PHYS-2020: General Physics II
Course Lecture Notes
Section VII

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Edition 4.0

Abstract

These class notes are designed for use of the instructor and students of the course PHYS-2020: General Physics II taught by Dr. Donald Luttermoser at East Tennessee State University. These notes make reference to the *College Physics, 10th Hybrid Edition* (2015) textbook by Serway and Vuille.

VII. Vibrations and Waves

A. Hooke's Law.

1. A mass connected to a spring will experience a force described by **Hooke's Law**:

$$\boxed{F_s = -kx\ .} \tag{VII-1}$$

a) $x \equiv$ displacement of the mass from the unstretched ($x = 0$) position.

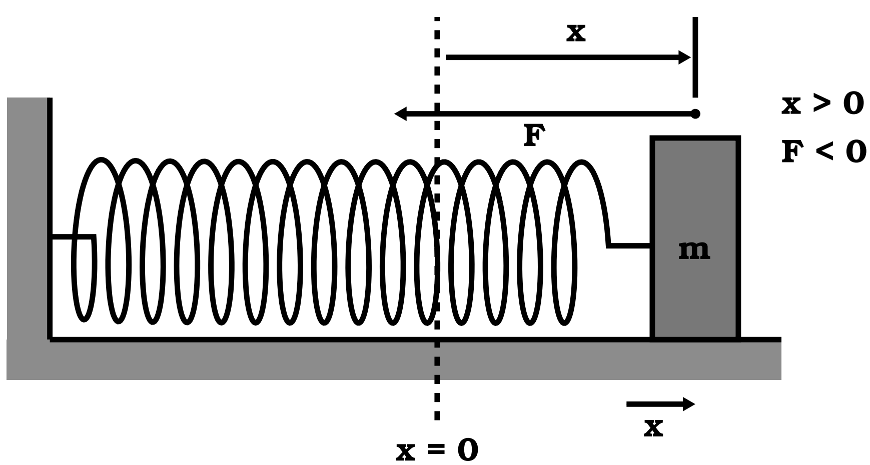
b) $k \equiv$ **spring constant**.

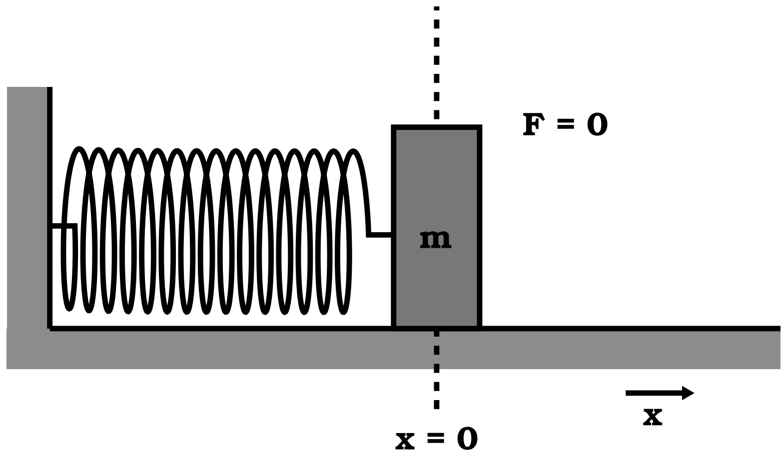
i) Stiff springs have large k values.

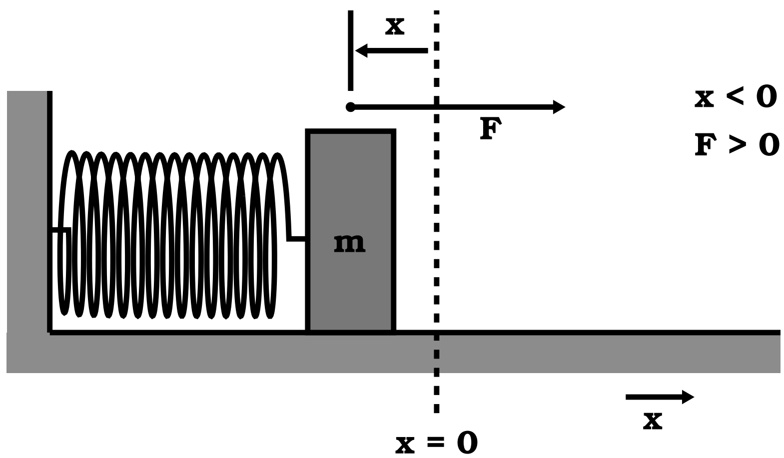
ii) Soft springs have small k values.

c) The negative sign signifies the F exerted by a spring is always directed opposite of the displacement of the mass.

d) The direction of the restoring force is such that the mass is either pulled or pushed toward the equilibrium position.







