PHYS-2020: General Physics II
Course Lecture Notes
Section VI

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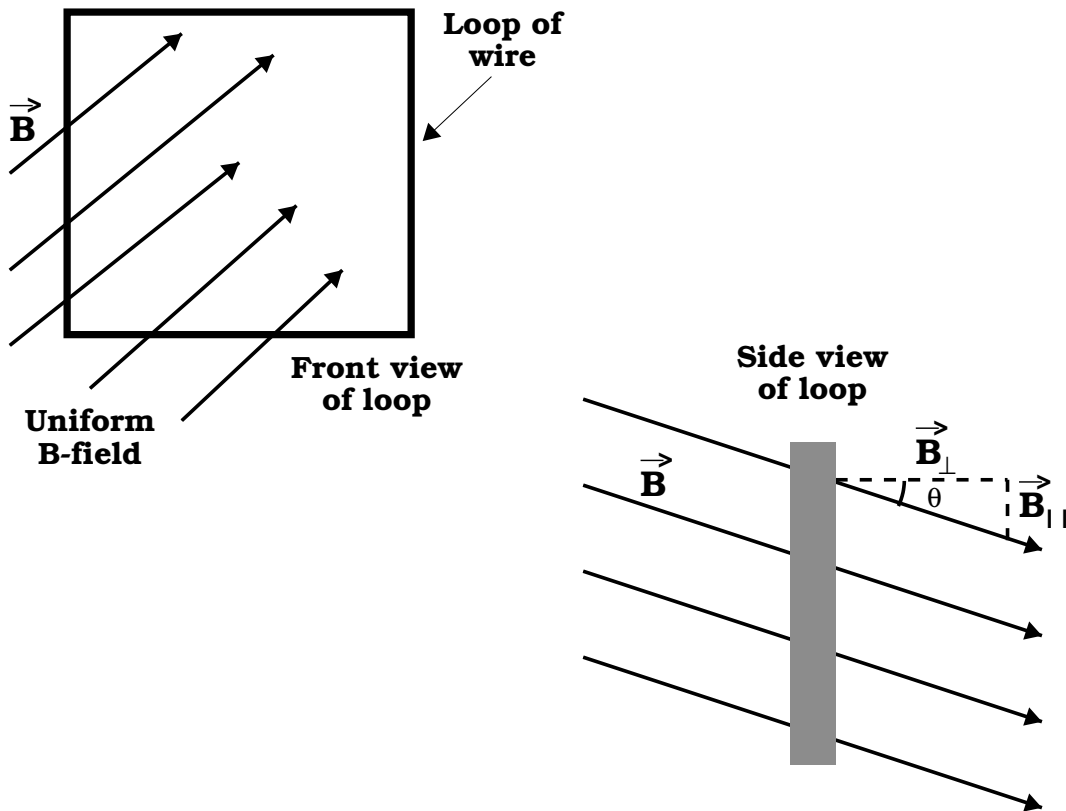East Tennessee State University

Edition 4.0

Abstract

These class notes are designed for use of the instructor and students of the course PHYS-2020: General Physics II taught by Dr. Donald Luttermoser at East Tennessee State University. These notes make reference to the *College Physics, 10th Hybrid Edition* (2015) textbook by Serway and Vuille.

VI. Induced Voltages and Inductance

A. Magnetic Flux.

1. Michael Faraday demonstrated that electric currents can be produced by a changing magnetic field $\implies$ a changing B-field can produce an **induced emf**.

2. Consider a loop of wire in a uniform B-field. The **magnetic flux** in the loop is the sum of the B-field strength across the entire (cross-sectional) surface area of the loop:

$$\Phi_B = \vec{B} \cdot \vec{A} = B_\perp A = BA\cos\theta \ . \qquad \text{(VI-1)}$$

Loop of wire

$\vec{B}$

Uniform B-field

Front view of loop

Side view of loop

$\vec{B}$

$\vec{B}_\perp$

θ

$\vec{B}_{||}$

a) Φ_B is the magnetic flux measured in T·m^2 = Wb (weber).

b) The value of the magnetic flux is proportional to the total number of lines passing through the loop.

