

c) Whether a fluid's flow is laminar or turbulent depends upon the viscosity of the fluid. The onset of turbulence can be determined by a dimensionless parameter called the **Reynolds number**:

$$\mathcal{R} = \frac{\rho v D}{\eta} \ , \tag{XI-31}$$

where ρ is the density of the fluid and v the velocity of the fluid flowing through a pipe of diameter D.

i) $\mathcal{R} < 2000$: Flow is laminar.

ii) $2000 < \mathcal{R} < 3000$: Flow is unstable $\Longrightarrow$ flow is laminar, but any small disturbance in the flow will cause turbulence to immediately form.

iii) $\mathcal{R} > 3000$: Flow is turbulent.

Example XI–6. A straight horizontal pipe with a diameter of 1.0 cm and a length of 50 m carries oil with a coefficient of viscosity of 0.12 N·s/m^2. At the output of the pipe, the flow rate is 8.6×10^{-5} m^3/s and the pressure is 1.0 atm. Find the gauge pressure at the pipe input.

Solution:

This is simple enough problem that we don't need to draw a figure. In Poiseuille's law, let $P_1 = P$ be the pressure of the pipe at input and $P_2 = P_\circ = 1.0$ atm be the pipe pressure on output. Using Eq. (XI-29) and converting all units to SI ($R = 0.5D = 0.5$ (1.0 cm $\times 0.01$ m/cm) $=$ 5.0×10^{-3} m), the gauge pressure ($P - P_\circ$) is

$$\begin{aligned} P - P_\circ &= \frac{8Q\eta L}{\pi R^4} \\ &= \frac{8(8.6 \times 10^{-5}\ \mathrm{m^3/s})(0.12\ \mathrm{N \cdot s/m^2})(50\ \mathrm{m})}{\pi (5.0 \times 10^{-3}\ \mathrm{m})^4} \\ &= \boxed{2.1 \times 10^6\ \mathrm{Pa}\ = 21\ \mathrm{atm}\ .} \end{aligned}$$
