Abstract

These class notes are designed for use of the instructor and students of the course Physics 2018: Great Ideas in Science. This edition was last modified for the Fall 2007 semester.
II. Stellar Evolution: The Origin of the Elements

A. Stars – What are they?

1. Stars are gaseous bodies that generate their own light via thermonuclear reactions.

2. Stars are the basic building blocks of the structure of the Universe.
   a) Stars organize themselves into giant collections of stars known as galaxies.
   b) Our star, the Sun, is a member of the Milky Way Galaxy which is a spiral galaxy made up of over 200 billion stars.
   c) The Milky Way is approximately 100,000 light years in diameter and the Sun is about 30,000 light years from the center of the Galaxy. (Note that a light year is the distance that light travels in one year.)
   d) The Sun’s orbital velocity about the center of the Galaxy is 220 km/s, which at this distance, takes about 250 million years to complete one orbit about the center of the Galaxy.

3. Astronomers have a variety of classification schemes for stars.
   a) Spectral type which is based on the strength of certain dark lines seem in a star’s spectrum. Spectral type is related to a star’s temperature.
   b) Luminosity class which is a measure of the star’s brightness.
c) **Population class** which is measure of the **metallicity** – the abundance of elements heavier than helium.

4. All three of these classification schemes above can be deduced from a star’s spectrum. A spectrum is a plot of the star’s energy flux as a function of wavelength (or frequency, or energy). Note that the energy flux is essential the brightness of the spectrum at a particular wavelength.

5. If we assume that star’s shine like a blackbody radiator, it is easy to show from the Stefan-Boltzmann radiation law that a star’s luminosity, \( L \), is related to the star’s radius, \( R \), and average photospheric (i.e., ‘surface’) temperature, the so-called **effective temperature**, \( T_{\text{eff}} \), via

\[
L = 4\pi \sigma R^2 T_{\text{eff}}^4 ,
\] (II-1)

or in terms of solar (\( \odot \)) values

\[
\frac{L}{L_\odot} = \left( \frac{R}{R_\odot} \right)^2 \left( \frac{T_{\text{eff}}}{T_{\text{eff}}(\odot)} \right)^4 ,
\] (II-2)

where the effective temperature of the Sun is \( T_{\text{eff}}(\odot) = 5770 \) K.

**B. Spectral Classification.**

1. The first large-scale classification of stellar spectra was undertaken by Mrs. W.P. Fleming, Antonia Maury, and Annie Jump Cannon in the 1920’s at Harvard College Observatory and became known as the Henry Draper (HD) catalog.

a) Over 400,000 stars were classified.

b) Stars were grouped by hydrogen (Balmer) line strengths with designations A-S.
c) Classes C, D, E, H, I, J, L, P, and Q were dropped for one reason or another or merged into other classes (see Jaschek and Jaschek 1987, *The Classification of Stars*, Cambridge Press).

d) The R and N stellar classifications corresponded to carbon stars and now have been merged into one classification designated C (although not the same as the original C stars, which were spectroscopic binaries). Many people however still use the R and N classification (including me) to describe carbon stars. R stars are the hotter of the two and correspond to the oxygen-rich K stars in temperature. The N-type carbon stars correspond to the coolest oxygen-rich M stars in temperature. Carbon stars differ from oxygen-rich stars in that there visual spectrum is dominated by carbon molecule (*i.e.*, C$_2$, CN, and CH) absorption bands.

e) S stars are another special class similar in temperature to late K and M stars. The S star’s spectrum is dominated by LaO, VO, and ZrO molecular bands.

2. Groups were rearranged from the hottest (called *early-type stars*) to the coolest (called *late-type stars*) and 10 subdivisions for each group introduced. Each “spectral type” is determined by various line strengths and line ratios. A brief overview is displayed in Table II-1. A more complete description can be found in Jaschek and Jaschek (1987) and Kaler (1989, *Stars and Their Spectra*, Cambridge Press).

3. Later it was found by Saha, that the sequence of spectral types from hottest to coolest stars should follow:

   O B A F G K M (R N S)
Table II–1: Spectral Classifications

<table>
<thead>
<tr>
<th>Spectral Type</th>
<th>Special Classes</th>
<th>Temperatures</th>
<th>Spectral Characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td>O 5–9</td>
<td>—</td>
<td>50,000–28,000 K</td>
<td>He II lines. He II (λ4686) &amp; N III (λλ4630–34) in emission. He II &amp; N III in absorption, H in emission. Wolf-Rayet (carbon sequence), O-star spectrum, wide emission lines.</td>
</tr>
<tr>
<td></td>
<td>f</td>
<td>&quot;</td>
<td>&quot;</td>
</tr>
<tr>
<td></td>
<td>e, WC5–WC8</td>
<td>&quot;</td>
<td>&quot;</td>
</tr>
<tr>
<td></td>
<td>WN6–WN8</td>
<td>&quot;</td>
<td>&quot;</td>
</tr>
<tr>
<td>B 0–9</td>
<td>—</td>
<td>28,000–9,900 K</td>
<td>He I lines, H lines strengthen. He I, H emission lines. B-star spectrum; Si II, Mn II, Cr II, Eu II, Sr II are strong.</td>
</tr>
<tr>
<td></td>
<td>e</td>
<td>&quot;</td>
<td>&quot;</td>
</tr>
<tr>
<td></td>
<td>p</td>
<td>&quot;</td>
<td>&quot;</td>
</tr>
<tr>
<td>A 0–9</td>
<td>—</td>
<td>9,900–7,400 K</td>
<td>H lines strong, Ca II H &amp; K strengthen toward later type. A-star spectrum; Si II, Mn II, Cr II, Eu II, Sr II are strong. A-star spectrum + Fe lines unusually strong.</td>
</tr>
<tr>
<td></td>
<td>p</td>
<td>&quot;</td>
<td>&quot;</td>
</tr>
<tr>
<td></td>
<td>m</td>
<td>&quot;</td>
<td>&quot;</td>
</tr>
<tr>
<td>F 0–9</td>
<td>—</td>
<td>7,400–6,000 K</td>
<td>Metals, H lines weaken, Ca II strengthen — emission “bumps” appear in H &amp; K line cores at F4.</td>
</tr>
<tr>
<td>G 0–9</td>
<td>—</td>
<td>6,000–4,900 K</td>
<td>Ca II very strong, Na I D line strengthens, metals. (G2-K4) spectrum, Ba II (λ4554) very strong. (G5-K5) spectrum, CH band (λ4300) very strong.</td>
</tr>
<tr>
<td></td>
<td>Ba, CH</td>
<td>&quot;</td>
<td>&quot;</td>
</tr>
<tr>
<td>K 0–7</td>
<td>—</td>
<td>4,900–3,900 K</td>
<td>Ca II strong, Ca I (λ4227) strengthens, molecular bands form.</td>
</tr>
<tr>
<td>M 0–10</td>
<td>—</td>
<td>3,900–2,500 K</td>
<td>TiO bands dominate visual spectrum, neutral metals very strong.</td>
</tr>
<tr>
<td>S 0–9</td>
<td>—</td>
<td>4,200–2,500 K</td>
<td>(K5-M) type stars except ZrO, VO, &amp; LaO molecular bands strong,</td>
</tr>
<tr>
<td>R 0–9</td>
<td>—</td>
<td>4,800–3,200 K</td>
<td>C2, CN, CH strong, =C0-C5 classification.</td>
</tr>
<tr>
<td>N 0–7</td>
<td>—</td>
<td>3,200–2,500 K</td>
<td>C2, CN, CH strong, strong violet flux depression, =C6-C9 classification.</td>
</tr>
</tbody>
</table>
a) Classes R, N, and S are special as described above.

b) The weakness of the H lines in O stars is due to most of the hydrogen being completely ionized.

c) The weakness of the H lines in M stars is due to their cool atmospheres, most of the electrons are in the ground state, with virtually none in the 2nd level where the Balmer lines arise.

C. Luminosity Classification.

1. Absorption lines that appear in a star’s spectrum arise in the outer layers of a star’s atmosphere. Spectral lines are broadened (i.e., made thicker = stronger) via a variety of processes. One of these processes is particle collisions.

   a) Since collisional rates are a function of density and density depends on the surface gravity of a star; bigger, lower gravity stars will tend to have sharper lines than high gravity stars for a given spectral type.

   b) A broader line “for the same spectral type” generally implies higher gravity.

