PHYS-2020: General Physics II
Course Lecture Notes
Section XIV

Dr. Donald G. Luttermoser
East Tennessee State University

Edition 4.0

Abstract

These class notes are designed for use of the instructor and students of the course PHYS-2020: General Physics II taught by Dr. Donald Luttermoser at East Tennessee State University. These notes make reference to the *College Physics, 10th Hybrid Edition* (2015) textbook by Serway and Vuille.

XIV. Optical Instruments

A. Common Optical Tools.

1. A **camera** consists of a lens (or series of lenses) that focus an image onto a light sensing detector (*e.g.*, film, CCD, etc.).

a) The diameter of the lens can be *stopped-down* to smaller sizes by adjusting the **aperture** opening.

i) These *stops* are technically referred to as the ***f*-number**.

$$f\text{-number} \equiv \frac{f}{D} \qquad \text{(XIV-1)}$$

f = focal length of lens.
D = diameter of the lens or aperture (whichever is smaller)

ii) The *larger* the f number, the *less* light is allowed into the camera.

iii) Cameras typically have the following f-stops: $f/2.8$ (lets the most light in), $f/4$, $f/5.6$, $f/8$, $f/11$, $f/16$ (lets the least amount of light in).

b) The magnification of the image follows the *thin lens* formulae (*e.g.*, Eq. XII-10) with an image size given by

$$h' = \frac{h}{p} f \; . \qquad \text{(XIV-2)}$$

c) The amount of light that falls on the detector is determined by the f-stop <u>and</u> the shutter speed.

i) For fast moving objects, one should use shutter speeds 1/1000th or 1/500th of a second.

ii) For indoor, low-light levels, speeds greater than 1/60th of a second are required $\Longrightarrow$ speeds longer than 1/60 sec usually require the camera to be mounted on a tripod to keep the camera steady.

iii) Astronomical photographs typically require the shutter to be open from minutes to hours in order to record the image.

2. The organic analogy to the camera is the **eye**. In the case of the human eye we have the following characteristics:

a) The **iris** is equivalent to the aperture of the camera $\Longrightarrow$ the opening is called the **pupil**.

b) The **cornea** is a transparent *lens cap* for the eye's **lens**.

c) The **retina** records the image and sends the signal to the brain via the **optic nerve** $\Longrightarrow$ analogous to the film in a camera or a CCD chip. The retina is composed of 2 types of receptor cells:

i) **Rods** are able to detect low light levels in black-and-white.

ii) **Cones** come in 3 types that respond respectively to red, green, and blue light.

d) Defects of the eye:

i) **Hyperopia**: Image of a nearby object forms behind the retina (farsighted).

ii) **Myopia**: Maximum focal length of eye lens is shorter than diameter of eye (nearsighted).

iii) **Astigmatism**:A point source produces a *line* image on the retina.

iv) **Cataracts**: Lens and/or cornea becomes partially or totally opaque.

v) **Glaucoma**: Abnormal increase in fluid pressure inside the eyeball.

e) Corrective lenses are often used to correct many defects of the eye. The *power* of such a lens is measured in **diopters**:

$$\mathcal{P}(\text{diopters}) = \frac{1}{f(\text{m})} \qquad \text{(XIV-3)}$$

$\Longrightarrow f$ is the lens focal length in meters.

i) The **near point** is the closest distance for which the lens can accommodate to focus light on the retina. This distance changes as the eye ages:

Age 10: near point = 18 cm.
Age 20: near point = 25 cm.
Age 40: near point = 50 cm.
Age 60: near point = 500 cm.

ii) The **far point** is the farthest distance for which the lens of a relaxed eye can focus light on the retina. People with normal vision can focus objects that are very far away (*e.g.*, the Moon). For these people the far point is essentially infinity.

3. The **Simple Magnifier** increases the image size of an object with a single lens (*i.e.*, magnifying glass), with a magnification of

$$m \equiv \frac{\theta}{\theta_\circ} \qquad \text{(XIV-4)}$$

$\Longrightarrow \theta =$ angle the object subtends with the lens
$\Longrightarrow \theta_\circ =$ angle the object subtends to the naked eye .

B. Microscopes and Telescopes.

1. Both microscopes and *refracting* telescopes have a series of 2 (or more) lenses that are used to magnify an image.

a) The **objective** lens is the *light collector* $\Longrightarrow$ the bigger the objective, the more light you receive, the fainter the object you can see.

b) The **eyepiece** lens is the main component of magnification.

c) A **reflecting** telescope has a mirror (usually parabolic in shape) to collect the light and is called the **primary mirror** instead of the objective lens.

