PHYS-2020: General Physics II
Course Lecture Notes
Section IV

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Edition 4.0

Abstract

These class notes are designed for use of the instructor and students of the course PHYS-2020: General Physics II taught by Dr. Donald Luttermoser at East Tennessee State University. These notes make reference to the *College Physics, 11th Edition* (2018) textbook by Serway and Vuille.

IV. Direct Current Circuits

A. Sources of *emf.*

1. Direct current means that the current travels only in one direction in a circuit $\Longrightarrow$ DC for short.

2. The source that maintains the constant current in a closed circuit is called a source of "**emf**."

a) One can think of such a source as a "charge pump" $\Longrightarrow$ forces electrons to move in a direction <u>opposite</u> the E-field, hence current, inside a source.

b) The emf, $\mathcal{E}$, of a source is the work done per unit charge $\Longrightarrow$ measured in volts.

c) A battery is a good example of a source of emf.

3. Since batteries have internal resistance, r, the potential difference across the terminals of a battery are

$$\Delta V = \mathcal{E} - Ir \ . \tag{IV-1}$$

a) $\mathcal{E}$ is equal to the terminal voltage when the current is zero.

b) From Ohm's law, the potential difference across the external resistance R, called the **load resistance**, is $\Delta V = IR$, so

$$IR = \mathcal{E} - Ir$$

or

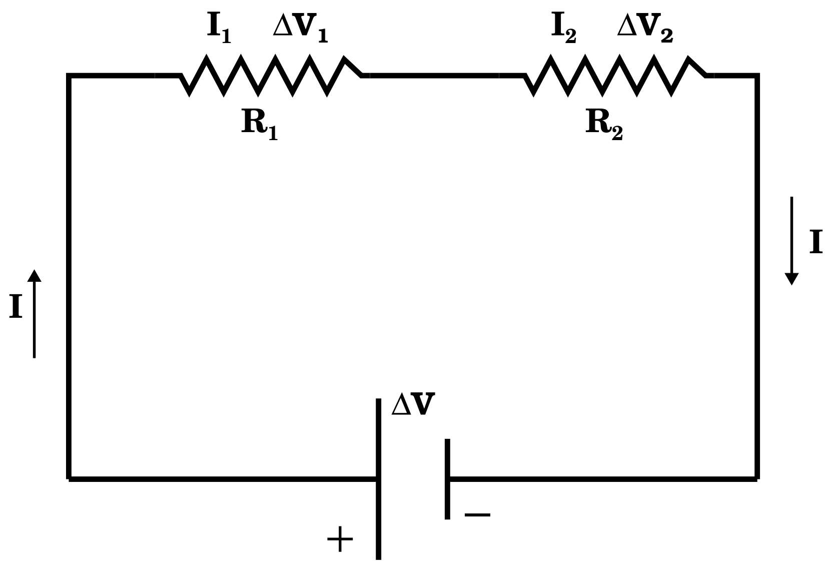
$$\mathcal{E} = IR + Ir = I(R + r) \ . \tag{IV-2}$$

c) If $r \ll R$, then $\mathcal{E} \approx IR$ (as we have been assuming to date).

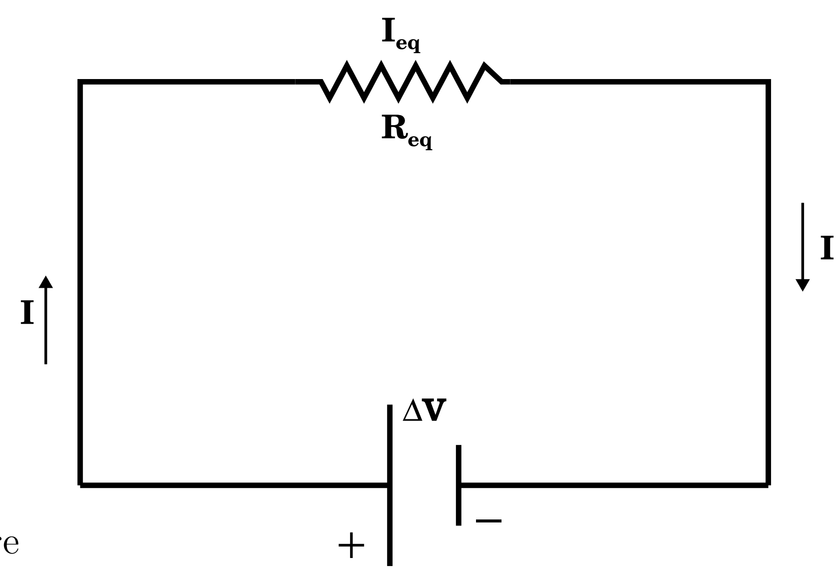
B. Resistors in Series.