2. As we just saw, luminosity depends both on the radius and temperature of a star.

3. Table II-2 displays the luminosity classification system.

4. The Morgan-Keenan (M-K) classification of a star is simply the spectral type along with the luminosity class and defines a star’s location on an H-R diagram (i.e., the Sun is a G2 V star, α Boo a K1 III star, see below).
Table II–2: Luminosity Classifications

<table>
<thead>
<tr>
<th>Luminosity Class</th>
<th>Type</th>
<th>Absolute Visual Mag</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>B0</td>
</tr>
<tr>
<td>Ia</td>
<td>Luminous supergiants</td>
<td>-6.7</td>
</tr>
<tr>
<td>Ib</td>
<td>Less luminous supergiants</td>
<td>-6.1</td>
</tr>
<tr>
<td>II</td>
<td>Bright giants</td>
<td>-5.4</td>
</tr>
<tr>
<td>III</td>
<td>Normal giants</td>
<td>-5.0</td>
</tr>
<tr>
<td>IV</td>
<td>Subgiants</td>
<td>-4.7</td>
</tr>
<tr>
<td>V</td>
<td>Main sequence (dwarfs)</td>
<td>-4.1</td>
</tr>
<tr>
<td>sd (VI)</td>
<td>Subdwarfs (pop II dwarfs)</td>
<td></td>
</tr>
<tr>
<td>wd (VII)</td>
<td>White dwarfs</td>
<td>10.2</td>
</tr>
</tbody>
</table>

D. Stellar Populations.

1. Chemical abundances are determined from line strengths. Typically, an elemental abundance is tabulated in the form of \([X/H]\),

\[
[X/H] \equiv \log \left( \frac{n(X)}{n(H)} \right)_\odot - \log \left( \frac{n(X)}{n(H)} \right)_\star, \tag{II-3}
\]

where X is the element in question.

a) To make matters more confusing, elemental abundance is sometimes scaled to the hydrogen abundance, with the logarithm of the hydrogen abundance normalized to 12.00. For instance, the solar abundance for sodium in this system is 6.33. This means that the actual abundance, \(\alpha = \frac{n(X)}{n(H)}\), is \(2.14 \times 10^{-6} \) (log \(\alpha = 6.33 - 12.00 = -5.67\)).

b) Confusing things even further, the abundance is tabulated with respect to the total number density and \textbf{not} the hydrogen density. One can adjust from one to the other by realizing that \(n(H)/n(\text{total}) = 0.908\) in the Sun. Except for the peculiar hydrogen deficient stars, it is always assumed that stellar hydrogen abundance is equivalent to the solar value.
2. Astronomers have classified stars via their metallicities (i.e., metal abundances), the so-called stellar populations:

a) Population I stars:
   i) High metal abundance \( (Z \approx 0.01) \). Note that \( Z \) is called the metallicity and is equal to
   \[
   Z = \frac{\sum_{i=3}^{92} m(X_i)}{m(\text{total})}, \tag{II-4}
   \]
   where \( m(X_i) \) is the mass of the metal (i.e., element heavier than helium) \( X_i \) (e.g., \( X_6 \) is carbon (C), \( X_8 \) is oxygen (O), etc.) in a star and \( m(\text{total}) \) is the total mass of the star (= summ of the mass of all the particles that make up the star).

   ii) Exist in the disk of the Galaxy.

   iii) Have fairly circular orbits with low inclinations to the galactic plane.

   iv) They are younger stars with \( t_{\text{age}} < 5 \times 10^9 \) yrs.

b) Population II stars:
   i) Low metal abundance \( (Z \lesssim 0.001) \).

   ii) Exist primarily in the halo of the Galaxy.

   iii) Have highly elliptical orbits with high inclinations to the galactic plane.

   iv) They are older stars with \( t_{\text{age}} > 1.2 \times 10^{10} \) yrs.
c) **Population III stars:**
   
   i) Zero metal abundance \((Z = 0)\).
   
   ii) They no longer exist in the Galaxy.
   
   iii) These were the first stars to form out of the primordial baryons formed during the Big Bang.

3. Besides representing metallicity with the “Z” index (i.e., mass fraction of metals to all particles), metallicity also is defined by a star’s Fe abundance with respect to the Sun following Eq. (II-3):

   \[
   [\text{Fe/H}] \equiv \log_{10} \left( \frac{n(\text{Fe})}{n(\text{H})} \right) - \log_{10} \left( \frac{n(\text{Fe})}{n(\text{H})} \right)_\odot .
   \]  

   a) The Sun’s Fe abundance is about \(\log_{10} \left( \frac{n(\text{Fe})}{n(\text{H})} \right)_\odot = 10^{-5}\).
   
   b) If a star has solar metallicity, then \([\text{Fe/H}] = 0.0\).
   
   c) Population I stars range from \(-0.5 < [\text{Fe/H}] < +0.5\).
   
   d) Population II stars have \([\text{Fe/H}] < -0.8\).
   
   e) The few stars that have \(-0.8 < [\text{Fe/H}] < -0.5\) are typically called old disk population stars, though they are typically grouped with the Population I stars.
   
   f) Population III stars will have \([\text{Fe/H}] = 0\) if they are ever observed in the youngest galaxies at the farthest reaches of the Universe.
E. The Hertzsprung-Russell Diagram

1. In the early 1900s, 2 astronomers found peculiar groupings of stars when they plotted their absolute magnitude (a star’s brightness at 10 parsecs distance) to their \((B - V)\) (blue - visual magnitudes) colors (Hertzsprung) or spectral type (Russell) \(\Rightarrow\) the Hertzsprung-Russell (HR) Diagram (see Figure II–1).

   a) **Main sequence** stars (~90% of all stars are of this type).

   b) **Giant** stars (most of them red in color).

   c) **Supergiant** stars (very luminous, hence large).

   d) **White dwarf** stars (very faint, hence small).

2. When plotting the HR diagram for a star cluster, the stars are virtually at the same distance, hence differences in *observed* brightness corresponds to actual luminosity differences (and not dis-


tance differences).

a) The shape of an HR diagram for a star cluster gives the age of the star cluster.

b) The main sequence turn-off point is a very precise gage of the cluster’s age  →  the point at which a main sequence star exhausts its supply of hydrogen in its core.

3. HR diagrams come in 2 types:

a) *Observed* HR diagrams, absolute magnitude \((M_V)\), or apparent magnitude \((V)\) for clusters, versus color index \((B - V)\) or spectral type.

i) The color index is the magnitude difference between 2 different filter measurements (see Table I–1). The color index, or color, is related to the temperature of the star.

ii) For very hot stars (e.g., O stars), \((U - B)\) (ultraviolet minus blue magnitudes) gives the most accurate temperature measurements. These stars appear bluish.

iii) For very cool stars (e.g., M stars), \((V - R)\) (visual minus red magnitudes) or \((R - I)\) (red minus infrared magnitudes) gives the most accurate temperatures. These stars appear reddish.

iv) For every other star, \((B - V)\) gives the most accurate temperatures.

v) Faint, bluish stars are on the lower left side of an HR diagram, and bright, reddish stars are on the
upper right.

b) *Theoretical* HR diagrams plot luminosity \( (L/L_\odot) \) versus effective temperature \( (T_{\text{eff}}) \) of the star.

i) Eddington, through the process of radiative diffusion, showed that \( L \propto M^4 \) on the main sequence, or
\[
\frac{L}{L_\odot} = \left( \frac{M}{M_\odot} \right)^4. \tag{II-6}
\]

ii) Since the time that a star spends on the main sequence depends on the amount of fuel the star has \( (M) \) divided by the rate at which it burns that fuel \( (L) \), the *main sequence lifetime* of a star is \( t_{\text{life}} \propto M/L \) or \( M^{-3} \), hence
\[
t_{\text{life}} = \left( \frac{M_\odot}{M} \right)^3 \times 10^{10} \text{ years}. \tag{II-7}
\]

iii) The y-axis goes from low (lower portion) to high luminosity (upper portion).

iv) The x-axis goes from hotter temperatures (left side) to lower temperatures (right side).

F. Stellar Nurseries.

1. Stars form from the dust and gas that lie between the stars \( \implies \) the *interstellar medium*.

2. Stellar nurseries are found within *giant molecular clouds* (GMC) as shown in Figure II-2. The nearest GMC to the solar system is the Orion complex.

1. J.H. Jeans first analyzed the gravitational instability of gas clouds in 1902 → the minimum size of a cloud which will *collapse* under its own gravity is called the **Jeans’ Length**.

2. It is nothing more than the conservation of energy. In an interstellar gas cloud, there are two energy sources: the thermal energy of the gas, $E_{th}$, and the gravitational energy of the clouds gravitational field, $E_g$. For a static cloud, energy conservation gives:

$$E_{th} + E_g = 0.$$  \hspace{1cm} (II-8)

a) The thermal energy is just the pressure multiplied by the volume of the material:

$$E_{th} = P \cdot V = \frac{\rho k_B T}{\mu m_H} \cdot \frac{4}{3} \pi R^3.$$  \hspace{1cm} (II-9)
b) The gravitational potential energy is
\[ E_g = -\frac{GM^2}{R} = -16\pi^2 G \rho^2 R^5. \quad \text{(II-10)} \]

c) For a constant density \( \rho \) and temperature \( T \) in the cloud, \( E_{\text{th}} \) increases with radius \( R \) like \( R^3 \), while \( E_g \propto R^5 \), a much stronger dependence on \( R \).

3. Using Eqs. (II-9) and (II-10) in Eq. (II-8) gives
\[ R_J = \left( \frac{k_B}{12\pi G \mu m_H} \right)^{1/2} \frac{T^{1/2}}{\rho^{1/2}}, \quad \text{(II-11)} \]
which is the Jeans’ Length. If the radius of a gas cloud is \( R > R_J \) (keeping \( \rho \) and \( T \) constant), the cloud becomes unstable and will collapse.

4. Defining \( M_J \equiv \frac{4\pi}{3} \rho R_J^3 \) as the Jeans’ Mass gives
\[ M_J = \frac{4\pi}{3} \left( \frac{k}{12\pi G \mu m_H} \right)^{3/2} \frac{T^{3/2}}{\rho^{1/2}} \quad \text{(II-12)} \]
and if \( M > M_J \) for a gas cloud, the cloud will collapse due to gravitational instabilities.