B. Faraday's Law of Induction.

1. The emf *induced* in a circuit equals the rate of change of the magnetic flux through the circuit:

$$\boxed{\mathcal{E} = -N\frac{\Delta\Phi_B}{\Delta t}} \text{ .} \tag{VI-2}$$

a) $\mathcal{E} \equiv$ emf, $N \equiv$ number of loops in the wire circuit, $\Delta\Phi_B/\Delta t \equiv$ change in magnetic flux over time.

b) Eq. (VI-2) is known as **Faraday's law of magnetic induction**.

c) The negative sign is included to indicate polarity of the induced emf.

d) Faraday's law, as it is written in Eq. (VI-2), represents the *average* induced emf. The instantaneous induced emf (using calculus), which describes the more precise version of Faraday's law, would be given by

$$\mathcal{E} = -N\lim_{\Delta t \to 0}\frac{\Delta\Phi_B}{\Delta t} = -N\frac{d\Phi_B}{dt} \text{ .}$$

2. The polarity mentioned in Faraday's law can be found from **Lenz's law**:

a) Any induced electromotive force will be in the direction such that the magnetic flux it creates will oppose the change in the flux that produced it.

b) More simply, the induced current tends to maintain the original flux through the circuit.

c) Lenz's Law is one consequence of the principle of conservation of energy. We can see why through the following example:

i) Move a permanent magnet towards the face of a closed loop of wire (*e.g.*, a coil or solenoid).

ii) An electric current is induced in the wire, because the electrons within it are subjected to an increasing magnetic field as the magnet approaches.

iii) This produces an emf that acts upon them. The direction of the induced current depends on whether the north or south pole of the magnet is approaching:

● An approaching north pole will produce an anti-clockwise current (from the perspective of the magnet).

● A south pole approaching the coil will produce a clockwise current.

Example VI–1. A long, straight wire lies in the plane of a circular coil with a radius of 0.010 m. The wire carries a current of 2.0 A and is placed along the diameter of the coil. (a) What is the net flux through the coil? (b) If the wire passes through the center of the coil and is perpendicular to the plane of the center of the coil, find the net flux through the coil.

(a) Face-on View   (b) Side View
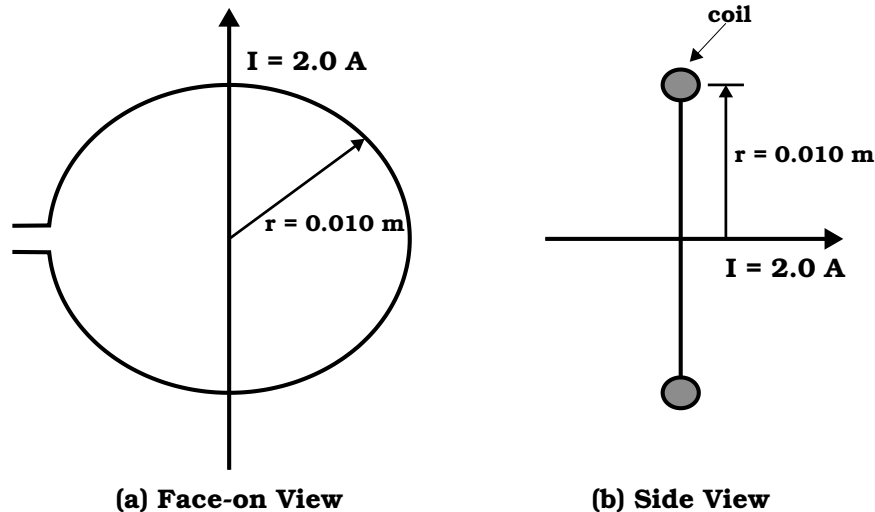
Solution (a):