1. The current through all resistors in a series circuit is the same.

$$I = I_1 = I_2 = \cdots \qquad \text{(IV-3)}$$



2. The reduced circuit above is



where

$$\begin{aligned}
\Delta V &= I_{\text{eq}} R_{\text{eq}} = I R_{\text{eq}} \\
\Delta V &= \Delta V_1 + \Delta V_2 = I_1 R_1 + I_2 R_2 = I R_1 + I R_2 \\
I R_{\text{eq}} &= I R_1 + I R_2 \; .
\end{aligned}$$

3. As such, we can express the equivalent resistance as

$$R_{\rm eq} = R_1 + R_2 \tag{IV-4}$$

for two series resistors or more generally (*e.g.*, for more than 2 resistors in series) as

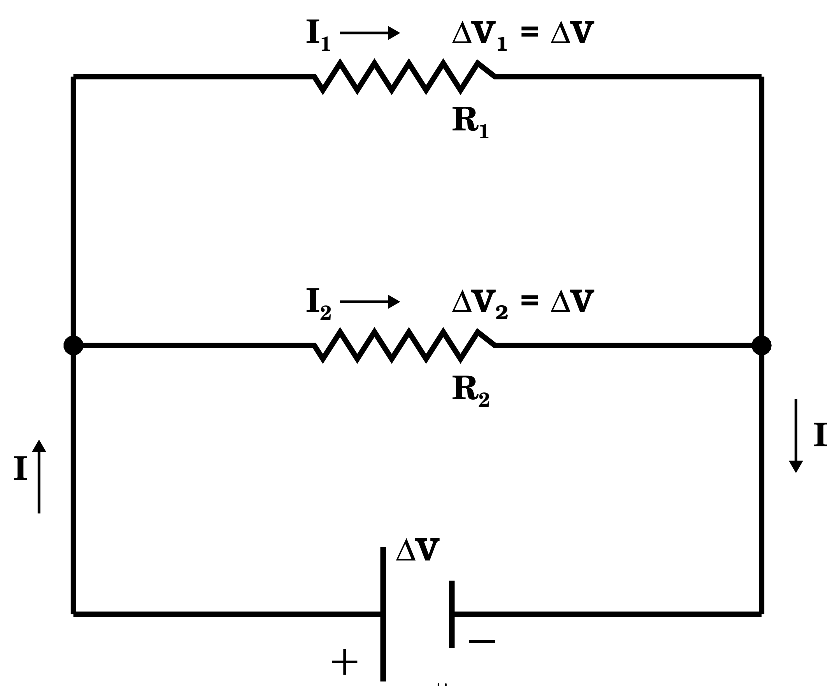
$$\boxed{R_{\rm eq} = \sum_{i=1}^{N} R_i \ ,} \quad \text{series circuits} \tag{IV-5}$$

where N is the total number of resistors.

4. Once the equivalent resistance is found, one can then calculate I from ΔV using Ohm's law.

C. Resistors in Parallel.

1. When resistors are connected in parallel, the potential difference across them are the same:



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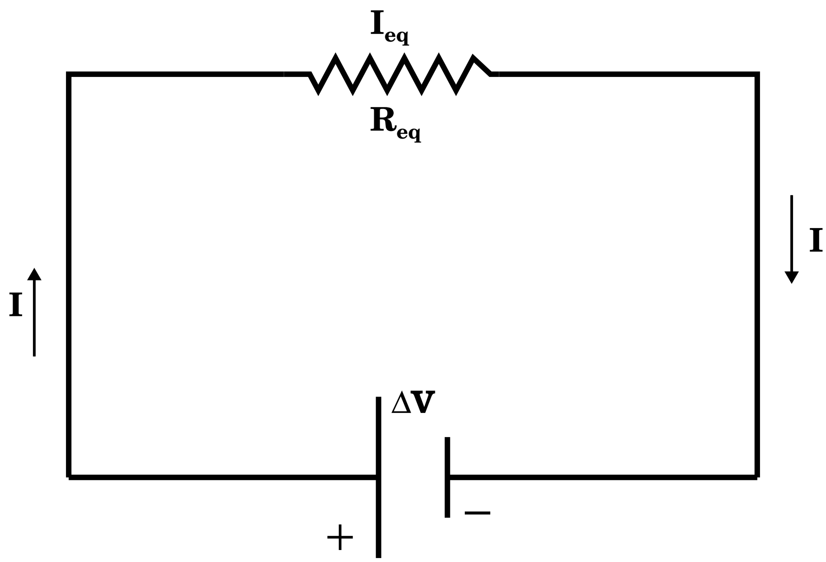