5. During this stage of star formation, the gas cloud is in free fall.

a) Throughout this free-fall stage, the temperature of the gas stays relatively constant \( \Rightarrow \) the collapse is said to be isothermal.

b) One can calculate the free-fall time by equating Newton’s 2nd Law of Motion to his Law of Gravitation:
\[ t_{ff} = \left( \frac{3\pi}{32} \frac{1}{G\rho_0} \right)^{1/2}, \quad \text{(II-13)} \]
where \( \rho_0 \) is the initial mass density of the cloud.
c) Since this free-fall time depends only on density, all parts of the cloud will collapse at the same as long as the cloud has uniform density \(\Rightarrow \text{homologous collapse}\).

6. A GMC will not necessarily collapse as a single unit, typically just a portion of it will collapse based upon the triggering mechanism (see below).

   a) The density of the collapsing cloud will increase by orders of magnitude during the free-fall time.

   b) Though we have developed the collapse criteria under the assumption of constant density, in reality, there will be pockets of inhomogeneities in the cloud.

   c) As a result, sections of the cloud will independently satisfy the Jeans’ mass limit and begin to collapse locally, producing smaller features within the original cloud.

   d) This cascading collapse could lead to the formation of large numbers of smaller objects.

H. Triggers of Star Formation (SF).

1. Observations.

   a) In the Milky Way Galaxy, SF occurs in GMCs.

   b) OB stars form in associations at edges of GMCs.

   c) OB association ionizes the surrounding gas producing an \(\text{H II region}\).

   d) Lower mass (T Tauri-type) stars form throughout the volume of the GMC.

   e) SF defines the optical spiral arms of the Milky Way.
2. What is the trigger? Any process that can cause a stable \((M < M_J)\) cloudlet to become unstable \((M > M_J)\).

   a) **Agglomeration**: Component cloudlets of GMC’s collide and sometime coallace until \(M > M_J\).

   b) **Shock Wave Compression**: A shock can be the trigger \(\implies\) it acts like a *snow plow* causing \(\rho\) to increase, and as a result, \(M_J\) drops (see Figure II-3).

      i) **Spiral Density Wave**: As Milky Way Galaxy rotates, its two spiral arms can compress a GMC, which then leads to star formation.

      ii) **Ionization Front**: O & B stars form very quickly once cloud collapse has started (see below). These produce H II regions from their strong ionizing UV flux, which initially expand outward away from the OB association. This ionization front heats the gas causing a shock to form. The shock can compress the gas such that \(M > M_J\), which once again, leads to star formation.

      iii) **Supernova Shocks**: O & B stars evolve very quickly on the main sequence and die explosively as supernovae. The shock sent out by such a supernova can excite further star formation.

I. The Free-Fall Stage of Stellar Birth.

   1. As a portion of a GMC begins to contract, cloud complexes with masses greater than \(\sim 50 M_\odot\) become unstable and fragment into smaller cloudlets (see Figure II-4). Each little cloudlet continues to collapse as described above.
2. As a large portion of the GMC collapses, many internal eddies and turbulent motions can exist within the cloud. As a result, when fragmentation to stellar-mass sizes occur, each little cloudlet has a rotation associated with it that was induced from one of these eddies as shown in Figure II-5.

3. As the cloudlet contracts, it spins faster due to the conservation of angular momentum $L = M R^2 \omega$, where $\omega$ is the angular velocity of the protostellar cloudlet $\Rightarrow$ since $M$ is constant, as $R$ gets smaller, $\omega$ gets larger. This increased spin causes the equatorial region to bulge outward which flattens the cloudlet. This continues until a central bulge with an equatorial disk forms as shown in Figure II-6.

a) The equatorial disk flattens rather quickly (hundreds of years) during this stage. At this point, the disk is now referred to as a proplyd or protoplanetary disk.

b) In this protoplanetary disk, dust grains begin to stick together from condensation and accretion, building in size to form planetesimals. These planetesimals conglomerate further into protoplanets (see Figure II-7).
Figure II–4: Collapsing cloud fragmentation to smaller collapsing “cloudlets.”

Figure II–5: Cloudlet contraction and spin as a result of internal eddies.
Figure II–6: Formation of protostar and protoplanetary disk.

i) In the outer protoplanetary disk, ice crystals condense out of the gas along with some dust grains.

ii) In the inner protoplanetary disk, it is too hot for ice crystals to form, only dust condenses out.

iii) The dust (and ice) begin to conglomerate together in a process known as advection (similar to building a snowman), building bigger and bigger particles. This processed continued until boulder to mountain sized objects existed ⇒ the planetesimals.

iv) Planetesimal is the name given to big rocks in orbit about a “protostar.” Due to their small size ($D < 1000$ km), they are not spherical in shape.

• The “rocky” planetesimals are now called asteroids.
Figure II–7: Inside the protoplanetary disk of the collapsing cloudlet.

- The “icy” planetesimals are now called comets.

v) Planetesimals occasionally smash into each other, sometimes destroying each other, sometimes sticking together to form even bigger planetesimals → this is a process known as accretion.

c) Numerous such disks have been seen in stellar nurseries with the Hubble Space Telescope. They are especially easy to see at IR wavelengths.

d) Unfortunately, we do not have time to elaborate on the formation of planets in this course.

4. During this time, the cloudlet is in free-fall — the so-called free-fall stage. Its luminosity continues to decrease as the cloud collapses, while temperature increases slightly. This stage ends when the protostar becomes opaque to its own radiation (primar-
ily IR photons). This happen when the cloudlet’s central bulge is about the size of Mercury’s orbit.

a) The free-fall time depends upon the density that the cloudlet had at initial formation due to fragmentation.

i) GMC temperatures are on the order of 50 K. Though the average density of a GMC is on the order of \(10^3\, \text{cm}^{-3}\), when cloudlet fragmentation occurs, the particle density is on the order of \(10^8\, \text{cm}^{-3}\) which corresponds to a mass density of \(2 \times 10^{-16}\, \text{gm cm}^{-3}\).

ii) For such initial conditions, the free-fall time to an opaque state is about 4700 years. Increasing the initial density by a factor of 10 would reduce the free-fall time to a mere 1500 years — a blink of an eye to the age of the Universe \(\Rightarrow\) cloudlets collapse rapidly to the protostar stage!

b) The higher-mass cloudlets have higher densities due to the higher gravitational force associated with them. As such, the more massive the cloudlet, the faster it will collapse.

5. Though there is a slight increase in temperature, this increase is very small, and as such, this free-fall stage is said to correspond to an isothermal collapse of the cloudlet.

J. The Hayashi Stage of Stellar Birth — The Protostars.

1. The contraction now continues at a much slower rate once the gas becomes opaque to IR photons.

a) This marks the beginning of the Hayashi stage. At this point, the collapsing cloudlet is now called a protostar.
b) During this stage, the gas becomes unstable to convection, and as a result, the protostar’s energy is transported via convection. This efficient energy transport mechanism increases the luminosity of the protostar considerably while the protostar maintains a relatively constant radius.

c) Temperatures begin to rise at a much more rapid rate as more material falls onto the protostar (though still not as much as later epochs).

d) However, since the gas is opaque to IR radiation (where $\lambda_{\text{max}}$ is located), the internal energy is unable to escape. At this point, the collapse is said to proceed adiabatically.

e) During this stage of stellar birth, the rate of evolution becomes controlled by the rate at which the protostar can thermally adjust to the collapse $\Rightarrow$ this is just the Kelvin-Helmholtz time.

f) For a $1 \, M_\odot$ protostar, this quasi-static collapse takes about $10^7$ years to complete until the next stage of stellar birth.

2. The evolution of the protostar has a variety of key events that take place which depend upon the mass of the protostar. These “events” were first described by Iben (1965, Astrophysical Journal, 141, 993). Here, we list these key events for a $1 \, M_\odot$ protostar. Later, we will describe the evolution of a low-mass and then a high-mass protostar.

a) With the steadying increasing effective temperature of the protostar, metals start to ionize which liberates free electrons. Some of these electron can interact with neutral hydrogen to produce the H$^-$ ion.
b) Opacity in the outer layers becomes dominated by the H\(^-\) bound-free (bf)- and free-free (ff)-opacities.

c) This increased opacity causes the convection instability criterion to be met and convection begins. This convection envelope becomes very deep since the temperatures are not yet high enough down deep to ionize hydrogen (hence reduce the H\(^-\) opacity).

d) When deep convection sets in, luminosity decreases sharply with effective temperature increasing only slightly.

e) At this point, temperatures at the center of the protostar are such that deuterium burning commences (i.e., the second step of the PP I chain).

i) The deuterium (\(^2\)H) that burns here is deuterium created during the Big Bang of the formation of the Universe (as described in Section I of these Astronomy Module notes).

ii) This reaction is favored over the first step of the PP I chain because it has a fairly large cross-section at low temperatures.

iii) Since \(^2\)H is not very abundant, these nuclear reactions have little effect on the overall collapse — they simply slow the rate of contraction slightly.

iv) At this point, the energy production rate is a combination of a nuclear term and a gravitational term:

\[
\varepsilon = \varepsilon_{\text{nuc}} + \varepsilon_{\text{grav}} .
\] (II-14)
f) As the central temperature continues to rise, increasing levels of ionization of H decrease the total opacity in that region (the cross section of H\(^{-}\) opacity is many orders of magnitude bigger than electron scattering and H bf- and H II ff-opacities).

i) At this point, a radiative core develops and progressively encompasses more and more of the star’s mass.

ii) The radiative core allows energy to escape into the convective envelope more readily, causing the luminosity of the star to increase again.

g) The path that a star takes on the H-R Diagram from the initial formation of the surface convection zone when luminosity drops by an order of magnitude to the point where luminosity begins to rise again is referred to as the **Hayashi track**.

