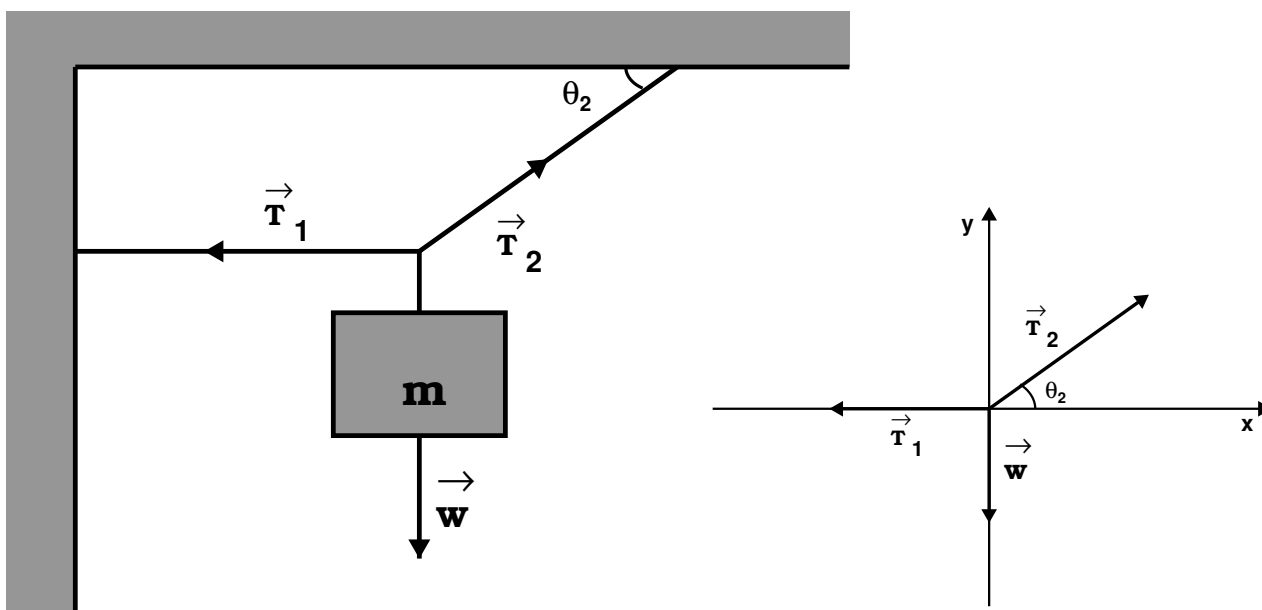


PHYS-2010: General Physics 1
Dr. Luttermoser's Classes
Sample Problems (with Solutions) for Final Exam

1. Consider the suspended object in the diagram below, where $\theta_2 = 36.8^\circ$ and the tension on the horizontal rope 1 is 597 N. (a) What is the tension on rope 2? (b) What is the mass of the object? (**Show all work and don't forget to draw a free body diagram!** Note that you can consider the mass to be located at the junction of ropes 1 and 2.)



Solution (a):

Since we are in equilibrium in both the x and y directions:

$$\sum F_x = -T_1 + T_2 \cos \theta_2 = 0 \quad (1)$$

$$\sum F_y = T_2 \sin \theta_2 - w = 0 \quad (2)$$

Since $\vec{T}_1$ lies completely in the x direction, we will use Eq. (1) to determine the value for $\vec{T}_2$:

$$T_2 \cos \theta_2 = T_1, \quad T_2 = \frac{T_1}{\cos \theta_2} = \frac{597 \text{ N}}{\cos 36.8^\circ} = \boxed{746 \text{ N} .}$$

Solution (b):

Now use Eq. (2) and note that $w = mg$:

$$w = mg = T_2 \sin \theta_2, \quad m = \frac{T_2 \sin \theta_2}{g} = \frac{(746 \text{ N}) \sin 36.8^\circ}{9.80 \text{ m/s}^2} = \boxed{45.6 \text{ kg} .}$$

2. A satellite is in a circular geosynchronous orbit (*i.e.*, an orbital period of exactly one day) above the Earth's equator. Such a satellite has an orbital radius of 42,164 km. (a) What is the altitude of this orbit in km? (b) What is the orbital velocity of this satellite in km/s? (c) If the centripetal force on this satellite is 362 N, what is the mass of this satellite in kg?