$$\Delta V = \Delta V_1 = \Delta V_2 \ . \qquad \text{(IV-6)}$$

2. The current entering a parallel circuit is equal to the sum of all currents traveling through each resistor:

$$I = I_1 + I_2 \qquad \text{(IV-7)}$$

in our 2 resistor diagram above.

3. For the reduced circuit above:

$$I = \frac{\Delta V}{R_{\rm eq}} \ , \quad I_1 = \frac{\Delta V_1}{R_1} \ , \quad I_2 = \frac{\Delta V_2}{R_2} \ ,$$

so

$$\frac{\Delta V}{R_{\rm eq}} = \frac{\Delta V_1}{R_1} + \frac{\Delta V_2}{R_2} = \frac{\Delta V}{R_1} + \frac{\Delta V}{R_2}$$

or

$$\frac{1}{R_{\rm eq}} = \frac{1}{R_1} + \frac{1}{R_2} \qquad \text{(IV-8)}$$

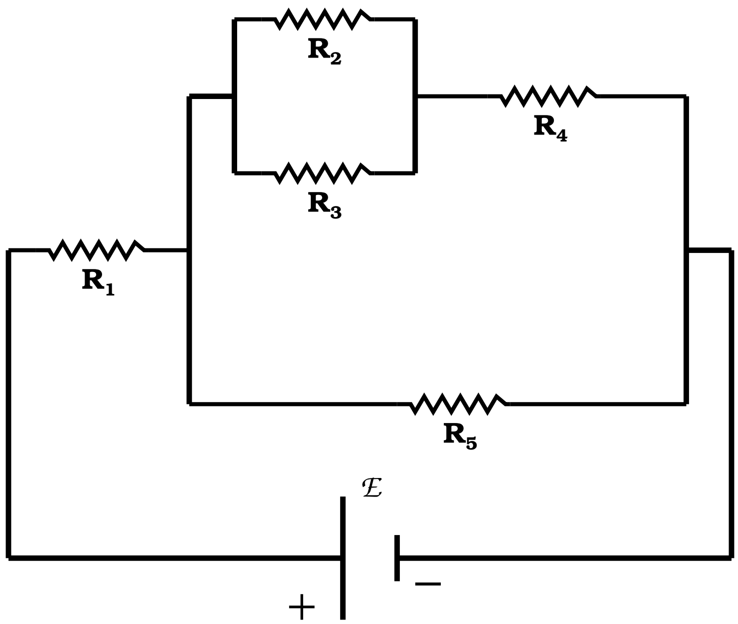
for two parallel resistors. If more that 2 resistors are in a parallel configuration, we use

$$\boxed{\frac{1}{R_{\rm eq}} = \sum_{i=1}^{N} \frac{1}{R_i} \ .} \qquad \text{(IV-9)}$$

parallel circuits

4. Once the equivalent resistance is found, this info can be used to deduce either the voltages or the individual currents across the resistors.

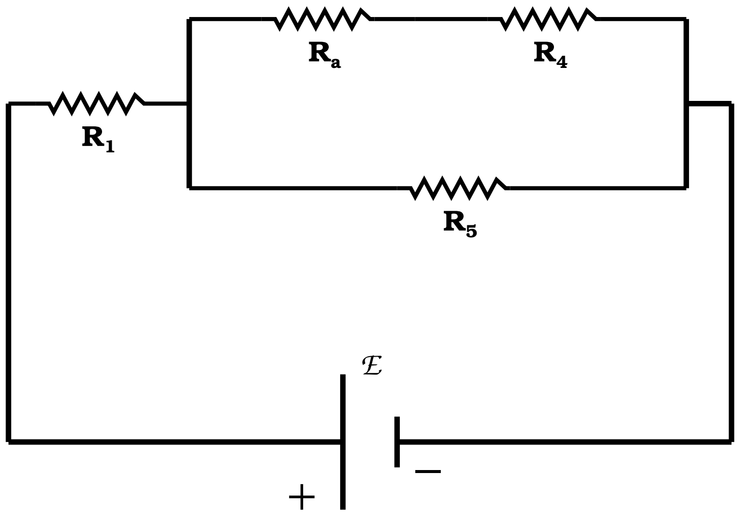
Example IV–1. Consider the circuit shown below, where $R_1 = 3.00\ \Omega$, $R_2 = 10.0\ \Omega$, $R_3 = 5.00\ \Omega$, $R_4 = 4.00\ \Omega$, and $R_5 = 3.00\ \Omega$. (a) Find the equivalent resistance of this circuit. (b) If the total power supplied to the circuit is 4.00 W, find the emf of the battery.