h) At about the time that the luminosity begins to increase again, the central temperatures are now high enough for the first two steps of the PP I reaction chain and the steps from \(^{12}\)C to \(^{14}\)N in the CNO cycle to begin in earnest, but not yet at their equilibrium rates. As time progresses, \(\varepsilon_{\text{grav}}\) has a smaller and smaller impact on \(L\) as compared to \(\varepsilon_{\text{nuc}}\).

i) The surface of the protostar now becomes hot enough to emit a significant amount of its energy flux as visible light photons (still being powered by gravitational contraction).

i) The pressure from this light starts to push dust sized particles out of the protoplanetary disk via radiation pressure.
ii) All that remains in the protoplanetary disk are larger planetesimals (rocky ones close in — the ‘asteroids,’ and icy ones farther out — the ‘comets’) and the protoplanets.

iii) This spring-cleaning phase is called the T-Tauri stage of the protostar evolution, named after the prototype of this class, T Tauri (see Figure II-8).

iv) This “spring cleaning” phase can continue well into the epoch when thermonuclear reactions start up in earnest, the so-called main sequence phase.

v) Since so much material exists in the disk, the momentum exchange between the photons and matter is not efficient. However at the poles, the density is much less which results in a strong bipolar outflow \(\Rightarrow\) bipolar jets form.

vi) These jets can interact with the surrounding material causing higher density clumps to form. Such clumps seen in a T Tauri star’s jets are call Herbig-Haro objects.

j) The rate of nuclear energy production has become so great that the central core is forced to expand somewhat, causing the gravitational energy term in Eq. (II-14) to become negative. This effect is apparent at the surface as the total luminosity decreases towards the main sequence value. This decrease in \(L\) forces \(T\) to decrease as well.

k) At this point, \(^{12}\text{C}\) is completely exhausted from the CN reactions of the CNO cycle which reduces the energy pro-
duction, the core adjusts by contracting which increases the temperature until the point when the 3rd step of the PP I can start occurring in earnest.

1) Now the temperatures become high enough to allow the last stages of the CNO cycle to proceed (though not at a very high rate), which replenishes the core with $^{12}\text{C}$.

m) When this occurs $\varepsilon_{\text{mc}} \gg \varepsilon_{\text{grav}}$, core contraction stops and the PP I chain achieves thermal equilibrium with the gas. At this point, **A STAR IS BORN.**

3. **Protostar Evolution of a 0.4 $M_\odot$ (i.e., Low Mass) Star.**

a) There are a few major differences between the evolution of a low-mass protostar ($\sim 0.4 M_\odot$) to that of a solar mass protostar:
b) Due to the lower temperatures in the core of these protostars (as compared to a \(1 \, M_\odot\) protostar), opacity stays relatively high in the core which prevents a radiative core from developing.

c) The protostar (and later star) stays completely convective throughout.

d) The core never gets hot enough to start the CN portion of the CNO cycle.

e) As a result of these events, the luminosity never turns around and starts to increase prior to nuclear reactions being in earnest \(\Rightarrow\) the protostar “falls” directly onto the main sequence.

f) Should a protostar’s mass be \(M < 0.08 \, M_\odot\), the central temperature never gets hot enough for the PP chain to get started (though deuterium burning can take place for a brief period of time). Collapse continues until electron degeneracy sets in (in about 100 billion years!). Such objects are called \textbf{brown dwarfs}.

4. Protostar Evolution of a \textbf{15 \, M_\odot} (\textit{i.e.}, High Mass) \textbf{Star}.

a) Because high mass stars \((M \gtrsim 5 \, M_\odot)\) reach high enough temperatures to ignite the first steps of the PP chain and the CN portion of the CNO cycle so quickly, their Hayashi tracks are very short.

b) These stars leave the Hayashi track at a much higher luminosity than their solar-type cousins and retain a relatively constant high luminosity as the protostar collapses due to the rapid increase in central and effective temperatures —
this can easily be shown with the blackbody radiation relation of Eq. (II-2) \(\Rightarrow\) the effective temperature increases by approximately a factor of 10 as the radius of the protostar decreases by a factor of 100 after the protostar leaves the Hayashi track.

c) Once the thermonuclear reactions reach their thermal equilibrium rates, the high temperature of the core allows the CNO cycle to completely dominate the PP chain as the primary source of energy.

d) Due to the strong temperature dependence of the CNO cycle, convective instability remains in effect in the core and the energy is transported outward via convection.

e) In the envelope, temperatures are high enough to keep H (and He) ionized all of the way to the photosphere which prevent H\(^-\) formation from occurring. Due to the lower opacities in the envelope, convective instability does not take place and the envelope becomes radiative from the Hayashi track all of the way through to thermonuclear ignition (\textit{i.e.}, main sequence).

5. In the outer layers of the most massive stars, the radiation pressure from the photon flow from the center of the star is enormous. The \textbf{Eddington limit}, \(L_{Ed}\), is defined as the maximum luminosity a star can have without blowing itself apart from radiation pressure.

a) From the radiative transfer equation, the pressure gradient near the surface of a star can be written as

\[
\frac{dP}{dr} \approx -\frac{k\rho}{c} \frac{L}{4\pi r^2}.
\]
b) HSE demands that
\[ \frac{dP}{dr} = -\frac{GM\rho}{r^2}. \]

c) Combining these two equation and solving for the luminosity, we get
\[ L_{\text{Ed}} = \frac{4\pi Gc}{k} M, \quad (\text{II-15}) \]
where this Eddington luminosity is the maximum luminosity a star can have and still satisfy HSE.

d) For massive stars, the effective temperatures are 40,000 K or greater which is enough to ionize hydrogen in their photospheres, and as such, electron scattering is the dominant opacity (i.e., \( k \)) source. Making use of this and expressing the Eddington luminosity in terms of solar luminosities, we can write
\[ \frac{L_{\text{Ed}}}{L_{\odot}} \simeq 3.8 \times 10^4 \frac{M}{M_{\odot}}. \quad (\text{II-16}) \]

e) Eddington also showed that a gas sphere in radiative equilibrium will follow the following mass-luminosity relationship (see next section):
\[ \frac{L}{L_{\odot}} \simeq \left( \frac{M}{M_{\odot}} \right)^{3.5}. \quad (\text{II-17}) \]

f) Equating this radiative-equilibrium luminosity with the Eddington limit gives us the maximum mass a star can have without blowing itself apart from radiation pressure:
\[ M \simeq 70 M_{\odot}. \quad (\text{II-18}) \]

g) Since gravitational instabilities tend to prevent stars from forming at such a high mass, there should not be too many stars that exceed the Eddington limit. The Wolf-Rayet stars, however, come very close to this Eddington limit.
K. Life on the Main Sequence.

1. Main sequence (MS) stars are also called hydrogen-core burners. Approximately 90% of a star’s nuclear burning lifetime is spent in this stage (hence 90% of all stars fall on the main sequence on the HR diagram). When a star first ignites hydrogen in its core, it resides on the zero-age main sequence (ZAMS).

2. This main sequence lifetime corresponds to the nuclear time scale:

\[ t_{\text{nuc}} = \frac{\text{Energy available from nuclear fuel}}{\text{Rate at which fuel is used up}} = \frac{\eta f X_{\text{init}} M c^2}{L} \]  \hspace{1cm} (II-19)

where \( \eta = 0.0071 \) is the \( ^4\text{H} \rightarrow \text{He} \) reaction efficiency introduced in the Nuclear Physics section of the Physics Module notes, \( X_{\text{init}} \) is the initial mass fraction of H, and \( f \) is the fraction of H actually burned on the MS.

a) So \( t_{\text{MS}} = t_{\text{nuc}} \) is the amount of time the star remains on the MS.

b) \( f \approx 0.1 \) for \( 1M_\odot \) stars, which gives

\[ t_{\text{MS}} \approx 10^{10} \text{ yr} \frac{M/M_\odot}{L/L_\odot}. \]  \hspace{1cm} (II-20)

c) \( f \) increases up to 0.3 for upper main sequence stars.

3. The mass-luminosity relation for stars on the main sequence.

a) Eddington showed in 1924, using a radiative diffusion approximation, that a spherical sphere of ideal gas which is in radiative equilibrium should follow a mass-luminosity relationship given by Eq. (II-17) \( \implies \) more massive stars are much more luminous than lower mass MS stars.
b) Later, based on main sequence stars in binary star systems in the solar neighborhood, a mass-luminosity relationship was empirically found for main sequence stars of

\[
\frac{L}{L_\odot} \approx \left( \frac{M}{M_\odot} \right)^4 \quad \text{for } L > L_\odot \quad \text{(II-21)}
\]

\[
\frac{L}{L_\odot} \approx \left( \frac{M}{M_\odot} \right)^{2.8} \quad \text{for } L \leq L_\odot \quad \text{(II-22)}
\]

4. Using the Eddington $M$-$L$ relationship, the MS lifetime then can be written as

\[
t_{\text{MS}} \approx \left( \frac{M}{M_\odot} \right)^{-2.5} \times 10^{10} \text{ yr} \quad \text{(II-23)}
\]

5. The energy transport mechanism that dominates the interior of a star changes along the MS as was described in the last section on stellar formation.

a) **High-mass stars** ($M \gtrsim 1.3 \, M_\odot$, which correspond to stars earlier than F5 V).

i) For stars more massive than 1.2–1.3 $M_\odot$, the CNO cycle dominates over the PP chain in the main sequence star’s energy production.