2. The oscillatory motion set up by such a system is called **simple harmonic motion** (SHM).

   a) SHM occurs when the net force along the direction of motion is a Hooke's Law type of force.

   b) SHM when $F \propto -x$.

   c) Terms of SHM:

   i) **Amplitude** (A): Maximum distance traveled by an object away from its equilibrium position.

   ii) **Period** (T): The time it takes an object in SHM to complete one cycle of motion.

iii) Frequency (f): Number of cycles per unit of time $\Longrightarrow f = 1/T$.

3. If a spring hangs downward in a gravitational field, we can use Newton's 2nd Law of Motion in conjunction with Hooke's Law to obtain

$$\begin{aligned} \sum F &= F_s - F_g = 0 \\ F_s &= F_g \\ k\,y &= m\,g \end{aligned} \qquad \text{(VII-2)}$$

(remember that signs associated with forces depend upon the orientation of the coordinate system).

4. In general, the equation of motion for a spring (*i.e.*, SHO – simple harmonic oscillator) is

$$\boxed{F = -kx = ma} \qquad \text{(VII-3)}$$

or

$$\boxed{a = -\frac{k}{m}x\ .} \qquad \text{(VII-4)}$$

Example VII–1. A spring oriented vertically is attached to a hard horizontal surface. The spring has a force constant of 1.46 kN/m. How much is the spring compressed when an object of mass $m =$ 2.30 kg is placed on top the spring and the system is at rest?

Solution:

Using Eq. (VII-2), we have $k = 1.46 \times 10^3$ N/m and $m = 2.30$ kg. Solving for y we get

$$\begin{aligned} k\,y &= m\,g \\ y &= \frac{mg}{k} = \frac{(2.30\ \text{kg})\,(9.80\ \text{m/s}^2)}{1.46 \times 10^3\ \text{N/m}} \\ &= 1.54 \times 10^{-2}\ \text{m} = \boxed{1.54\ \text{cm}\ .} \end{aligned}$$

B. Energy and Motion of a Simple Harmonic Oscillator (SHO).

1. The energy stored in a stretched/compressed spring (or other elastic material) is called **elastic potential energy** PE_s:

$$\boxed{\mathrm{PE}_s \equiv \frac{1}{2}kx^2 \ .} \qquad \text{(VII-5)}$$

2. Energy equation of a SHO:

$$W_{nc} = (\mathrm{KE} + \mathrm{PE}_g + \mathrm{PE}_s)_\mathrm{f} - (\mathrm{KE} + \mathrm{PE}_g + \mathrm{PE}_s)_\mathrm{i} \ . \qquad \text{(VII-6)}$$

a) $W_{nc} \equiv$ work done by a non-conservative force.

b) $_{\mathrm{i,\,f}} \equiv$ initial and final values.

c) KE $\equiv$ kinetic energy $= \frac{1}{2}mv^2$.

d) $\mathrm{PE}_g \equiv$ gravitational potential energy $= mgy$.

e) $\mathrm{PE}_s \equiv$ elastic potential energy given by Eq. (VII-5).

f) Note that if there are no non-conservative forces present, $W_{nc} = 0$ and the conservation of energy results:

$$\begin{aligned} (\mathrm{KE} + \mathrm{PE}_g + \mathrm{PE}_s)_\mathrm{i} &= (\mathrm{KE} + \mathrm{PE}_g + \mathrm{PE}_s)_\mathrm{f} \\ &= \text{constant} \end{aligned} \qquad \text{(VII-7)}$$

3. From these energy equations, we can deduce v as a function of x:

a) Assume the spring is horizontal ($h_\mathrm{i} = h_\mathrm{f} = 0$) and no non-conservative forces are present (*i.e.*, no friction):

i) $(\mathrm{PE}_g)_\mathrm{i} = (\mathrm{PE}_g)_\mathrm{f} = 0$, then

ii) $(\mathrm{KE} + \mathrm{PE}_s)_\mathrm{i} = (\mathrm{KE} + \mathrm{PE}_s)_\mathrm{f}$.