Solution (a):

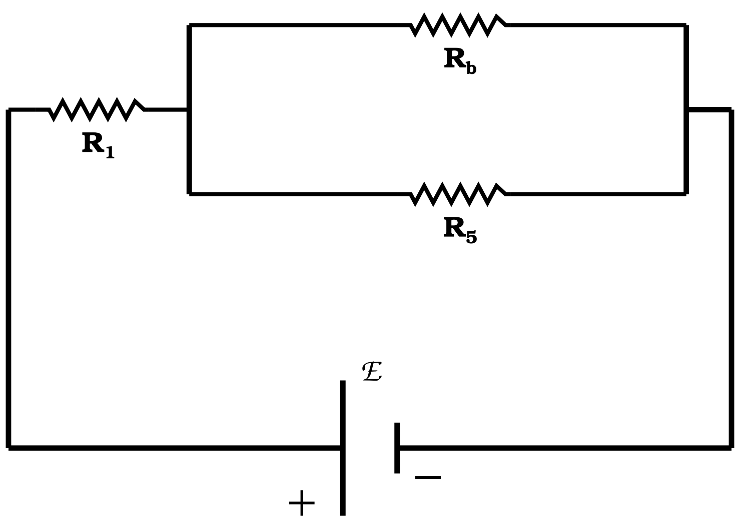
We have to reduce this circuit in steps. *But which do we reduce first?* Typically, one should look for those resistors that are in the most compact configuration and start there. For our circuit above, resistors R_2 and R_3 have the most compact configuration. Since these are in parallel, we use Eq. (IV-9) and call the equivalent resistance R_a as shown in the next figure.

$$\frac{1}{R_a} = \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{10.0\,\Omega} + \frac{1}{5.00\,\Omega}$$
$$\frac{1}{R_a} = \frac{1}{10.0\,\Omega} + \frac{2}{10.0\,\Omega} = \frac{3}{10.0\,\Omega}$$
$$R_a = 3.33\,\Omega\ .$$

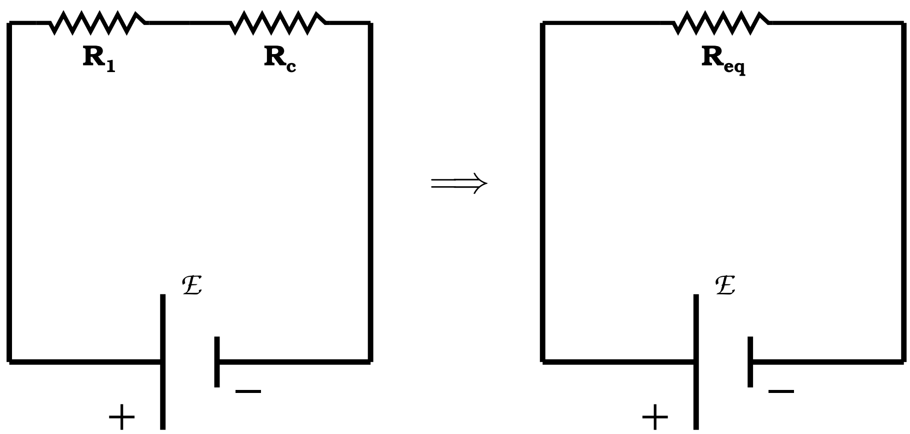


We see from the diagram above that we now need to reduce the 2 series resistors R_a and R_4 with Eq. (IV-5) giving

$$R_b = R_a + R_4 = 3.33\,\Omega + 4.00\,\Omega = 7.33\,\Omega \ .$$



The diagram below show the next stages on route to the final equivalent resistor: Reduce the parallel R_b and R_5 resistors followed the series R_c and R_1 resistors as shown in the next diagram.