ii) Since the CNO cycle has such a large temperature dependence (anywhere from $T^{13}$ to $T^{20}$) in comparison to the PP chain, temperature gradients are very large in the cores of these stars.

iii) Due to this large temperature gradient, convective instability is established and energy in the core is transported outward by convection over radiation transport $\implies$ hence these stars have **convective cores**.
iv) The temperatures in the outer envelope keep most of the hydrogen completely ionized all of the way to the photosphere of these stars. As such, the dominant opacities in the envelope of these stars is electron scattering, H I bf, and H II ff.

v) The cross sections of these opacity sources is small in comparison to H$^{−}$ bf and H$^{−}$ ff. As such, photons are able to flow outward much easier in the envelope of these stars with respect to low mass stars.

vi) As a result, these stars have radiative envelopes.

b) Low mass stars ($\sim 0.4 \, M_\odot < M < \sim 1.3 \, M_\odot$) stars.

i) The PP chain is the dominant energy production source in these stars. With its lower temperature dependence (anywhere from $T^{3.5}$ to $T^{6}$) the temperature gradient in the core of these stars is not high enough to cause convective instability $\Rightarrow$ energy is transported via radiation.

ii) As a result, these stars have radiative cores.

iii) The outermost regions of the envelopes of these stars have temperatures low enough for neutral hydrogen to exist in sufficient quantities to allow large H$^{−}$ bf and ff opacities (due to there large cross sections).

iv) These large opacity sources are very efficient in blocking photon flow which sets up convective instabilities.
v) As a result, these stars have **convective envelopes**.

vi) As we go to cooler and cooler main sequence stars, the convective envelope gets deeper and deeper due to the ability of \( \text{H}^\text{−} \) to form from the under-ionization of hydrogen.

c) **Very low mass stars** \((M \lesssim 0.4 M_\odot)\), which corresponds to M4-M5 V stars) are completely convective due to the fact that sufficient \( \text{H}^\text{−} \) opacity exists throughout much of the envelope to effectively block much of the energy trying to escape from below. The core still stays convective since the PP chain dominates in these stars too.


   a) The high mass limit on the main sequence is about 60 \( M_\odot \) (O5 V).

      i) Physics dictates that this limit should be about 90 \( M_\odot \) since for masses larger than this, thermal oscillations in their cores lead to dramatic variations in nuclear energy production rates which prohibits the formation of stable stars.

      ii) However, gravitational fragmentation typically does not produce protostars larger than about 50-60 \( M_\odot \). As such, 90 \( M_\odot \) stars typically will not form in the first place.

      iii) One final thing can occur that prevents us from seeing high mass stars — collapse is so rapid from protostar state that they either:
• Collapse to a black hole before there is even a chance to ignite H.

• Should these objects ever make it to the main sequence, they supernova within a few 100 to a thousand years!

b) The low mass limit is about $0.08 \, M_\odot$ (M8 – M10 V).

i) Temperatures never get high enough to ignite H at an equilibrium rate to produce He.

ii) Objects with $M < 0.08 \, M_\odot$ are called brown dwarfs.

iii) Objects with $M < 10 \, M_{\text{Jupiter}} (= 0.01 \, M_\odot)$ are called planets.

L. Fundamental Time Scales on which Stars Evolve.

1. There are 3 basic time scales that are important to stars during their evolution:

<table>
<thead>
<tr>
<th>NAME</th>
<th>CONDITIONS</th>
<th>SOLAR</th>
</tr>
</thead>
<tbody>
<tr>
<td>dynamic time</td>
<td>non-HSE</td>
<td>$\sim 1/2$ hour</td>
</tr>
<tr>
<td>thermal (K-H) time</td>
<td>HSE, non-TE</td>
<td>$\sim 30$ million years</td>
</tr>
<tr>
<td>nuclear time</td>
<td>HSE, TE</td>
<td>$\sim 10$ billion years</td>
</tr>
</tbody>
</table>

2. While on the MS, a star’s luminosity will slowly increase and temperature slowly decrease. This gives the main sequence a thickness when viewed on the HR diagram.

a) As such, stars do not stay at their ZAMS position for their entire H-core burning life.

b) As the hydrogen is fused into helium, $\mu$ (the mean molecular weight) in the core increases.
i) This will cause the pressure to drop (assuming ideal gas).

ii) Following the dynamic time scale, the core compresses as a result which increases the density and temperature.

iii) This in turn increases the thermonuclear reaction rates which increases the surface luminosity.

c) For stars of 1.2-1.3 $M_\odot$ or less, this increased luminosity, increases the surface temperature of the star. These stars get brighter and hotter during their main sequence lifetime.

d) For higher mass stars where H is ionized throughout the entire interior, this added flux increases the photon momentum on the free electrons in the outer layers, which expands these layers and results in a cooling in the photosphere. These stars get brighter and cooler while on the main sequence.

e) Note that these changes are extremely gradual, and as such, stars on the main sequence can be considered to be in hydrostatic and thermal equilibrium.

3. Once the nuclear time scale has been exceeded, hydrogen in the region of thermonuclear reactions (i.e., the core) becomes exhausted. This core goes out of TE and HSE and begins to collapse.
Table II–3: Stellar Parameters for ZAMS Stars.

<table>
<thead>
<tr>
<th>Spectral Class</th>
<th>( M/M_\odot )</th>
<th>( R/R_\odot )</th>
<th>( L/L_\odot )</th>
<th>( T_{\text{eff}} ) (K)</th>
<th>( t_{\text{MS}} ) (years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>O5</td>
<td>60</td>
<td>12</td>
<td>( 7.9 \times 10^3 )</td>
<td>44,500</td>
<td>( 7.6 \times 10^5 )</td>
</tr>
<tr>
<td>B0</td>
<td>17.5</td>
<td>7.4</td>
<td>( 5.2 \times 10^4 )</td>
<td>30,000</td>
<td>( 3.4 \times 10^6 )</td>
</tr>
<tr>
<td>A0</td>
<td>2.9</td>
<td>2.4</td>
<td>54</td>
<td>9520</td>
<td>( 5.4 \times 10^8 )</td>
</tr>
<tr>
<td>F0</td>
<td>1.6</td>
<td>1.5</td>
<td>6.5</td>
<td>7200</td>
<td>( 2.5 \times 10^9 )</td>
</tr>
<tr>
<td>G0</td>
<td>1.05</td>
<td>1.1</td>
<td>1.5</td>
<td>6030</td>
<td>( 7.0 \times 10^9 )</td>
</tr>
<tr>
<td>K0</td>
<td>0.79</td>
<td>0.85</td>
<td>0.42</td>
<td>5250</td>
<td>( 1.9 \times 10^{10} )</td>
</tr>
<tr>
<td>M0</td>
<td>0.51</td>
<td>0.61</td>
<td>( 7.7 \times 10^{-2} )</td>
<td>3850</td>
<td>( 6.6 \times 10^{10} )</td>
</tr>
<tr>
<td>M5</td>
<td>0.21</td>
<td>0.27</td>
<td>( 1.1 \times 10^{-2} )</td>
<td>3240</td>
<td>( 1.9 \times 10^{12} )</td>
</tr>
</tbody>
</table>

M. Ascent on the Red Giant Branch — Marching Towards Helium Ignition.

1. For this section, we will investigate stellar evolution from main sequence up until the ignition of helium in the core for stars in three mass classifications. We start with the very Very Low-Mass Stars \((M < 0.4M_\odot)\):  
   a) These stars are completely convective while on the main sequence. As such, since H is continuously being replenished into the core from the outer envelope, these stars have \( f = 1 \) in Eq. (II-19) and burn H in their cores until the star is completely exhausted of H.

   b) Once the PP chain runs out of fuel, the He-rich star goes out of HSE and begins to collapse.

   c) As the star gets smaller, it gets hotter, ionizing much of the interior of the star.

   d) The star collapses until the free electrons start to “feel” the Pauli Exclusion Principle — two electrons cannot exist in the same quantum state in the same place at the same time.
i) Once this occurs, the equation of state is no described by the ideal gas law given by

\[ P = \frac{\rho k_B T}{\mu m_\text{H}} , \]

where \( P \) is the gas pressure, \( \rho \) is the mass density, \( T \) is the temperature, \( k_B \) is Boltzmann’s constant, \( \mu \) is the mean molecular weight, and \( m_\text{H} \) is the mass of a hydrogen atom.

ii) Instead, degenerate electron pressure describes the equation of state:

\[ P_e = K \rho^{5/3} , \]  \hspace{1cm} (II-24)

where \( K \) is a constant \( \implies \) pressure is a function of density alone and is independent of temperature.

iii) This degenerate electron pressure is strong and causes the gravitational contraction to cease.

iv) The star will exist from this point forward until the end of time as a **helium-rich white dwarf**.