As shown in the diagram above, every field line that comes up through the area A on one side of the wire, goes back down through area A on the other side of the wire. Thus, the net flux through the coil is ⎡ zero. ⎤

Solution (b):

The magnetic field is parallel to the plane of the coil as shown in the diagram above, so $\theta = 90.0°$. Therefore, $\Phi_B = BA\cos\theta = BA\cos 90.0° = $ ⎡ 0. ⎤

Example VI–2.   A 25-turn circular coil of wire has a diameter of 1.00 m. It is placed with its axis along the direction of Earth's magnetic field of 50.0 μT, and then in 0.200 s it is flipped $180°$. An average emf of what magnitude is generated in the coil?

Solution:

First, the change in flux results from the change of orientation of the loop with the magnetic field. Using Eq. (VI-1) we get:

$$\Delta\Phi_B = BA\cos\theta_2 - BA\cos\theta_1 = BA[\Delta(\cos\theta)] \ ,$$

where $B = 50.0 \times 10^{-6}$ T, $A = \pi D^2/4 = \pi(1.00 \text{ m})^2/4 = 0.785 \text{ m}^2$, $\Delta(\cos\theta) = \cos 0° - \cos 180° = 1 - (-1) = 2$. As such, the average

emf generated for $N = 25$ loops from this changing magnetic flux over a $\Delta t = 0.200$ s time interval is given by Eq. (VI-2):

$$
\begin{aligned}
|\mathcal{E}| &= N\frac{\Delta \Phi_B}{\Delta t} = \frac{NBA[\Delta(\cos\theta)]}{\Delta t} \\
&= \frac{25(50.0 \times 10^{-6} \text{ T})(0.785 \text{ m}^2)(2)}{0.200 \text{ s}} \\
&= 9.82 \times 10^{-3} \text{ V} = \boxed{9.82 \text{ mV} .}
\end{aligned}
$$

C. Motional emf.

1. The potential difference is maintained across a conductor moving through a B-field. If the motion is reversed, the polarity of the potential difference is also reversed.
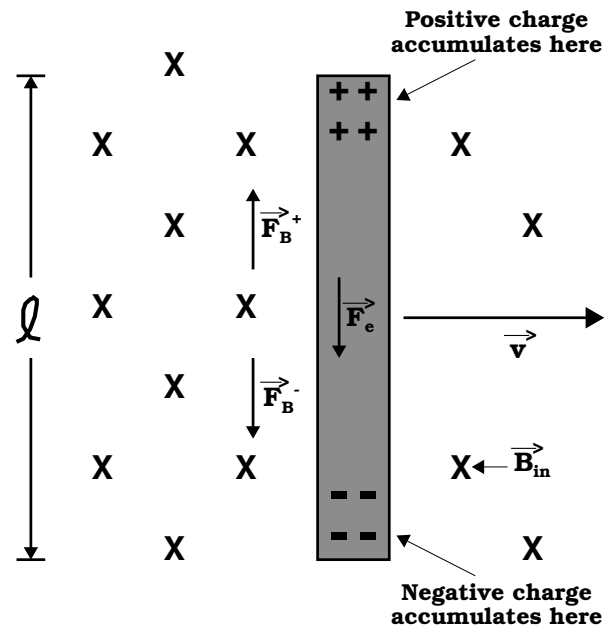
a) Note that the magnetic force, $F_B = qvB$, points downward for electrons and upward for positive ions in the diagram above.

b) The charge separates which sets up an electric field with force, $F_e = qE$.

c) The charges at either end build up until downward F_B balances upward F_e for the electrons (and opposite this for the positive ions), then

$$F_B = F_e \implies qvB = qE \quad \text{or} \quad E = vB .$$

d) Since $\vec{E}$ is constant, we can write $\Delta V = \mathcal{E} = E\ell$ (where $\Delta V \equiv$ potential difference $=$ emf), or