2. These instruments magnify objects following the relations:

a) For compound microscopes:

$$M = M_1 m_e = -\frac{L}{f_o}\left(\frac{25\text{ cm}}{f_e}\right) , \qquad \text{(XIV-5)}$$

where

$$\begin{aligned} M_1 &= \text{magnification of the objective} \\ m_e &= \text{magnification of the eyepiece} \\ L &= \text{distance between objective and eyepiece (in cm)} \\ f_o &= \text{focal length of the objective (in cm)} \\ f_e &= \text{focal length of the eyepiece (in cm).} \end{aligned}$$

b) For a telescope:

$$m = \frac{f_o}{f_e} \ . \qquad \text{(XIV-6)}$$

i) Note that one can get as high as a magnification as you want by just getting eyepieces with smaller and smaller focal lengths.

ii) The higher the magnification, the fainter the image $\Longrightarrow$ rule of thumb for useful magnifications: 50 power for each inch of the objective.

$\Longrightarrow$ a 2.4" telescope's maximum useful power is 120!

Example XIV–1. Astronomers often take photographs with the objective lens or mirror of a telescope alone, without an eyepiece. (a) Show that the image size h' for this telescope is given by $h' = fh/(f-p)$ where h is the object size, f the objective focal length, and p the object distance. (b) Simplify the expression in part (a) if the object distance is much greater than the objective focal length. (c) The "wingspan" of the International Space Station is 108.6 m, the overall width of its solar panel configuration. When it is orbiting at an altitude of 407 km, find the width of the image formed by the telescope objective of focal length 4.00 m.

Solution (a):

Using the thin lens equation (Eq. XII-10), we get

$$\begin{aligned}
\frac{1}{p} + \frac{1}{q} &= \frac{1}{f} \\
\frac{1}{q} &= \frac{1}{f} - \frac{1}{p} \\
\frac{1}{q} &= \frac{p}{pf} - \frac{f}{pf}
\end{aligned}$$

$$q = \frac{pf}{p-f} .$$

Inserting this into the magnification equation (Eq. XII-11), we get

$$M = \frac{h'}{h} = -\frac{q}{p} = -\frac{f}{p-f} .$$

Therefore the image size will be

$$h' = Mh = -\frac{fh}{p-f} = \boxed{\frac{fh}{f-p} .}$$

Solution (b):

If $p \gg f$, then $f - p \approx -p$ and

$$\boxed{h' \approx -\frac{fh}{p} .}$$

Solution (c):

Suppose the telescope observes the space station while at the zenith (*i.e.*, the point directly overhead), then

$$\begin{aligned} h' &\approx -\frac{fh}{p} = -\frac{(4.00 \text{ m})(108.6 \text{ m})}{407 \times 10^3 \text{ m}} \\ &= -1.07 \times 10^{-3} \text{ m} = \boxed{-1.07 \text{ mm} .} \end{aligned}$$

C. Spatial Resolution.

1. The ability of an optical system to distinguish between closely spaced *objects* is limited due to the wave nature of light.

a) Light gets **diffracted** when passing through apertures (see Figures 25.13 and 25.14 in the textbook).

b) When the central maximum of one image falls on the first minimum of another image, the images are said to just be resolved $\Longrightarrow$ **Rayleigh's criterion**.

2. The limiting angle which an object is viewed depends both on the size of the aperture and the wavelength of light.

   a) For a slit:
$$\theta_m = \frac{\lambda}{a} \ , \qquad \text{(XIV-7)}$$
   where a is the width of the slit opening. Both the wavelength λ and a must be measured in the same units, then θ_m, the **resolving angle**, is in radians.

   b) For a circular aperture:
$$\theta_m = 1.22 \frac{\lambda}{D} \ , \qquad \text{(XIV-8)}$$
   where D is the circular aperture diameter. Again, λ and D must be in the same units to give θ_m in radians.

3. Turbulence in the Earth's atmosphere limits the spatial resolution of Earth-based telescopes to about 1 arcsecond!

Example XIV–2. A converging lens with a diameter of 30.0 cm forms an image of a satellite passing overhead. The satellite has two green lights (wavelength 500 nm) spaced 1.00 m apart. If the lights can just be resolved according to the Rayleigh criterion, what is the altitude of the satellite?