2. Low-Mass Stars (0.4\(M_\odot \lesssim M \lesssim 4M_\odot\)):

a) Here we just mention a few key points about the evolution of these stars from H-core exhaustion to He-core ignition.

b) Note that the upper limit of this mass regime is a bit uncertain and is very sensitive to the metal abundance of a given star. Some authors put the upper limit for this regime at around 3 \(M_\odot\).

c) Once the hydrogen abundance \((X)\) drops below \(\sim 1\text{-}2\%\) in the core of these stars, the PP chain/CNO cycle can not
longer sustain itself. The core is now dominated by helium “ash,” however, because the core is still hot, the second law of thermodynamics dictates that this heat energy has to flow outward to cooler regions.

d) Due to this, the core goes out of TE and HSE and begins to contract. This contraction causes the density, pressure, and temperature to increase in the core.

e) This contraction heats the core (though initially not enough to start helium fusion) and also heats the H-rich shell just above it.

i) As soon as $T \gtrsim 10^7$ K, H fusion starts in that shell.

ii) The star is now a H-shell burner.

iii) Due to the high opacities inside the star, it takes a while for this core-collapse, H-shell burning information to get to the outer envelope of the star.

f) This H-shell burning increases the energy input into the outer envelope.

i) However, most of the work from this added energy goes towards increasing the pressure in the outer envelope $\Rightarrow$ causing the envelope to expand.

ii) As it expands, the extreme outer layers of the star cool since the energy flux must be conserved and the surface area is getting larger. From the laws of blackbody radiation, as the surface cools, it gets redder and redder.
iii) The star gets larger (and more luminous) and cooler producing an orange subgiant star. The F5 star Procyon is at this stage in its evolution (although Procyon, being a bit more massive than the Sun, is moving from the white region of the main sequence to the yellow region of the subgiant luminosity class).

iv) The star is now said to be ascending the red giant branch.

g) While the outer envelope is expanding, the core continues to contract and heat. The H-burning shell is producing more helium ash which falls on the core, increasing the core’s mass as a result.

i) Helium fusion will not begin until $T > 10^8$ K.

ii) Prior to these temperatures being obtained in the core, electrons are forced closer and closer together until the gas becomes degenerate.

h) These star quickly experience a run-away thermonuclear event in their cores know as the “helium flash,” or to be more specific, the Helium-Core Flash.

i) The electron degeneracy pressure counterbalances gravity (as described above), hence resists compression due to gravity.

ii) As shown in Eq. (II-24), pressure ($P$) in a degenerate gas is independent of temperature ($T$).
iii) The temperature of the helium atoms continues to rise as matter is dumped from the H-burning shell above — but pressure remains constant

⇒ the normal pressure-temperature-gravity thermostat is off since the ideal gas law is no longer in effect!

iv) \( T \) reaches \( 10^8 \) K so helium starts fusing which increases \( T \) more (without increasing \( P \)) ⇒ increase of \( T \) increases reaction rates, which further increases \( T \)!

⇒ Resulting in runaway thermonuclear reactions!

⇒ Hence, the **Helium-Core Flash**.

At this time, the core luminosity increases to \( 10^{11} L_\odot \), comparable to that of an entire galaxy in just a few seconds time! Most of this energy never reaches the surface however, instead it is absorbed by the gas in the envelope, possibly causing some mass to be lost as this time from the envelope.

v) Soon, \( T \) gets high enough to “lift” degeneracy ⇒ HSE is reinstated ⇒ the (now) red giant becomes stable and is called a **red giant clump star** (if a Population I star) or a **horizontal branch star** (if a Population II star).

vi) The star now has two energy sources, H-shell burning (primarily from the CNO cycle) and He-core burning (via the triple-\( \alpha \) process) and will remain in this state for the final 10% of its thermonu-
3. **High-Mass Stars** \((M \gtrsim 4 \, M_{\odot})\) follow the same evolutionary sequence as the Low-Mass Stars, except the core never becomes degenerate during collapse. Once temperatures get above \(10^8\) K, the triple-\(\alpha\) process begins gradually until the rates obtain an equilibrium value — no runaway occurs in these stars, hence no He-core flash.

N. **Post-Red Giant Evolution.**

1. The path a star takes on its road to stellar death depends upon its initial mass. Note that the following masses correspond to the mass of the star while on the main sequence.

2. \(M < 0.08 M_{\odot}\): These objects never ignite hydrogen and hence never become main sequence stars. These objects are called **brown dwarfs**. They are bright at infrared wavelengths from energy liberated from contraction.

3. \(0.08 M_{\odot} < M < 0.4 M_{\odot}\): These stars are totally convective, and as such, helium ash gets uniformly mixed throughout the star. No inert He core develops, hence no H-shell burning and no expansion into a red giant. After hydrogen is exhausted, the entire star contracts until the helium becomes degenerate (see the white dwarf section) and the star becomes a helium-rich **white dwarf**.

4. \(0.4 M_{\odot} < M < 3 M_{\odot}\): These stars ignite He in a degenerate core \((i.e.,\) the He-flash). During the core He-fusion stage, the star sits in the red giant **clump** (Population I stars) or the **horizontal branch** (Population II stars) on the HR diagram. He-fusion creates \(^{12}\text{C}\) and \(^{16}\text{O}\) ash and once the He-fusion stops, the carbon-oxygen core continues to collapse.
a) This causes energy to be dumped into the He-rich layer just above the core which ignites \( \Rightarrow \) star now has a He-burning shell and H-burning shell. This extra energy source expands the outer layers of the star further — the star becomes a bit more red and a lot more luminous.

b) During this time, the star moves up the **asymptotic giant branch** on the HR diagram.

c) The outer layers of the star can become unstable during this phase and it can begin to pulsate \( \Rightarrow \) **Mira-type variable**.

d) Strong stellar winds begin at this phase which produce **planetary nebula**. The mass loss continues until only the core of the star remains.

e) The contracting carbon-oxygen core becomes degenerate before carbon fusion can begin — the star is now dead as a carbon-oxygen rich white dwarf.

5. \( 3M_\odot < M < 8M_\odot \): These stars ignite \( ^{12}\text{C} \) in a degenerate core. Unlike the He-flash, this **carbon detonation** completely destroys the star in a **supernova** explosion.

6. \( M > 8M_\odot \): These stars burn elements up to iron (Fe) in their cores. While this is going on, strong stellar winds are pushing much of the star’s outer envelope into the ISM. After Fe is formed in the core, no further energy can be obtained from thermonuclear reactions. As a result, the collapse of the core cannot be halted by the onset of further thermonuclear reactions. The core can do one of 2 things during its final collapse:
a) Achieve nuclear densities \((e^- + p \longrightarrow n + \nu)\) and bounce when neutron degeneracy sets in \(\Longrightarrow\) a *neutron star* forms.

b) The core collapses past the neutron degeneracy state (if \(M_{\text{core}} > 3M_\odot\)) down to a *black hole*.

### O. Stellar Pulsations and Mass Loss.

1. The first variable star discovered was \(o\ Cet\) in 1596. Astronomers were so surprised by this finding that they renamed the star *Mira* (the wonderful).

a) This was the discovery of the first *long period variable* (LPV) star.

i) Most LPV stars change in brightness on a somewhat periodic time frame anywhere from 150 to 500 days and change by a factor of over 1000 in their brightness. These LPV stars are called *Mira-type* (M) *variables* after their prototype \(o\ Cet\).

ii) Some LPVs are less periodic in their variability and are called *semiregular (SR) variables*. These stars typically change in brightness by factors less than 100 of their average brightness.

iii) The remaining stars classified as LPV stars have no periodicity and typically change in brightness by less than 10 from maximum to minimum light. These LPVs are called *irregular (L) variables*.

b) All of these types of variable stars are red giant stars on the asymptotic giant branch of their evolution.
c) The brightness variability is caused by an actual pulsation of the star!

2. Another type of variable star was discovered in 1784, the **Cepheid variables**, with δ Cephei as the prototype.
   a) These stars are more luminous and are hotter than the long period variables.
   b) Their variability period can range anywhere from 1 to 60 days, depending on their overall luminosity.
   c) These are pulsating stars too.
   d) Cepheids follow a **period-luminosity relationship**. The luminosity of the star scales *somewhat* linearly with the pulsational period → by measuring a Cepheid's period, we can deduce its absolute magnitude $M_V$, then by comparing $M_V$ to its apparent magnitude $V$, we can deduce its distance $d$ via the distance modulus formula:

   $$V - M_V = 5 \log d - 5 . \quad (II-25)$$

e) Population I star cepheids (called **Type I** or **classical cepheids**) have a slightly different period-luminosity relationship than the Population II star cepheids (called Type II cepheids or **W Virginis stars**).

3. Lower mass versions of Cepheids exist called **RR Lyrae** type variables, which change in brightness with period shorter than 1 day. These stars are horizontal branch stars, and as such, all have the approximate same luminosity (e.g., 100$L_\odot$), hence can also be used as distance indicators.
4. All of these different types of variables pulsate due to their internal structure — they all lie on an instability strip on the HR diagram.

a) The pulsations are due to the kappa effect, kappa for opacity, which results from an ionization zone that lies just beneath the photospheres of these stars.

b) Miras pulsate from a hydrogen ionization zone just beneath the surface of the star.

c) Cepheids and RR Lyr’s pulsate from a helium ionization zone just beneath the surface.

P. Stellar Corpses I: White Dwarfs.

1. Stars will end up in one of 3 states: white dwarf, neutron star, or black hole. Stars that are initially \( \sim 4 \, M_\odot \) or less on the main sequence will wind up as a white dwarf.

2. As the carbon-oxygen core continues to collapse after He-core burning, the outer envelope of the star continues to expand away, helped along by strong stellar winds.

a) The outer envelope detaches itself from the collapsing core and becomes a shell of material surrounding the core.

b) When the shell becomes thin enough so that the hot core can be seen through the shell, UV photons emitted from the core fluoresce the expanding shell \( \Rightarrow \) a planetary nebula forms.

c) This shell will continue to expand, getting thinner and thinner as it gets larger until it begins to intermingle with the surrounding ISM.
d) Since the outer envelope of the star was enhanced with C, N, and O from the nuclear reactions due to the shell burning phases, the ISM gets enhanced with these elements.

e) Future stars that will be born out of the ISM gas will thus have higher abundances of C, N, and O due to this previous epoch of stellar evolution.