Solution (a):

$$T = 1 \text{ day} \left(\frac{24 \text{ hr}}{1 \text{ d}} \right) \left(\frac{3600 \text{ s}}{1 \text{ hr}} \right) = 8.64 \times 10^4 \text{ s}, \quad R_{\oplus} = 6.378 \times 10^6 \text{ m} = 6.378 \times 10^3 \text{ km}.$$

$$r = R_{\oplus} + h$$

$$h = r - R_{\oplus} = 4.2164 \times 10^4 \text{ km} - 6.378 \times 10^3 \text{ km} = \boxed{3.5786 \times 10^4 \text{ km}}.$$

Solution (b):

First convert r to meters:

$$r = 4.2164 \times 10^4 \text{ km} \times \frac{10^3 \text{ m}}{1 \text{ km}} = 4.2164 \times 10^7 \text{ m}.$$

$$\begin{aligned} v_{orb} &= \sqrt{\frac{GM_{\oplus}}{r}} = \sqrt{\frac{(6.673 \times 10^{-11} \text{ N m}^2/\text{kg}^2) (5.9763 \times 10^{24} \text{ kg})}{4.2164 \times 10^7 \text{ m}}} \\ &= 3.075 \times 10^3 \text{ m/s} \times \frac{1 \text{ km}}{10^3 \text{ m}} = \boxed{3.075 \text{ km/s}}. \end{aligned}$$

Solution (c):

$$\begin{aligned} F_c &= \frac{m v_{orb}^2}{r} \\ m &= \frac{F_c r}{v_{orb}^2} = \frac{(362 \text{ N}) (4.2164 \times 10^7 \text{ m})}{(3.08 \times 10^3 \text{ m/s})^2} \\ &= \boxed{1.61 \times 10^3 \text{ kg}}. \end{aligned}$$

3. Aluminum pellets, 1240 in number, each with an identical mass, are heated to a temperature of 242°C. These pellets are placed into 8.44 liters of 20.0°C water. Upon reaching equilibrium, the temperature of the system is 25.2°C. (a) What is the mass of each pellet in grams? (b) How much heat was gained by the water in kilocalories? ($\rho_w = 1000 \text{ kg/m}^3$, $c_w = 4187 \text{ J/(kg } ^\circ\text{C)}$, $c_{Al} = c_p = 900 \text{ J/(kg } ^\circ\text{C)}$).

Solution (a):

$$\rho_w = \frac{m_w}{V}, \quad V = 8.44 \text{ li} \left(\frac{10^{-3} \text{ m}^3}{1 \text{ li}} \right) = 8.44 \times 10^{-3} \text{ m}^3$$

$$m_w = \rho_w V = (10^3 \text{ kg/m}^3)(8.44 \times 10^{-3} \text{ m}^3) = 8.44 \text{ kg}$$

The total mass of the aluminum pellets (m_{Al}) is equal to the total number of the pellets ($N = 1240$) times the mass of the individual identical pellets (m_p):

$$m_{Al} = N m_p .$$

Now solve the calorimetry equation for the mass of an individual pellet:

$$\begin{aligned} Q_{gain} &= Q_{loss} \\ m_w c_w (T - T_w) &= m_{Al} c_p (T_p - T) \\ m_w c_w (T - T_w) &= N m_p c_p (T_p - T) \\ m_p &= \frac{m_w c_w (T - T_w)}{N c_p (T_p - T)} \\ m_p &= \frac{(8.44 \text{ kg})(4187 \text{ J/(kg } ^\circ\text{C)})(25.2^\circ\text{C} - 20.0^\circ\text{C})}{1240 (900 \text{ J/(kg } ^\circ\text{C)})(242^\circ\text{C} - 25.2^\circ\text{C})} \\ m_p &= \frac{1.838 \times 10^5 \text{ J}}{2.419 \times 10^8 \text{ J/kg}} \\ m_p &= 7.595 \times 10^{-4} \text{ kg} \times 10^3 \text{ gm/kg} = \boxed{0.76 \text{ gm} .} \end{aligned}$$

Solution (b):

$$\begin{aligned} Q_{gain} &= m_w c_w (T - T_w) \\ &= (8.44 \text{ kg})(4187 \text{ J/(kg } ^\circ\text{C)})(25.2^\circ\text{C} - 20.0^\circ\text{C}) \\ &= 1.838 \times 10^5 \text{ J} \times \frac{1 \text{ kcal}}{4187 \text{ J}} = \boxed{44 \text{ kcal} .} \end{aligned}$$

4. A 2.50 liter plastic container is filled with 2.44 grams of O_2 gas at a temperature of $54.2^\circ F$.
 (a) How many moles of O_2 gas are contained in this container? (b) How many O_2 molecules are in this container? (c) What is the pressure in this container? (d) What is the root-mean-square speed of these O_2 molecules? (e) What is the thermal energy of this O_2 gas?