b) Now extend the spring a distance A from the equilibrium position and release from rest ($v = 0$).

i) $\mathrm{KE_i} = \frac{1}{2} m v_\mathrm{i}^2 = 0$.

ii) $(\mathrm{PE}_s)_\mathrm{i} = \frac{1}{2} k A^2 \qquad (x = A)$.

c) From the equations above and setting $v_\mathrm{f} = v$ and $x_\mathrm{f} = x$, we can write

$$0 + \frac{1}{2} k A^2 = \frac{1}{2} m v^2 + \frac{1}{2} k x^2$$

and solving for v gives

$$\boxed{v = \pm \sqrt{\frac{k}{m} \left(A^2 - x^2\right)}\ .} \qquad \text{(VII-8)}$$

i) If $x = \pm A$, then $v = 0$.

ii) If $x = 0$, then $v = \pm \sqrt{k/m}\, A$.

4. SHO motion is very similar to circular motion.

a) An "orbit" is analogous to a SHO "cycle."

b) Remember from circular motion,

$$\begin{aligned} v_\mathrm{orbit} &= \frac{\text{circumference of orbit}}{\text{period of orbit}} = \frac{2\pi R}{T} \\ T &= \frac{2\pi R}{v} = \frac{2\pi A}{v} \quad (R = \text{ amp. of orbit } = A). \end{aligned}$$

c) The radius of the orbit is analogous to the position of a SHO when passing through its equilibrium position, thus

$$T = \frac{2\pi A}{A\sqrt{(k/m)}}\ .$$

Note that only the '+' solution is used for velocity (Eq. VII-8) since "period" is always positive. Simplifying gives

$$\boxed{T = 2\pi\sqrt{\frac{m}{k}}\ .} \qquad \text{(VII-9)}$$

d) Eq. (VII-9) is the oscillation period of a SHO. The frequency is then

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad \text{(VII-10)}$$

$\Longrightarrow$ the unit of frequency is **hertz** (Hz) = 1/s.

5. The position of a SHO as a function of time (see §13.5 of the textbook) is given by

$$\boxed{x = A\cos(\omega t)\ .} \qquad \text{(VII-11)}$$

a) $\omega \equiv$ angular speed $= 2\pi/T = 2\pi f$.

b) In terms of frequency:

$$x = A\cos(2\pi f t)\ . \qquad \text{(VII-12)}$$

6. The derivation of velocity and acceleration as a function of time is complicated using algebra. However with calculus, the derivation is easy:

$$v = \frac{dx}{dt} = -\omega A\sin(\omega t) \qquad \text{(VII-13)}$$

$$a = \frac{dv}{dt} = -\omega^2 A\cos(\omega t)\ . \qquad \text{(VII-14)}$$

a) The velocity is 90° out of phase with displacement.

b) The acceleration is 90° out of phase with velocity and 180° out of phase with displacement.

Example VII–2. An object-spring system oscillates with an amplitude of 3.5 cm. If the spring constant is 250 N/m and the object has a mass of 0.50 kg, determine (a) the mechanical energy of the system, (b) the maximum speed of the object, and (c) the maximum acceleration.

Solution (a):

Since there are no non-conservative forces involved here, the initial mechanical energy = final mechanical energy = total mechanical energy. If we choose the maximum extension point as our reference point, we have $x = A = 3.5$ cm $= 0.035$ m and $v = 0$. Since no information was given as to whether the spring is positioned vertically or horizontally, we will assume it is horizontal to make the problem easier. As such, set $y = h = 0$ and $\mathrm{PE}_g = 0$ which gives the total mechanical energy as

$$\begin{aligned} E &= \mathrm{KE} + \mathrm{PE}_g + \mathrm{PE}_s = \frac{1}{2}mv^2 + mgy + \frac{1}{2}kx^2 \\ &= 0 + 0 + \frac{1}{2}kA^2 = \frac{1}{2}(250\ \mathrm{N/m})(0.035\ \mathrm{m})^2 \\ &= \boxed{0.15\ \mathrm{J}\ .} \end{aligned}$$

Solution (b):