3. The laws of quantum mechanics (QM) dictate the characteristics of electrons.

a) Electrons have a variety of quantum states associated with them:
   
i) *Principle* quantum number \( (n) \Rightarrow \) which electronic shell an electron is in.

   ii) *Orbital* angular momentum quantum number \( (\ell) \) \( \Rightarrow \) IDs the subshell.

   iii) *Spin* angular momentum quantum number \( (s) \) \( \Rightarrow \) \( s = +\frac{1}{2} \) for counterclockwise spin, \( s = -\frac{1}{2} \) for clockwise spin.

   iv) *Total* angular momentum quantum number \( (j = \ell + s) \).

b) One fundamental law in QM is the Pauli Exclusion Principle (PEP): no two electrons can exist in the same quantum state (i.e., have the same quantum numbers) at the time in the same place.

4. As the stellar core continues to collapse, the free electrons there are forced so close together that they all try to sit in the same
quantum state at the same location.

a) This cannot happen since it violates the PEP — the electrons resist further compression and become degenerate.

b) The core becomes stable due to degenerate electron pressure.

c) When this happens, the core is now called a white dwarf (WD).

d) White dwarfs range in mass from 0.6 $M_\odot$ to 1.4 $M_\odot$.

e) The radius of a WD scales as $R \propto 1/M^{1/3}$ $\implies$ the more massive a white dwarf, the smaller it is.

f) Most of the WDs are about the size of the Earth giving an enormous density for these objects ($\rho = 10^9$ kg/m$^3$) $\implies$ 1 teaspoon of WD material on the surface of the Earth would weight 5.5 tons, as much as an elephant!

5. White dwarfs are in HSE since electron degeneracy pressure balances gravity. However, if the mass of a WD exceeds 1.4 $M_\odot$, this degeneracy pressure will be too weak to counterbalance gravity!

a) This mass limit is called the Chandrasekhar limit.

b) All white dwarfs have $M < 1.4M_\odot$.

c) If $M > 1.4M_\odot$, the core will continue to collapse to the next type of stellar corpse — the neutron star.

6. White dwarfs have the following spectral classifications. The first letter ‘D’ denotes degenerate and the second letter denotes the the primary spectral type in the optical spectrum.

a) DA: Only hydrogen Balmer lines are seen — no helium or metal lines present.
b) **DB**: Only He I lines are seen — no hydrogen or metal lines present.

c) **DC**: Continuous spectrum, no lines deeper than 5% in any part of the electromagnetic spectrum.

d) **DO**: He II strong, He I and H lines present.

e) **DZ**: Metal lines only, no H or He lines.

f) **DQ**: Carbon features, either atomic or molecular in any part of the electromagnetic spectrum.

7. The nearest white dwarfs to the solar system are Sirius B and Procyon B, each a companion of the brighter star we see in the night sky.

Q. Stellar Corpses II: Neutron Stars.

1. Stars more massive than $8 \, M_{\odot}$ on the main sequence will be able to successively burn heavier and heavier elements as their core collapses.

2. The final type of stellar burning to go on in a massive core is Si (silicon) burning. The ash from this burning is Fe (iron).

a) Fe is the most stable of all the chemical elements $\implies$ you can neither get energy out of an Fe nucleus from either **nuclear fusion** (*i.e.*, bringing lighter elements together to form heavier ones) or **nuclear fission** (*i.e.*, the breaking apart of heavier elements into lighter ones).

b) Once an Fe core forms, its collapse cannot be halted by further nuclear reactions.
c) The Fe core become degenerate as it collapses but is too massive for the electron degeneracy pressure to hold up the weight of the star.

d) Since the Pauli Exclusion Principle dictates that electrons cannot exist in the same state, the only place for them to go is into the Fe nuclei!

3. The electrons interact with protons in an inverse $\beta$-decay:

$$e^- + p \rightarrow n + \nu,$$

forming a stellar core of pure neutrons!

4. Once the neutrons are formed, they stop the collapse of the core via the strong nuclear force $\Rightarrow$ neutron degeneracy pressure holds up the weight of the core $\Rightarrow$ a neutron star (NS) is born!

a) This halting of the collapse of the core is rather sudden and causes the core to bounce and rebound a bit. This bounce sets up a shock wave which propagates outward and blows apart the outer envelope of the star in a supernova explosion!

b) A tremendous amount of energy is released during this explosion over a fraction of a second, increasing the luminosity of the star by a factor of $10^8$!

c) However this luminosity increase only corresponds to 1% of the total energy released, most of the energy comes out in the form of neutrinos!

d) After the explosion, the outer envelope continues to expand, creating a supernova remnant which can last for a few million years (e.g., the Crab nebula).
5. Supernovae are classified based primarily upon their spectra and their light curves (*i.e.*, how brightness changes with time). The primary classification is based upon the appearance of hydrogen Balmer lines.

a) **Type I** supernovae do not show hydrogen Balmer lines and reach a maximum brightness of $M_V \approx -19$. They are classified into 3 subclasses:

i) **Type Ia** supernovae display a strong Si II line at 6150 Å. These are white dwarfs in close binary star systems that exceed the Chandrasekhar limit and explode from a runaway thermonuclear reaction in the carbon-oxygen core of the white dwarf. These types of supernovae are seen in both elliptical galaxies and throughout spiral galaxies.

ii) **Type Ib** supernovae display strong helium lines. These types of supernovae are only seen in the arms of spiral galaxies near star forming regions. Hence, this implied that short-lived massive stars in binary systems are probably involved. These explosions are similar to that of a Type II supernova, only in a binary star system where the outer H envelope has been transferred to the secondary star in the system before the Fe-core bounce.

iii) **Type Ic** supernovae display weak helium lines and no Si II is seen. Other than this, they are observed in the same location in galaxies as Type Ib. These types of supernovae are likely the same as Type Ib, except the helium-shell has been transferred to the companion in addition to the hydrogen envelope.
b) **Type I**\textsuperscript{1/2} supernovae display weak hydrogen lines. These are isolated stars of main sequence mass of the range 4-8 $M_{\odot}$ where carbon detonates in a degenerate core and completely disrupts the star. Hydrogen lines are weak due to earlier mass loss on the AGB which results in a lower mass hydrogen envelope in comparisons to the Type II supernovae.

c) **Type II** supernovae display strong hydrogen lines and reach a maximum brightness of $M_{V} \approx -17$. These supernovae result from the explosion of an isolated star due to an Fe-core bounce. They are classified into 2 subclasses base upon the shape of their light curve.

i) **Type II-L** supernovae display a linear decline in their light curves after maximum emission is reached.

ii) **Type II-P** supernovae display a plateau in the light curve between 30 and 80 days after maximum light. This plateau results from additional light generated from the radioactive decay of the large amounts of $^{56}$Ni that was created in the explosion. These supernova are likely more massive than the Type II-L supernova which accounts for the larger amounts of this radioactive isotope of nickel in these explosions.

6. During the explosion, material is ejected off of the core at very high velocities. Fast moving free neutrons given off by all of the nucleosynthesis occurring in the outward moving shock interact with nuclei in the envelope to make different chemical elements \( \rightleftharpoons \) such nuclear reactions are called **rapid neutron capture** or the **r-process**.
a) Most of the heavy elements on the periodic table are made from this r-processed material during supernovae (the iron in your car was made in a supernova, whereas the carbon in your DNA was made via the triple-α process in a now extinct AGB star!).

b) Additional nuclear reactions can occur too from slow moving neutrons, the so-called slow neutron capture or s-process. This process also occurs during normal thermonuclear burning while a star is on the main sequence, RGB, RGC (or horizontal branch), and AGB.

7. Supernovae send the envelope of the star, with its nuclear-processed material, out into the ISM at supersonic speed. As this material collides with the ISM material, shocks form which ionizes the gas and produces enough opacity to “light up” the gas $\implies$ a supernova remnant is seen. Supernova remnants emit light at virtually all wavelengths. They are especially bright at radio wavelengths.

8. There are two other types of stellar explosions — the low energy stellar explosions:

a) A nova is the result of gradual mass transfer onto a WD in a binary star system.

i) Thermonuclear runaway occurs on the surface of the WD before maximum mass limit is reached $\implies$ blows off accreted H.

ii) Maximum brightness: $M_V = -5$.

b) X-ray bursters are the nova analogy except matter is dumped on a neutron star instead of a white dwarf (as is
9. All stars rotate while on the main sequence. As the core of a star collapses, it spins faster and faster due to the conservation of angular momentum.

a) By the time a stellar core reaches NS size (\( R_{\text{NS}} = 30 \text{ km} \), the size of a city), it is spinning hundreds of times a second!

b) Pulsars are rapidly spinning neutron stars whose magnetic axis is not aligned with its polar axis.

c) Pulsars flash very rapidly as a result due to the lighthouse effect of the beamed light coming from the magnetic axes sweeping in the line-of-sight towards Earth.

d) Pulsars were first discovered in radio waves in 1967 \( \implies \) first thought to be signals from extraterrestrial intelligence — they were initially called LGM’s for Little Green Men as a result.

10. The small size and high mass \((1.4M_{\odot} < M_{\text{NS}} < 3M_{\odot})\) for neutron stars give them an exceedingly large density \( \implies \rho = 10^{17} \text{ kg/m}^3 \), 1 teaspoon of NS material on Earth would weight over 500 million tons!