Solution (a):

Since O_2 has $16+16 = 32$ nucleons, it has a molecular mass of 32.0 gm/mol. As such, following Example XIII-3 in the notes, the number of O_2 moles in this container is

$$n = \frac{2.44 \text{ gm}}{32.0 \text{ gm/mol}} = \boxed{7.63 \times 10^{-2} \text{ mol .}}$$

Solution (b):

We just need to make use of Avogadro's number to calculate the number of O_2 molecules:

$$\begin{aligned} \# \text{ of molecules} &= n N_A = (7.63 \times 10^{-2} \text{ mol}) (6.02252 \times 10^{23} \text{ molecules/mol}) \\ &= \boxed{4.59 \times 10^{22} \text{ molecules .}} \end{aligned}$$

Solution (c):

$$V = 2.50 \text{ li} \left(\frac{10^{-3} \text{ m}^3}{1 \text{ li}} \right) = 2.50 \times 10^{-3} \text{ m}^3.$$

$$T_F = \frac{9}{5} T_C + 32$$

$$\frac{9}{5} T_C = T_F - 32$$

$$T_C = \frac{5}{9} (T_F - 32) = \frac{5}{9} (54.2 - 32) = 12.3^\circ C$$

$$T_C = T - 273.15$$

$$T = T_C + 273.15 = 285.5 \text{ K}$$

Now use Form 1 of the ideal gas law to solve for pressure:

$$\begin{aligned} PV &= nRT \\ P &= \frac{nRT}{V} = \frac{(7.63 \times 10^{-2} \text{ mol}) (8.3143 \text{ J/(mol} \cdot \text{K)}) (285.5 \text{ K})}{2.50 \times 10^{-3} \text{ m}^3} \\ &= \boxed{7.24 \times 10^4 \text{ Pa .}} \end{aligned}$$

Solution (d):

From Eq. (XIII-13) of the course notes, we first need to calculate the mass of the species in question:

$$m = 2 m_{\text{O}} = 2 \times 16.0 \text{ amu} \times \frac{1.66 \times 10^{-27} \text{ kg}}{1 \text{ amu}} = 5.31 \times 10^{-26} \text{ kg} .$$

Plugging this into Eq. (XIII-13), we get

$$v_{rms} = \sqrt{\frac{3k_B T}{m}} = \sqrt{\frac{3(1.38 \times 10^{-23} \text{ J/K})(285.5 \text{ K})}{5.31 \times 10^{-26} \text{ kg}}} = \boxed{472 \text{ m/s} .}$$

Solution (e):

Using Eq. (XIII-9) we get,

$$\text{TE} = \frac{3}{2} k_B T = \frac{3}{2} (1.38 \times 10^{-23} \text{ J/K})(285.5 \text{ K}) = \boxed{5.91 \times 10^{-21} \text{ J} .}$$

5. A cylinder with a piston filled with 3360 cubic centimeters of a monatomic ideal gas is compressed isobarically as 678 J of work is applied to it. The cylinder has an inner diameter of 13.2 cm and the piston moves a total of 8.22 cm as the volume compresses. Initially the temperature of the gas in the cylinder is 32.6°C and is 87.6°C at the end of the compression. (a) What is the pressure of the gas in atmospheres? (b) What is the change in internal energy of the gas? (c) How much heat is lost during the compression process? (Assume there are no gas leaks in this cylinder.)

Solution (a):

First convert input parameters to SI units:

$$\begin{aligned} T_o &= T_{oC} + 273.15 = 32.6 + 273.15 = 305.8 \text{ K} \\ T &= T_C + 273.15 = 87.6 + 273.15 = 360.8 \text{ K} \\ \Delta T &= T - T_o = 360.8 \text{ K} - 305.8 \text{ K} = 55.0 \text{ K} \\ V_o &= 3360 \text{ cm}^3 \times \left(\frac{1 \text{ m}}{100 \text{ cm}} \right)^3 = 3.36 \times 10^{-3} \text{ m}^3 \\ W &= 678 \text{ J}, \quad D = 0.132 \text{ m}, \quad \Delta x = -0.0822 \text{ m}, \end{aligned}$$

Note that Δx is negative since the gas is being compressed, hence the volume must be decreasing. The change in volume of the cylinder is

$$\Delta V = A \Delta x = \frac{\pi D^2}{4} \Delta x = \frac{\pi (0.132 \text{ m})^2}{4} (-0.0822 \text{ m}) = -1.12 \times 10^{-3} \text{ m}^3.$$

Now using Eq. (XIV-4), the pressure of the gas in this container is

$$\begin{aligned} W &= -P\Delta V \\ P &= -\frac{W}{\Delta V} = -\frac{678 \text{ J}}{-1.12 \times 10^{-3} \text{ m}^3} \\ &= 6.03 \times 10^5 \text{ Pa} \left(\frac{1 \text{ atm}}{1.013 \times 10^5 \text{ Pa}} \right) = \boxed{5.95 \text{ atm}}. \end{aligned}$$