The maximum speed occurs at the equilibrium position, $x = 0$:

$$\begin{aligned} E &= \mathrm{KE} + \mathrm{PE}_g + \mathrm{PE}_s = \frac{1}{2}mv^2 + mgy + \frac{1}{2}kx^2 \\ &= \frac{1}{2}mv^2 + 0 + 0 = \frac{1}{2}mv_{\mathrm{max}}^2 \\ v_{\mathrm{max}}^2 &= \frac{2E}{m} \\ v_{\mathrm{max}} &= \sqrt{\frac{2E}{m}} = \sqrt{\frac{2(0.15\ \mathrm{J})}{0.50\ \mathrm{kg}}} \\ &= \boxed{0.77\ \mathrm{m/s}\ .} \end{aligned}$$

Solution (c):

The acceleration is simply found from Newton's 2nd law:

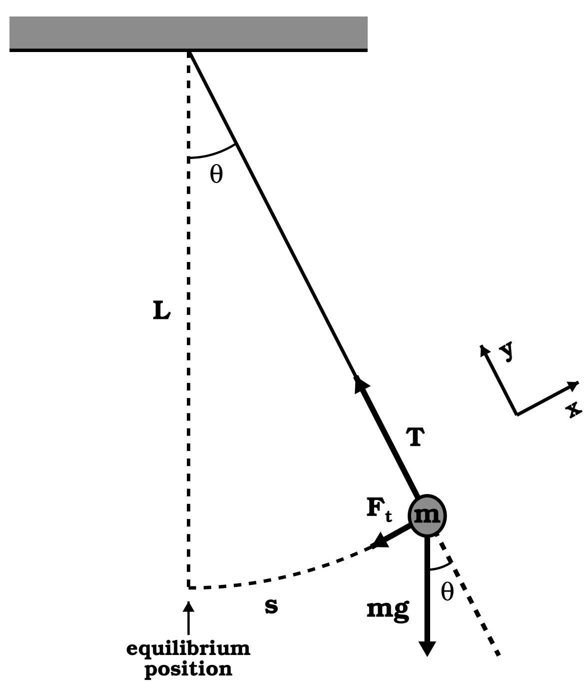
$$a = \frac{\sum F}{m} = \frac{-kx}{m} \ .$$

As can be seen from this equation, the acceleration will take on its maximum value when $x = -x_{\rm max} = -A$, so

$$\begin{aligned} a_{\rm max} &= \frac{-k(-A)}{m} = \frac{kA}{m} = \frac{(250 \text{ N/m})(0.035 \text{ m})}{0.50 \text{ kg}} \\ &= \boxed{18 \text{ m/s}^2 \ .} \end{aligned}$$

C. Pendulum Motion.

1. Consider a pendulum bob of mass m hanging from a support at a distance L from the pivot point. If the pendulum bob moves θ degrees from the equilibrium position, we have



a) In the diagram on the previous page, we define a Cartesian coordinate system which is centered on the moving bob with the y axis pointing in the *radial* direction (*i.e.*, towards the pivot point) and the x axis pointing in the negative of the *tangential* force direction. Note that $\vec{F}_r \perp \vec{F}_t$ (*i.e.*, the radial force is perpendicular to the tangential force).

b) Summing the forces in the tangential (x) direction gives

$$F_t = \sum F_x = -mg \sin\theta \ . \tag{VII-15}$$

c) Summing the forces in the radial direction gives

$$F_r = \sum F_y = T - mg\cos\theta = 0 \ , \tag{VII-16}$$

since the bob always remains a fixed distance L away from the pivot point.

2. The tangential force acts to restore the pendulum to its equilibrium position.

a) Sets up an oscillation.

b) Motion is <u>not</u> simple harmonic since the force doesn't follow $F \propto -x \Longrightarrow$ instead it follows $F \propto -\sin\theta$.

3. If the pendulum oscillates at small angles ($\theta < 15°$), then

$$\boxed{\sin\theta \approx \theta \quad \text{(with } \theta \text{ measured in radians)}\ .} \tag{VII-17}$$

a) Then $F \propto -\theta$ and the motion becomes simple harmonic!

b) Mathematically we have

$$F_t = -mg\,\theta \ , \quad (\theta \text{ small}). \tag{VII-18}$$

c) Note that $\theta = s/L$ (from angular measure of General Physics I), so we also can write

$$F_t = -\left(\frac{mg}{L}\right) s \ , \quad (s \ll L). \qquad \text{(VII-19)}$$

i) Similar to Hooke's Law, however 'k' is replaced by 'mg/L'.

ii) From analogy with Hooke's Law, we can write the period of a pendulum as

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{mg/L}} \ ,$$

or

$$\boxed{T = 2\pi\sqrt{\frac{L}{g}} \ .} \qquad \text{(VII-20)}$$

iii) Note that the period of a pendulum (under small oscillation) is independent of mass $\Longrightarrow$ Galileo showed this empirically long before Newton's Laws were developed!