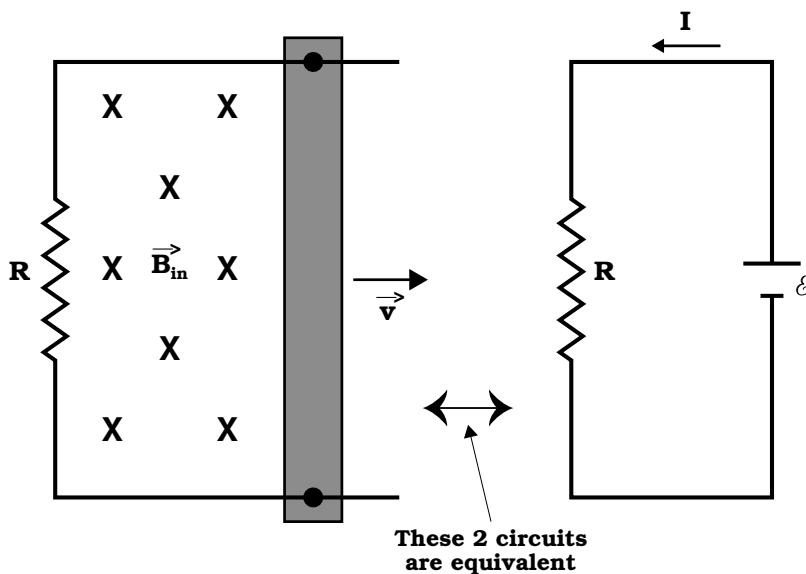$$\boxed{\Delta V = \mathcal{E} = E\ell = B\ell v .} \qquad \text{(VI-3)}$$

2. Let's assume this bar moves a distance Δx in time Δt, then a cross-sectional area $A = \Delta x \cdot \ell$ is created as the bar moves in the B-field (see diagram below):
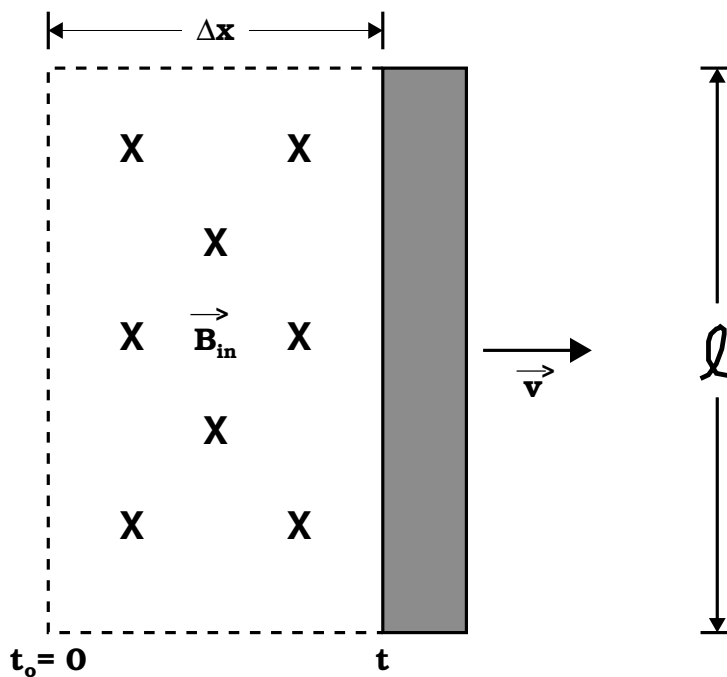
a) As the bar moves, the magnetic flux increases:

$$\Delta \Phi_B = BA = B\ell \, \Delta x .$$

b) If we were to hook this bar into a circuit, we can derive an induced emf due to the moving bar.