Using the previous three circuits as a guide, we have

$$\frac{1}{R_c} = \frac{1}{R_b} + \frac{1}{R_5} = \frac{1}{7.33\,\Omega} + \frac{1}{3.00\,\Omega} = 0.4697\,\Omega^{-1}$$
$$R_c = 2.13\,\Omega$$

$$R_{\rm eq} = R_1 + R_c = 3.00\,\Omega + 2.13\,\Omega = \boxed{5.13\,\Omega\ .}$$

Solution (b):

Since the power is given to us, we can use Eq. (III-17) to determine the emf of the battery. If we assume that the internal resistance of the battery is negligible to the load resistance, we have $\Delta V = \mathcal{E}$, so

$$\mathcal{P} = \frac{(\Delta V)^2}{R_{\rm eq}}$$
$$\Delta V = \sqrt{\mathcal{P} \cdot R_{\rm eq}} = \sqrt{(4.00\ \text{W})(5.13\,\Omega)}$$
$$= \boxed{4.53\ \text{V}\ .}$$

D. Kirchhoff's Rules and Complex DC Circuits.

1. For complex DC circuits, it is not always possible to reduce all of the individual elements of the circuit into a single reduced circuit.

2. In such cases, one should use **Kirchhoff's rules** to figure out the currents and potential differences across each element in the circuit.

a) The sum of the current entering any junction must equal the sum of the currents leaving the junction $\Longrightarrow$ the **junction rule** (see Figure 18.12 in your textbook).

b) The sum of the potential differences across all of the elements around any closed circuit loop must be zero $\Longrightarrow$ the **loop rule** $\Longrightarrow$ this is nothing more than the conservation of energy (see Figure 18.14 in your textbook).

i) Going across a battery from + to − is equivalent to saying you are going in the $-\Delta V$ or $-\mathcal{E}$ direction.

ii) Going across a battery from − to + is equivalent to going in the $+\Delta V$ or $+\mathcal{E}$ direction.

iii) Going in the opposite direction to current is equivalent to saying you are going in the $+IR$ direction.

iv) Going in the same direction as current is equivalent to saying you are going in the $-IR$ direction.

v) Current flows from $+V$ to $-V$ in a circuit.

3. Problem-Solving Strategies for Complex DC Circuits.

a) To solve a particular circuit problem, you need as many independent equations as unknowns.

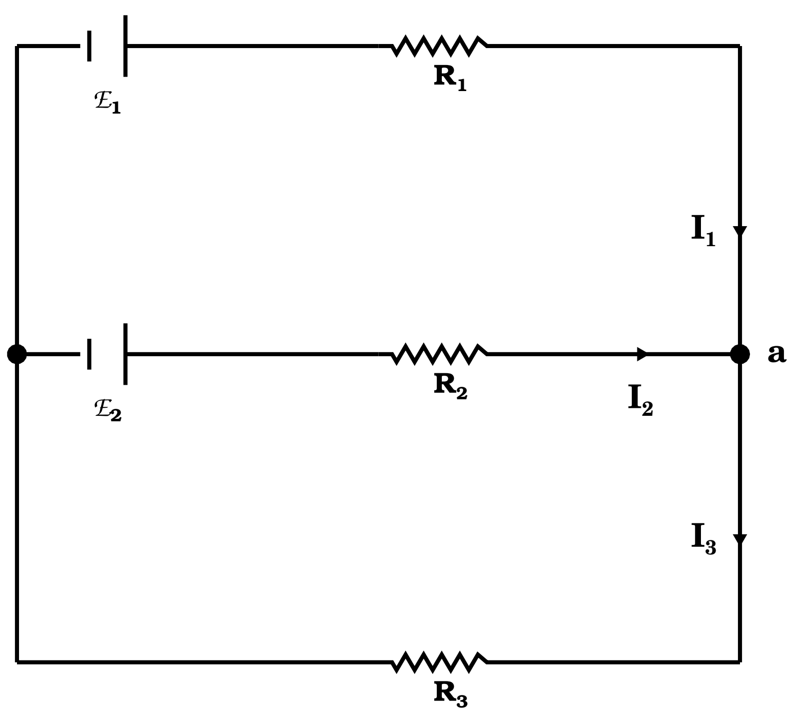
b) First, draw the circuit diagram and label all elements. Assign directions to the currents in each part of the circuit (remember, current flows from $+V$ to $-V$).

c) Apply the junction rule to any junction in the circuit.

d) Now, apply Kirchhoff's loop rule to all loops in the circuit.

e) Solve the simultaneous set of equations for the unknown quantities [realizing the point made in (a)].