Example VII–3. A man enters a tall tower, needing to know its height. He notes that a long pendulum extends from the ceiling almost to the floor and that its period is 15.5 s. (a) How tall is the tower? (b) If this pendulum is taken to the Moon, where the free-fall acceleration is 1.67 m/s^2, what is the period there?

Solution (a):

This problem is simple enough that we need no figure. Here, we only have to use Eq. (VII-20) and solve for L, the length of the pendulum (which is nearly the same length as the height of the building as indicated in the question).

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$
\begin{aligned}
T^2 &= \frac{4\pi^2 L}{g} \\
L &= \frac{gT^2}{4\pi^2} = \frac{(9.80\ \mathrm{m/s^2})(15.5\ \mathrm{s})^2}{4\pi^2} \\
&= \boxed{59.6\ \mathrm{m}\ .}
\end{aligned}
$$

Solution (b):

On the Moon, where $g = 1.67\ \mathrm{m/s^2}$, the period will be

$$
T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{59.6\ \mathrm{m}}{1.67\ \mathrm{m/s^2}}} = \boxed{37.5\ \mathrm{s}\ .}
$$

D. Types of Waves.

1. Waves can either move in space (*e.g.*, water waves), the so-called **traveling waves**, or be stationary in an enclosure, the so-called **standing waves**.

a) Let's define the y direction as the direction in which the oscillation occurs (*i.e.*, $\Delta y_{\mathrm{max}} = A$, the amplitude of the wave) and the x direction as the direction of propagation (for traveling waves) or the length of the enclosure (for standing waves). We also will define a node in the wave as those positions where $\Delta y = 0$ (for a sine wave, this will occur at $0° = 0$ rad, $180° = \pi$ rad, $360° = 2\pi$ rad, etc.)

b) Standing waves do not change in time. An enclosure can only contain those standing waves that have an integer number wavelengths that can fit inside the enclosure with node points lying at the boundaries of the enclosure.

c) Traveling waves move in space at a velocity given by

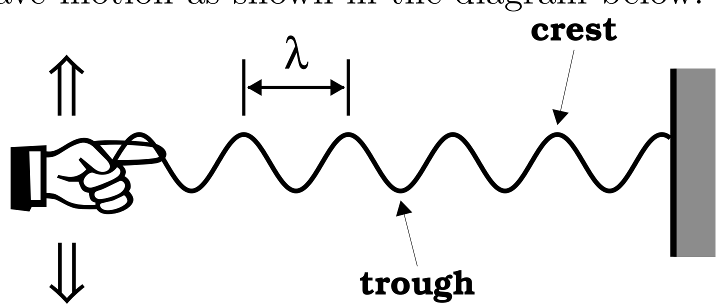
$$
v = \lambda f\ , \tag{VII-21}
$$

where λ is the wavelength (*i.e.*, distance between wavecrests) and f is the frequency (*i.e.*, number of wavecrests past a given point per second) of the wave.

d) Note that a single 'hump' (*i.e.*, a **pulse**) can propagate along a medium too, and as such, is also considered a traveling wave, even though it has no definite wavelength associated with it. Examples of wave pulses are tsunami (often incorrectly called *tidal waves*) and shock waves.

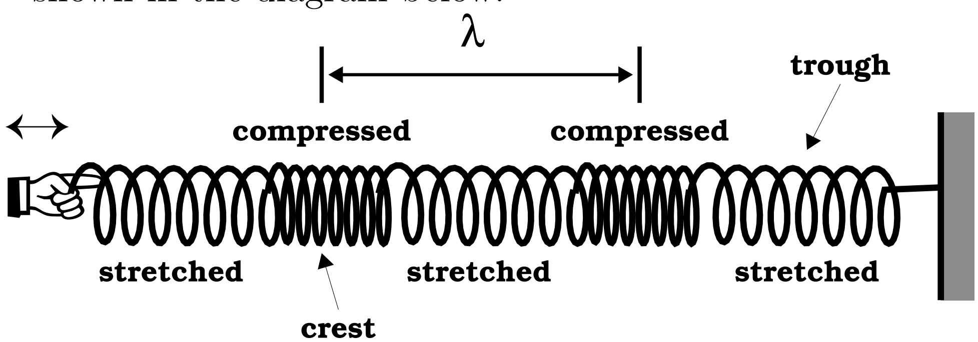
2. Two types of traveling waves exist in nature:

a) **Transverse waves**: Each segment of the medium (*e.g.*, a rope or water) that is disturbed moves *perpendicular* to the wave motion as shown in the diagram below:



$\Longrightarrow$ water waves, guitar strings, and E/M radiation are examples of transverse waves.

b) **Longitudinal waves**: The elements of the medium undergo displacements *parallel* to the direction of motion as shown in the diagram below:



$\Longrightarrow$ sound waves are longitudinal waves.

3. The speed of a transverse wave on a string is

$$v = \sqrt{\frac{F}{\mu}} \,, \qquad \text{(VII-22)}$$

where $F = T$ is the tension on the string (or rope) and μ is the mass per unit length of the string.

Example VII–4. An astronaut on the Moon wishes to measure the local value of g by timing pulses traveling down a wire that has a large object suspended from it. Assume a wire of mass 4.00 g is 1.60 m long and has a 3.00 kg object suspended from it. A pulse requires 36.1 ms to traverse the length of the wire. Calculate g_{Moon} from these data. (You may neglect the mass of the wire when calculating the tension in it.)

Solution:

The given parameters (converted to SI units) are $m_w = 4.00 \times 10^{-3}$ kg, $L = 1.60$ m, $m = 3.00$ kg, and $\Delta t = 3.61 \times 10^{-2}$ s. The mass per unit length of the wire is

$$\mu = \frac{m_w}{L} = \frac{4.00 \times 10^{-3}\ \text{kg}}{1.60\ \text{m}} = 2.50 \times 10^{-3}\ \text{kg/m} \,,$$

and the speed of the pulse is

$$v = \frac{L}{\Delta t} = \frac{1.60\ \text{m}}{3.61 \times 10^{-2}\ \text{s}} = 44.3\ \text{m/s} \,.$$

Using Eq. (VII-22), we can calculate the tension in the wire as

$$F = T = v^2\mu = (44.3\ \text{m/s})^2(2.50 \times 10^{-3}\ \text{kg/m}) = 4.91\ \text{N} \,.$$

Since we are not told that the mass is accelerating up or down, we will assume that the tension of the wire is being counterbalanced by the force of gravity downward, hence we will assume that the mass is in static equilibrium. Then summing the forces in the y direction gives:

$$\begin{aligned}
\sum F_y &= T - mg_{\mathrm{Moon}} = 0 \\
mg_{\mathrm{Moon}} &= T \\
g_{\mathrm{Moon}} &= \frac{T}{m} = \frac{4.91\ \mathrm{N}}{3.00\ \mathrm{kg}} = \boxed{1.64\ \mathrm{m/s}^2}\ .
\end{aligned}$$

E. The Interference and Reflection of Waves.

1. Two waves can meet and pass through each other without being destroyed or even altered.

a) We must add interacting waves together using the principle of superposition $\Longrightarrow$ If two or more traveling waves are moving through a medium, the resultant wave is found by adding together the displacements of the individual waves point by point.

b) This superposition principle is only valid when the individual waves have small amplitudes of displacement.

2. The interaction of two or more waves is called **interference**.

a) **Constructive interference** occurs if the waves are *in phase* with each other $\Longrightarrow$ wavecrests line up with wavecrests and troughs line up with troughs. This produces a larger amplitude resultant wave which is the sum of the amplitudes of the individual waves.

b) **Destructive interference** occurs if the waves are *out of phase* with each other $\Longrightarrow$ wavecrests line up with troughs and troughs line up with crests which *nullifies* the wave $\Longrightarrow$ a wave of zero amplitude results.

3. Waves also can bounce off of immovable objects.

a) If the "attached" end of a string is fixed to the immovable object (like a wall), a wave-pulse will become inverted upon reflection.

b) If the "attached" end of a string is free to move on the immovable object (*e.g.*, string attached to a ring surrounding a pole where the ring is free to slide up and down the pole), a wave-pulse will not be inverted upon reflection (see Figures 13.34 and 13.35 of your textbook).