These 2 circuits are equivalent

c) The magnitude of the induced emf is then

$$|\mathcal{E}| = \left| -N \frac{\Delta \Phi_B}{\Delta t} \right| = \frac{\Delta \Phi_B}{\Delta t} \quad (N = 1, \text{ one loop})$$
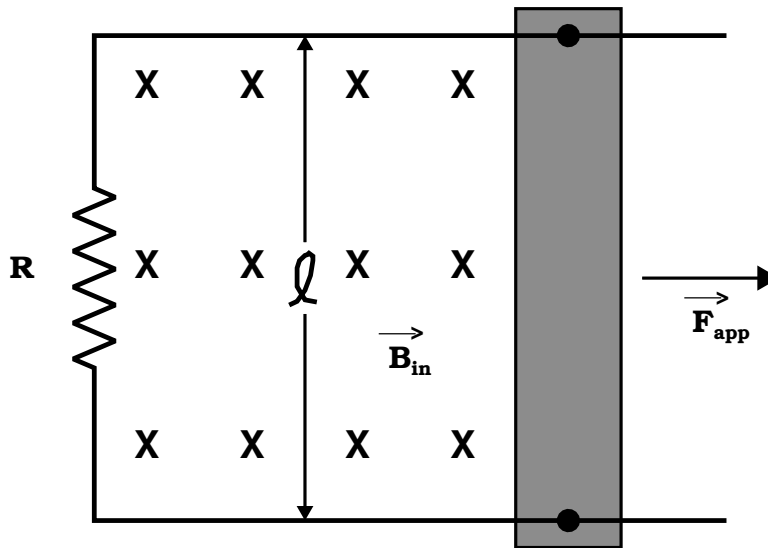
or

$$|\mathcal{E}| = B\ell \frac{\Delta x}{\Delta t} = B\ell v \ . \qquad \text{(VI-4)}$$

d) Induced emf is sometimes called **motional emf**.

e) If the resistance of the circuit is R, then

$$\boxed{I = \frac{|\mathcal{E}|}{R} = \frac{B\ell v}{R} \ .} \qquad \text{(VI-5)}$$

Example VI–3. Consider the arrangement shown the diagram on the next page. Assume that $R = 6.00\,\Omega$ and $\ell = 1.20$ m, and that a uniform 2.50 T magnetic field is directed *into* the page. At what speed should the bar be moved to produce a current of 0.500 A in the resistor?

Solution:

Using Eq. (VI-4), $\mathcal{E} = B\ell v$ in conjunction with Ohm's law, $\mathcal{E} = IR$, we can write

$$v = \frac{\mathcal{E}}{B\ell} = \frac{IR}{B\ell} = \frac{(0.500 \text{ A})(6.00 \, \Omega)}{(2.50 \text{ T})(1.20 \text{ m})} = \boxed{1.00 \text{ m/s} \, .}$$

D. Generators and Motors.

1. Whereas electric motors convert electrical energy to mechanical energy, **generators** perform the reverse effect $\Longrightarrow$ mechanical energy into electrical energy.

    a) This is how most of the power plants on Earth produce electricity: A turbine is turned via some mechanical process. The turbine is connected to loops of wire embedded in an external B-field. As the loops rotate, the motional emf drives a current.

        i) Hydroelectric power plants produce electricity from water flowing down a potential energy gradient set up by the Earth's gravitational field: Gravitational

energy $(\Delta(\text{PE})_g = mg\Delta y) \rightarrow$ mechanical energy $(\text{KE} = \frac{1}{2}mv^2) \rightarrow$ electrical energy $(E \propto I^2)$.

ii) Steam turbines use the flow of steam, generated either in a coal burning furnace or nuclear reactor, through a turbine instead of liquid water.

b) Generators come in 2 types, where the mechanics of both are similar to each other — loops of wire rotating in a B-field:

i) Alternating current (AC) generators produce an AC current.

ii) Direct current (DC) generators produce an DC current.