Example IV–2. (a) Can the circuit shown below be reduced to a single resistor connected to the batteries? (b) Find the magnitude of the current and its direction in each resistor.



Solution (a):

No. This multi-loop circuit does not have any resistors in series (*i.e.*, connected so all the current in one must pass through the other) nor in parallel (connected so the voltage drops across one is always the same as that across the other). Thus, this circuit cannot be simplified any further, and Kirchoff's rules must be used to analyze it.

Solution (b):

As shown in the diagram above, the batteries and resistors have the following values: $R_1 = 30.0\ \Omega$, $R_2 = 5.00\ \Omega$, $R_3 = 20.0\ \Omega$, $\mathcal{E}_1 = 20.0$ V, and $\mathcal{E}_2 = 10.0$ V. Assume the currents going through each of the resistors are traveling in the direction shown in the figure above. The directions have been selected based upon the voltages of each battery and where it's located in the circuit. If our guessed directions are wrong, the algebra will tell us by producing a negative current for those resistors where we made the wrong guess. Using Kirchhoff's junction rule at point 'a' gives

$$I_3 = I_1 + I_2 \ . \qquad (1)$$

Applying Kirchhoff's loop rule on the upper loop gives (going in a clockwise loop)

$$\begin{aligned} +\mathcal{E}_1 - R_1 I_1 + R_2 I_2 - \mathcal{E}_2 &= 0 \\ +20.0\text{ V} - (30.0\,\Omega)\,I_1 + (5.00\,\Omega)\,I_2 - 10.0\text{ V} &= 0 \\ -(30.0\,\Omega)\,I_1 + (5.00\,\Omega)\,I_2 &= -10.0\text{ V} \\ -\left(\frac{30.0\,\Omega}{5.00\,\Omega}\right) I_1 + \left(\frac{5.00\,\Omega}{5.00\,\Omega}\right) I_2 &= -\frac{10.0\text{ V}}{5.00\,\Omega} \\ 6.00\,I_1 - I_2 &= 2.00\text{ A} \qquad (2) \end{aligned}$$

and for the lower loop, we get (again going in a clockwise loop)

$$+\mathcal{E}_2 - R_2 I_2 - R_3 I_3 = 0$$

$$
\begin{aligned}
+10.0\ \text{V} - (5.00\,\Omega)\,I_2 - (20.0\,\Omega)\,I_3 &= 0 \\
-(5.00\,\Omega)\,I_2 - (20.0\,\Omega)\,I_3 &= -10.0\ \text{V} \\
\left(\frac{5.00\,\Omega}{5.00\,\Omega}\right) I_2 + \left(\frac{20.0\,\Omega}{5.00\,\Omega}\right) I_3 &= \frac{10.0\ \text{V}}{5.00\,\Omega} \\
I_2 + 4.00\,I_3 &= 2.00\ \text{A}\ . \qquad (3)
\end{aligned}
$$

Solving Eqs. (1), (2), and (3) simultaneously gives

$$
\begin{aligned}
(3)\quad I_3 &= 0.500\ \text{A}\ - 0.250\,I_2 \\
(2)\quad I_1 &= 0.333\ \text{A}\ + 0.167\,I_2 \\
(1)\quad I_2 &= I_3 - I_1 = (0.500\ \text{A}\ - 0.250\,I_2) - (0.333\ \text{A}\ + 0.167\,I_2) \\
I_2 &= 0.167\ \text{A} - 0.417\,I_2 \\
I_2 + 0.417\,I_2 &= 0.167\ \text{A} \\
1.417\,I_2 &= 0.167\ \text{A} \\
I_2 &= 0.118\ \text{A} \\
(2)\quad I_1 &= 0.333\ \text{A}\ + 0.167\,(0.118\ \text{A}) = 0.353\ \text{A} \\
(3)\quad I_3 &= 0.500\ \text{A}\ - 0.250\,(0.118\ \text{A}) = 0.471\ \text{A}
\end{aligned}
$$