Solution (b):

Using Eq. (XIV-5), the change in internal energy for a monatomic ideal gas is

$$\Delta U = \frac{3}{2} n R \Delta T.$$

Before we can answer this, we have to determine the number of moles contained in the gas. Since the cylinder does not lose any gas during the compression, the number of moles contained in the gas will remain constant. Hence, we can determine this number from Form 1 of the ideal gas law using the initial conditions:

$$\begin{aligned} PV &= nRT \\ n &= \frac{PV_o}{RT_o} = \frac{(6.03 \times 10^5 \text{ Pa})(3.36 \times 10^{-3} \text{ m}^3)}{(8.3143 \text{ J}/(\text{mol} \cdot \text{K}))(305.8 \text{ K})} = 0.797 \text{ mol}. \end{aligned}$$

Using this in the equation above we get:

$$\Delta U = \frac{3}{2} (0.797 \text{ mol})(8.3143 \text{ J}/(\text{mol} \cdot \text{K}))(55.0 \text{ K}) = \boxed{547 \text{ J}}.$$

Solution (c):

Making use of the First Law of Thermodynamics (Eq. XIV-1), the heat lost is

$$Q = \Delta U - W = 547 \text{ J} - 678 \text{ J} = \boxed{-131 \text{ J} .}$$

6. A 45.6 kg rock falls from rest into 32.4 liters of 32.2°C fresh water from a height of 254 m. Half of the kinetic energy at impact goes into heating the water. (a) What is the temperature of the water (in °C) after impact? (b) What is the change in entropy of the water after impact? (Note that $\rho_w = 1000 \text{ kg/m}^3$ and $c_w = 4187 \text{ J/(kg } ^\circ\text{C)}$.)

Solution (a):

To determine the amount of heat generated, we need to first calculate the kinetic energy of the rock as it hits the water using the conservation of mechanical energy (note that $v_o = 0$, $y_o = 254 \text{ m}$, and $y = 0$):

$$\begin{aligned} \text{KE}_i + \text{PE}_i &= \text{KE}_f + \text{PE}_f \\ \text{KE}_f &= \text{KE}_i + \text{PE}_i - \text{PE}_f \\ &= \frac{1}{2}mv_o^2 + mgy_o - mgy \\ &= 0 + (45.6 \text{ kg})(9.80 \text{ m/s}^2)(254 \text{ m}) - 0 \\ &= 1.14 \times 10^5 \text{ J} \\ Q &= \frac{1}{2} \text{KE}_f = \frac{1}{2} (1.14 \times 10^5 \text{ J}) = 5.68 \times 10^4 \text{ J} . \end{aligned}$$

Now we use the calorimetry equation to find the new temperature of the water. We first need to the mass of the water:

$$\begin{aligned} \rho_w &= \frac{m_w}{V}, \quad V = 32.4 \text{ li} \left(\frac{10^{-3} \text{ m}^3}{1 \text{ li}} \right) = 3.24 \times 10^{-2} \text{ m}^3 \\ m_w &= \rho_w V = (10^3 \text{ kg/m}^3)(3.24 \times 10^{-2} \text{ m}^3) = 32.4 \text{ kg} \end{aligned}$$

The new temperature T is

$$\begin{aligned} Q &= m_w c_w (T - T_o) \\ T - T_o &= \frac{Q}{m_w c_w} \\ T &= T_o + \frac{Q}{m_w c_w} = 32.2^\circ\text{C} + \frac{5.68 \times 10^4 \text{ J}}{(32.4 \text{ kg})(4187 \text{ J/(kg } ^\circ\text{C)})} \\ &= 32.2^\circ\text{C} + 0.419^\circ\text{C} = \boxed{32.6^\circ\text{C}} . \end{aligned}$$

Solution (b):

The change in entropy (ΔS) can be determined from the Second Law of Thermodynamics (Eq. XIV-34):

$$\Delta S = \frac{Q}{T} ,$$

where T will be the average in the change of temperature (in K):

$$\begin{aligned} T_o &= T_{oC} + 273.15 = 32.2 + 273.15 = 305.4 \text{ K} \\ T &= T_C + 273.15 = 32.6 + 273.15 = 305.8 \text{ K} \\ T_{ave} &= (T + T_o)/2 = (305.8 \text{ K} + 305.4 \text{ K})/2 = 305.6 \text{ K} , \end{aligned}$$

so

$$\Delta S = \frac{Q}{T_{ave}} = \frac{5.68 \times 10^4 \text{ J}}{305.6 \text{ K}} = \boxed{186 \text{ J/K .}}$$