2. The induced emf created by a generator is given by

$$\boxed{\mathcal{E} = NBA\omega \sin \omega t \ .} \qquad \text{(VI-6)}$$

a) All of the terms in this equation have already been defined except ω, which is the angular speed of the rotating loops in the generator ($\omega = 2\pi f$, where f is the frequency $[\text{s}^{-1}]$).

b) Note that when $\omega t = 90°$ or $270°$, the maximum emf is produced, hence

$$\mathcal{E}_{\text{max}} = NBA\omega \ . \qquad \text{(VI-7)}$$

3. The average induced emf produced by a generator, the so-called **root-mean-square** emf, is given by

$$\mathcal{E}_{\text{rms}} = \frac{\mathcal{E}_{\text{max}}}{\sqrt{2}} = \frac{1}{\sqrt{2}}NBA\omega \ . \qquad \text{(VI-8)}$$

4. As mentioned above, an electric motor is a generator ran in reverse. As the coil in a motor rotates, the changing magnetic flux through it induces an emf, which acts to reduce the current in the coil.

a) The phrase **back emf** is used for an emf that tends to reduce the applied current and is given by

$$\mathcal{E}_{\text{back}} = \mathcal{E} - IR , \qquad\qquad (\text{VI-9})$$

where I is the actual current that arises due to the reduced emf.

b) The back emf increases in magnitude as the rotational speed of the coil increases.

c) When a motor is first turned on, there is no back emf initially and the current is at its maximum value, being limited only by the resistance of the coil.

d) As the coil begins to rotate, the induced back emf opposes the applied voltage $\mathcal{E}$ and the current in the coil is reduced.

Example VI–4.   A motor has coils with a resistance of 30 Ω and operates from a voltage of 240 V. When the motor is operating at its maximum speed, the back emf is 145 V. Find the current in the coils (a) when the motor is first turned on and (b) when the motor has reached maximum speed. (c) If the current in the motor is 6.0 A at some instant, what is the back emf at that time?

Solution (a):

Immediately after the switch is closed, the motor coils are still stationary and the back emf is zero. Thus,

$$I_{\text{max}} = \frac{\mathcal{E}}{R} = \frac{240 \text{ V}}{30\,\Omega} = \boxed{8.0 \text{ A} .}$$

Solution (b):

At maximum speed, $\mathcal{E}_{\text{back}} = 145$ V and

$$I = \frac{\mathcal{E} - \mathcal{E}_{\text{back}}}{R} = \frac{240 \text{ V} - 145 \text{ V}}{30\,\Omega} = \boxed{3.2 \text{ A}} .$$

Solution (c):

Finally we have a time when $I = 6.0$ A, so

$$\mathcal{E}_{\text{back}} = \mathcal{E} - IR = 240 \text{ V} - (6.0 \text{ A})(30\,\Omega) = \boxed{60 \text{ V}} .$$

E. Self-Inductance.

1. After a switch is closed in a circuit, the current produces its own magnetic flux through the loop of the circuit.

    a) As the current increases towards its equilibrium value, the flux changes in time and induces an emf in the loop.

    b) This effect is called **self-inductance**:

    $$\boxed{\mathcal{E} = -L\frac{\Delta I}{\Delta t}} . \qquad\qquad \text{(VI-10)}$$

    c) Once again, as was the case of Eq. (VI-2), Eq. (VI-10) represents the *average* induced emf, the instantaneous induced emf is given by

    $$\mathcal{E} = -L \lim_{\Delta t \to 0} \frac{\Delta I}{\Delta t} = -L\frac{dI}{dt} ,$$

    where dI/dt is the time derivative of the current.

2. The proportionality constant, L, in Eq. (VI-10) is called the **inductance**.

a)   Its SI unit is the **henry** (H):

$$1 \text{ H } = 1 \text{ V} \cdot \text{s/A} .$$

b)   If we equate Eq. (VI-10) with Eq. (VI-2), we get

$$N\frac{\Delta\Phi_B}{\Delta t} = L\frac{\Delta I}{\Delta t} \qquad \text{(VI-11)}$$

or

$$\boxed{L = N\frac{\Delta\Phi_B}{\Delta I} = \frac{N\Phi_B}{I}} \;, \qquad \text{(VI-12)}$$

where we have set $\Phi_B = 0$ and $I = 0$ at $t = 0$.