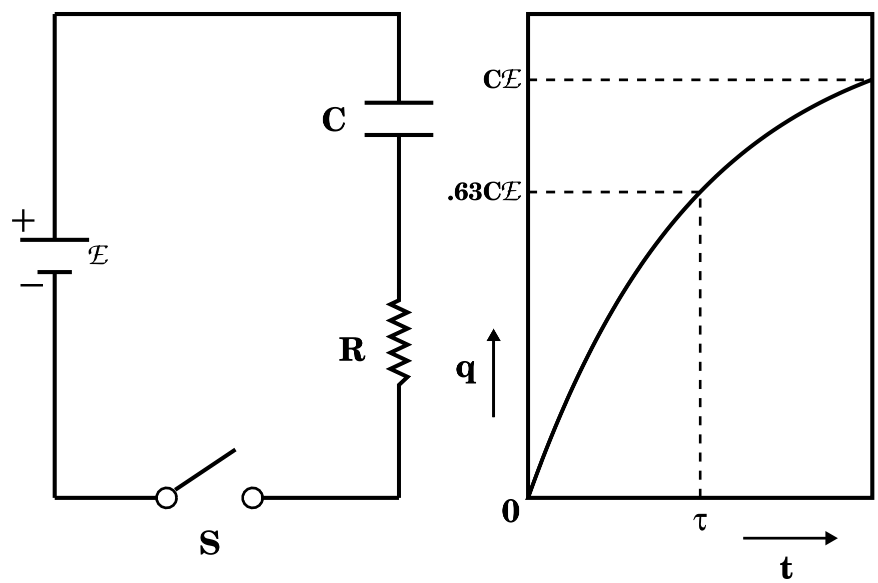
Hence, our answer for the currents through each resistor is

$$
\boxed{I_1 = 0.353\ \text{A}\ , \quad I_2 = 0.118\ \text{A}\ , \quad I_3 = 0.471\ \text{A}\ .}
$$

Since all of our currents are positive, we guessed correctly for direction of the currents going through each resistor (indicated in the figure above).

E. RC Circuits.

1. So far we have been concerned with direct circuits with constant currents. However, if capacitors are included with the circuit, the current can vary with time as the capacitor charges.



a) The charge on the capacitor will increase in time when the switch S is closed.

i) The maximum charge $q = Q$ is reached when $q = C\mathcal{E}$.

ii) The slope of the line on the graph shown above is equal to the current:

$$I = \frac{\Delta q}{\Delta t} \ .$$

b) Once the capacitor is fully charged, the current in the circuit goes to zero.

2. If this was a calculus-based general physics course, we could set up the differential equation showing how charge on the capacitor changes with time. The solution of such a differential equation takes the form:

$$q = Q\left(1 - e^{-t/RC}\right) \qquad \text{(IV-10)}$$

for an RC (resistor-capacitor) circuit.

a) $e = 2.718\ldots$ is the base of natural logarithms (note that 10 is the base of common logarithms).

b) When $t = RC$, $e^{-t/RC} = e^{-1} = 0.3679\ldots \Longrightarrow$ this time is called the e-**folding time** and is set equal to τ ('tau'), the **capacitance time constant**:

$$\boxed{\tau = RC\ .} \qquad \text{(IV-11)}$$

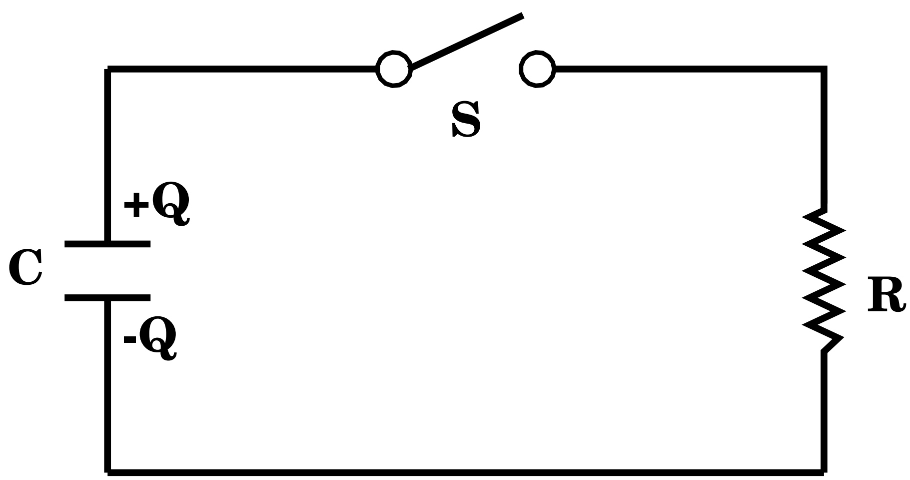
c) When $t = \tau$, $q = (1 - 0.37)\, Q = 0.63\, Q = 0.63\, C\, \mathcal{E}$.

d) Note that $[R] = \Omega = \text{V/A}$, and $[C] = \text{F} = \text{C/V}$. As such,

$$[\tau] = [R]\,[C] = \frac{\text{V}}{\text{A}} \cdot \frac{\text{C}}{\text{V}} = \frac{\text{C}}{\text{A}} = \frac{\text{C}}{\text{C/s}} = \text{s}\ .$$

Hence the product $R\,C$ has units of time (*i.e.*, seconds).