11. Neutron stars also have a maximum mass, if the mass of a stellar remnant exceeds \( 3M_{\odot} \), not even neutron degeneracy can hold up the stellar core. It collapses further to a black hole!
R. Stellar Corpses III: Black Holes.

1. Whereas Newton envisioned gravity as some *magical* force, Einstein describes gravity as a curvature in space and time (called the **space-time continuum**).

   a) The general theory is based upon the **principle of equivalence**. You cannot distinguish between acceleration in a gravitational field and accelerating due to a mechanical force (*i.e.*, \( F = ma \)) in a gravity-free environment.

   b) Space and time are interrelated — you cannot have one without the other!

   c) The Universe can be considered to consist of a *fabric* of space-time (*i.e.*, the vacuum) with pockets of matter that exist within this fabric, where this matter *bends* the fabric of space-time. The more mass density, the greater the bending or **warping** of space-time (see Figure II-9).

2. As a stellar core collapses, it gets denser and denser, and the escape velocity from the surface of the star goes increases. First, let’s look at the conservation of energy from Newtonian mechanics for a body of mass \( m \) moving in the gravitational field from rest \( (v = 0) \) at the surface \( (h = 0) \) to a height \( h \) above the surface (marked with a ‘\( \circ \)’ subscript) of a much larger body of mass \( M \) and radius \( R \):

\[
E = (KE + PE)_{\circ} = (KE + PE)_{h} = \text{constant}, \quad \text{(II-26)}
\]

where \( E \) is the total energy,

\[
KE = \frac{1}{2}mv^{2}, \quad \text{(II-27)}
\]

is the kinetic energy, and

\[
PE = -\frac{GMm}{R}, \quad \text{(II-28)}
\]

II–53
Figure II–9: The curvature of space-time gets greater and greater as a stellar core gets denser and denser as it collapses.
is the gravitational potential energy.

a) The conservation of energy becomes

$$\frac{mv^2}{2} - \frac{GMm}{R} = -\frac{GMm}{(R + h)}.$$  \hspace{1cm} (II-29)

b) We can solve this for $h$ such that

$$h = \left(\frac{v^2}{2g}\right) \left[\frac{R}{R - (v^2/2g)}\right],$$  \hspace{1cm} (II-30)

where $g = GM/R^2$ is the surface gravity of mass $M$.

c) One can see from this that as $v^2/2g \to R, h \to \infty$ — the mass $m$ escapes the gravitational field of $M$. When this occurs, the velocity required to send $h \to \infty$ is called the escape velocity:

$$v_{esc} = \sqrt{\frac{2GM}{R}}.$$  \hspace{1cm} (II-31)

3. The escape velocity for various objects:

a) At the Earth’s surface: $R = R_\oplus, M = M_\oplus$, so $v_{esc} = 11$ km/s.

b) At the Sun’s photosphere: $R = R_\odot, M = M_\odot$, so $v_{esc} = 620$ km/s.

c) At a WD surface: $R = R_\oplus, M = M_\odot$, so $v_{esc} = 6500$ km/s = 0.02 $c$.

d) At a NS surface: $R = 30$ km, $M = 2M_\odot$, so $v_{esc} = 230,000$ km/s = 0.77 $c$.

4. A black hole (BH) is defined when an object reaches a size such that its escape velocity equals the speed of light. The collapse
continues past this $v_{esc} = c$ size until the BH becomes infinitely small ($R \to 0$) $\implies$ it collapses to a **singularity**.

**Example II–1.** How big would the Sun have to be in order to become a black hole?

\[
v_{esc} = c, \quad M = M_\odot \quad v_{esc}^2 = \frac{2GM}{R}
\]

\[
R = \frac{2GM}{c^2} = 2.96 \times 10^3 \text{ m} = 2.96 \text{ km}
\]

a) A black hole is defined as a region in space-time where $v_{esc} \geq c$. The outer boundary of this region where $v_{esc} = c$ is called the **event horizon**.

b) The event horizon has a radius of

\[
R_S = \frac{2GM_{BH}}{c^2}, \quad \text{(II-32)}
\]

which is called the **Schwarzschild Radius**.

c) The vast majority of the region within a black hole (i.e., within the event horizon) is empty space. The mass that was once a stellar core has no size (i.e., zero volume). This infinitely dense object at the center of a black hole is called a **singularity**.

5. Stellar cores will become black holes only if $M_c > 3M_\odot$. No known force in nature will prevent its collapse.

6. Of course as the core collapses, its mass density increases without bound and we enter a regime where Newton’s Universal Law of
Gravity begins to break down and general relativity must be used to describe the gravitational field.

7. Actually, as \( R \to 0 \), we reach a point where the general theory is no longer valid either \( \implies \) we must await the theory of quantum gravity to be fully developed. General relativity breaks down when

\[
R \leq \ell_p = \sqrt{\frac{\hbar G}{c^3}} = 1.6 \times 10^{-33} \text{ cm} \quad (II-33)
\]

and for time intervals less than

\[
\tau_p \leq \frac{\ell_p}{c} = \sqrt{\frac{\hbar G}{c^5}} = 5.4 \times 10^{-44} \text{ sec}, \quad (II-34)
\]

where \( \hbar = h/2\pi \). The quantities \( \ell_p \) and \( \tau_p \) are called the Planck length and the Planck time, respectively. We will have more to say about these quantities when we discuss the Big Bang.

8. Anything ( \( i.e. \), matter or light) that gets inside the event horizon will never be heard from again in our space-time continuum. But what happens to objects as they approach the event horizon? Let’s assume we are observing a particle falling into a BH at a large distance \( r_\infty \) and that time passes at rate \( t_\infty \) at this position.

a) As the particle approaches the BH, it experiences normal Newtonian gravity when \( r_\infty > r \gg r_s \).

b) As \( r \to r_s \) (the strong field regime), general relativity has an enormous effect on the particle.

c) The interval between ticks for a clock in the frame of the particle, \( dt \), is given by

\[
dt = \left( 1 - \frac{2GM}{rc^2} \right)^{1/2} dt_\infty = \left( 1 - \frac{r_s}{r} \right)^{1/2} dt_\infty. \quad (II-35)
\]

\( \implies \) clocks appear to run slow near a BH when observed from a large distance.
d) A standard ruler of length $dr_\infty$ measured at point $r_\infty$ would appear to be of length

$$dr = \frac{dr_\infty}{(1 - \frac{2GM}{rc^2})^{1/2}} = \frac{dr_\infty}{(1 - \frac{r_s}{r})^{1/2}} \quad \text{(II-36)}$$

at a position $r$ in the vicinity of a BH.

e) A result of these two equations is that the position of a particle released at position $r_o$ in a BH’s gravitational field follows

$$r \simeq r_s + r_o e^{-ct_\infty/2r_s} \quad \text{(II-37)}$$

as seen by a distant observer $\implies$ the particle never reaches the event horizon as view by an external observer!

9. Although we won’t describe the details of the curvature of spacetime here, we can present a general overview. Instead of our
3-dimensional universe, assume that we live in a 2-dimensional universe. Then according to the general theory, mass bends this 2-dimensional universe into a 3rd dimension. BH’s bend space-time to such an extent that the BH actually rips a hole in the fabric of the Universe and reconnects with a distance part of the Universe or connects to a parallel universe in a different space-time continuum (see Figure II-10).

a) A connecting tunnel forms in space-time called an Einstein-Rosen bridge and also called a wormhole.

b) Our 3-dimensional Universe bends into a 4th dimension, called hyperspace.

c) Tidal forces increase without bound as you approach the singularity ⇒ anything with size, even nuclear particles, get ripped apart from these tidal forces as matter falls into the wormhole.

10. The only way to detect a BH is to observe the effect one has on a companion star in a binary star system (see Figure II-11). As material is stripped away from a normal star by the BH, it spirals down to the BH and heats up to very high temps (due to friction). Before crossing the event horizon, the gas heats to such a high temp that it emits X-rays. Black candidates must satisfy the following observational requirements:

a) An X-ray source in a binary star system whose X-rays vary in brightness over the time period of seconds (hence emitting region must be very small in size).

b) The unseen companion of the spectroscopic binary has a mass greater than $3 \, M_{\odot}$. 
c) The best candidates are the ones where the mass of the unseen companion is greater than the mass of the visible star.

11. The best of the stellar BH candidates are:

a) Cyg X-1 (optical component: 30 $M_\odot$, X-ray component: 7 $M_\odot$).

b) LMC X-3 (optical component: 5 $M_\odot$, X-ray component: 10 $M_\odot$).

c) V404 Cyg (optical component: 1 $M_\odot$, X-ray component: 12 $M_\odot$) — this is the best candidate.

d) SS 433 (BH with jets?).
Figure II–12: Summary of stellar evolution.

12. Black holes can also result from non-stellar evolutionary processes.

   a) The Big Bang may have created an enormous number of mini-black holes.

   b) All of the large galaxies in the Universe contain supermassive black holes (1 million to 1 billion solar masses in size) at their centers (indeed, our home galaxy, The Milky Way, has one)!
S. Stellar Evolution Summary.

1. We have established that stars change over time — they evolve!

2. All elements heavier than helium (that is, everything except H and He) were created in factories we call stars. There would be no small rocky planets, no moons, and no people, if it were not for stars producing the material from which these objects are made.

3. How a star evolves depends upon its initial mass of the collapsing material from which it formed. Figure II-12 summarizes the process of stellar evolution for all possible masses of stars.