3.   In the case of a solenoid, we can write the inductance in a few different ways. Using Eq. (V-18) in conjunction with Eqs. (VI-1) and (VI-12), we can write the inductance in any of the following ways:

$$L = \frac{\mu_\circ N^2 A}{\ell} = \mu_\circ n^2 A\ell = \mu_\circ n^2 V \;, \qquad \text{(VI-13)}$$

where $N =$ number of coil 'turns' across length ℓ in the solenoid, $n = N/\ell$ is the number of turns per unit length, A is the cross-sectional area of the solenoid, and V is the total volume inside the solenoid.

4.   From Eq. (VI-11) we see that changing the magnetic flux causes the current to change, which in turn from Eq. (VI-10), produces an induced emf. As a result of this, any charged particle brought close to such a electric device will experience a magnetic force imparted on it from said device of the form

$$F = \frac{q}{2\pi r}\frac{\Delta\Phi_B}{\Delta t} \;, \qquad \text{(VI-14)}$$

where q is the charge of the nearby particle which is at a distance r from the device (*i.e.*, in the case of a solenoid, the distance that the charge is from the central axis of the solenoid).

Example VI–5. An emf of 24.0 mV is induced in a 500-turn coil when the current is changing at a rate of 10.0 A/s. What is the magnetic flux through each turn of the coil at an instant when the current is 4.00 A?

Solution:

From Eq. (VI-10), the magnitude of the induced emf is $|\mathcal{E}| = L(\Delta I/\Delta t)$, as such, the self-inductance is

$$L = \frac{|\mathcal{E}|}{\Delta I/\Delta t} = \frac{24.0 \times 10^{-3} \text{ V}}{10.0 \text{ A/s}} = 2.40 \times 10^{-3} \text{ H} .$$

Then, from Eq. (VI-12), the magnetic flux through each turn is

$$\Phi_B = \frac{L \cdot I}{N} = \frac{(2.40 \times 10^{-3} \text{ H})(4.0 \text{ A})}{500} = \boxed{1.92 \times 10^{-5} \text{ T·m}^2 .}$$

F. RL Circuits.

1. A circuit element that has a large inductance, such as a closely wrapped coil of many turns, is called an **inductor**. The circuit symbol for an inductor is ⎯⏛⏛⏛⏛⏛⎯.

   a) Whereas the resistance is a measure of the opposition to the current, we can interpret L **as a measure of the opposition to the rate of change in current**.

   b) We shall always assume that the self-inductance of a circuit is negligible with respect to the inductance of the inductor in the circuit.

2. Just as we had in an RC circuit, current changes exponentially in an RL circuit following

$$I = \frac{\mathcal{E}}{R}\left(1 - e^{-Rt/L}\right) . \qquad \text{(VI-15)}$$

3. Once again, as we did for the RC circuit, we can introduce a **time constant** for an RL circuit defined by

$$\tau = \frac{L}{R} , \qquad\qquad\text{(VI-16)}$$

where τ is measured in seconds if L is measured in henries and R is measured in ohms.

G. Energy Stored in a Magnetic Field.

1. The emf induced by an inductor prevents a battery from establishing an instantaneous current in a circuit. The battery has to do work to produce a current.

2. The energy stored in an inductor due to its magnetic field is

$$\boxed{\text{PE}_L = U_L = \frac{1}{2}LI^2 ,} \qquad\qquad\text{(VI-17)}$$

which is analogous to the energy of a capacitor (see Eq. II-28):

$$\text{PE}_C = U_C = \frac{1}{2}C(\Delta V)^2 .$$

Example VI–6. How much energy is stored in a 70.0-mH inductor at an instant when the current is 2.00 A?

Solution:

We just need to use Eq. (VI-17) here

$$\text{PE}_L = U_L = \frac{1}{2}LI^2 = \frac{1}{2}(70.0 \times 10^{-3} \text{ H})(2.00 \text{ A})^2 = \boxed{0.140 \text{ J} .}$$