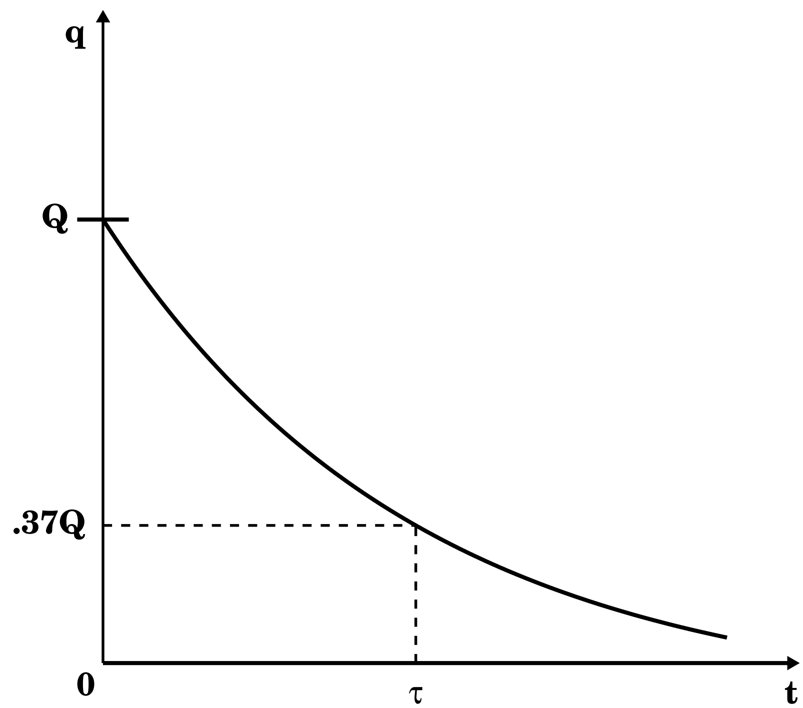
3. If we take the battery out of the above circuit after the capacitor has been charged:



a) When we close the switch, the capacitor begins to drain following:

$$q = Qe^{-t/RC} \ . \qquad \text{(IV-12)}$$

b) Graphically:



c) Note that the capacitor is now acting like a battery, however, a battery with a continually decreasing voltage:

$$\Delta V = \frac{q}{C} = \frac{Q}{C}\, e^{-t/RC} \ . \qquad \text{(IV-13)}$$

Example IV–3. Consider a series RC circuit for which $C =$ 6.0μF, $R = 2.0 \times 10^6\,\Omega$, and $\mathcal{E} =$ 20 V. Find (a) the time constant of the circuit and (b) the maximum charge on the capacitor after a switch in the circuit closes. (c) If the battery is immediately taken out of the circuit once the capacitor reaches its maximum charge, how long would it take for the capacitor to drain to one-tenth its maximum charge?

Solution (a):

The time constant for an RC circuit is given by Eq. (IV-11):

$$\tau = RC = (2.0 \times 10^6\,\Omega)(6.0 \times 10^{-6}\text{ F}) = \boxed{12\text{ s}\,,}$$

since $\Omega{\cdot}\text{F} = \text{s}$.

Solution (b):

The maximum charge Q is simply found from

$$Q = \mathcal{E}C = (20\text{ V})(6.0 \times 10^{-6}\text{ F}) = \boxed{1.2 \times 10^{-4}\text{ C}\,.}$$

Solution (c):

Here we use Eq. (IV-12) with $q = 0.10Q$. Solving for t gives

$$\begin{aligned}
q &= Q\,e^{-t/RC} = Q\,e^{-t/\tau} \\
e^{-t/\tau} &= \frac{q}{Q} \\
e^{t/\tau} &= \frac{Q}{q} \\
\frac{t}{\tau} &= \ln\left(\frac{Q}{q}\right) \\
t &= \tau\,\ln\left(\frac{Q}{q}\right) = (12\text{ s})\ln(10) = (12\text{ s})(2.30) \\
&= \boxed{28\text{ s}\,.}
\end{aligned}$$