Solution:

Since the lens corresponds to a circular aperture, we can just use Eq. (XIV-8) to determine the angular separation:
$$\theta = \theta_m = 1.22 \frac{\lambda}{D} = 1.22 \frac{500 \times 10^{-9} \text{ m}}{0.300 \text{ m}} = 2.03 \times 10^{-6} \text{ rad} \ .$$

The altitude can then be determined from the arc-length equation, $s = \theta r$ (Eq. VIII-1 from my General Physics I notes), where s is the size of the arc-length (here corresponding to the linear separation of the two lights) and r the radius of the circle (here corresponding to the altitude of the satellite). Using this arc-length formula and solving for r, the altitude of the satellite, we get

$$\begin{aligned} r &= \frac{s}{\theta} = \frac{1.00 \text{ m}}{2.03 \times 10^{-6} \text{ rad}} = 4.92 \times 10^5 \text{ m} \\ &= \boxed{492 \text{ km}} . \end{aligned}$$

D. Spectral Resolution.

1. Earlier we mentioned that prisms disperse light through refraction. **Diffraction gratings** also can disperse light through **diffraction** (see §XIII of these notes).

   a) Diffraction gratings have parallel lines *etched* ("scratched") on the surface of a glass (or some other transparent material) plate.

   b) These etched lines act like a series of parallel slits.

2. Diffraction is caused by *interference* of many light waves. By using the mathematics of interference patterns, the **grating equation** results:

$$\boxed{d \sin\theta = m\lambda \ , \quad m = 0, 1, 2, \ldots} \qquad \text{(XIV-9)}$$

   a) d is the **slit spacing** $= 1/N$, where N is the number of slits or lines per cm.

b) θ is the **grating angle** or angle of deviation.

c) m is the **order number**.

3. Spectral resolution, $\Delta\lambda_G$, is typically measured in Å or nm. **Spectral resolution** measures how close together 2 spectral lines can be separated in a spectrograph.

4. The **resolving power**, R, is related to the spectral resolution via

$$R \equiv \frac{\lambda}{\lambda_2 - \lambda_1} = \frac{\lambda}{\Delta\lambda_G} \, . \qquad \text{(XIV-10)}$$

a) The higher the resolving power, the finer the detail seen in a spectrum — the better the spectral resolution.

b) For a grating, the resolving power (a unitless number) is directly proportional to the line density on the grating:

$$R = Nm \ (\times 1 \text{ cm}), \qquad \text{(XIV-11)}$$

where N is measured in inverse centimeters.

Example XIV–3. A light source emits two major spectral lines, an orange line of wavelength 610 nm and a blue-green line of wavelength 480 nm. If the spectrum is resolved by a diffraction grating having 5000 lines/cm and viewed on a screen 2.00 m from the grating, what is the distance (in cm) between the two spectral lines in the second-order spectrum?

Solution:

The grating spacing is $d = 1/N = 1 \text{ cm}/5000 = 2.00 \times 10^{-4} \text{ cm} = 2.00 \times 10^{-6}$ m, and Eq. (XIV-9) gives the angular position of the second order ($m = 2$) spectral line as

$$\sin\theta = \frac{2\lambda}{d} \quad \text{or} \quad \theta = \sin^{-1}\left(\frac{2\lambda}{d}\right) \, .$$

For the given wavelengths, the angular positions are

$$\theta_1 = \sin^{-1}\left[\frac{2(610 \times 10^{-9}\ \text{m})}{2.00 \times 10^{-6}\ \text{m}}\right] = 37.6^\circ$$

$$\theta_2 = \sin^{-1}\left[\frac{2(480 \times 10^{-9}\ \text{m})}{2.00 \times 10^{-6}\ \text{m}}\right] = 28.7^\circ$$

If L is the distance from the grating to the screen, the distance on the screen from the central maximum to a second-order bright line is $y = L\tan\theta$. Therefore, for the two given wavelengths, the separation of the lines on the screen is

$$\begin{aligned}\Delta y &= L\left[\tan\theta_1 - \tan\theta_2\right] \\ &= (2.00\ \text{m})\left[\tan(37.6^\circ) - \tan(28.7^\circ)\right] = 0.445\ \text{m} \\ &= \boxed{44.5\ \text{cm}\ .}\end{aligned}$$