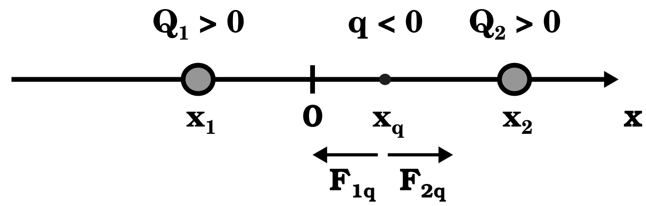
Physics 2020: Sample Problems for Exam 1

1. Two particles are held fixed on the x-axis. The first particle has a charge of $Q_1 = 6.88 \times 10^{-5}$ C and is located at $x_1 = -4.56$ m on the x-axis. The second particle has a charge of $Q_2 = 2.33 \times 10^{-4}$ C and is located at $x_2 = 8.05$ m on the x-axis. A third particle of charge $q < 0$ and mass $m = 0.765$ g is located at $+2.89$ m on the x-axis and is free to move. (a) If charge q experiences a net electric force of $+5.56 \times 10^{-4}$ N at this location from the fixed charges, what is the charge on this particle? (b) At what acceleration will this charge q move at this location? (**Show all work!**)

Solution:



Part (a): Since $Q_1 > 0$ and $q < 0$, the force (F_{1q}) between $Q_1 > 0$ and $q < 0$ points to the left (as shown in the figure above) due to the fact that these charges will attract each other. Since $Q_2 > 0$ and $q < 0$, the force (F_{2q}) between $Q_2 > 0$ and $q < 0$ points to the right, once again due to the fact that these charges will attract each other. As stated in the problem, the sum of these two forces is $+5.56 \times 10^{-4}$ N ($F_{\rm tot}$). Using this fact in conjunction with Coulomb's Law, we can do a little algebra to solve for q:

$$\begin{aligned}
F_{\rm tot} &= F_{2q} - F_{1q} \\
&= k_e \frac{|Q_2|\,|q|}{r_{2q}^2} - k_e \frac{|Q_1|\,|q|}{r_{1q}^2} = k_e\,|q| \left[\frac{|Q_2|}{r_{2q}^2} - \frac{|Q_1|}{r_{1q}^2}\right] , \\
|q| &= \frac{F_{\rm tot}}{k_e} \left[\frac{|Q_2|}{r_{2q}^2} - \frac{|Q_1|}{r_{1q}^2}\right]^{-1} .
\end{aligned}$$

$$\begin{aligned}
r_{1q} &= x_q - x_1 = 2.89 \text{ m} - (-4.56 \text{ m}) = 7.45 \text{ m} \\
r_{2q} &= (x_2 - x_q) = 8.05 \text{ m} - 2.89 \text{ m} = 5.16 \text{ m}
\end{aligned}$$

$$\frac{|Q_1|}{r_{1q}^2} = \frac{6.88 \times 10^{-5} \text{ C}}{(7.45 \text{ m})^2} = \frac{6.88 \times 10^{-5} \text{ C}}{55.5025 \text{ m}^2} = 1.24 \times 10^{-6} \text{ C/m}^2.$$

$$\frac{|Q_2|}{r_{2q}^2} = \frac{2.33 \times 10^{-4} \text{ C}}{(5.16 \text{ m})^2} = \frac{2.33 \times 10^{-4} \text{ C}}{26.6256 \text{ m}^2} = 8.75 \times 10^{-6} \text{ C/m}^2.$$

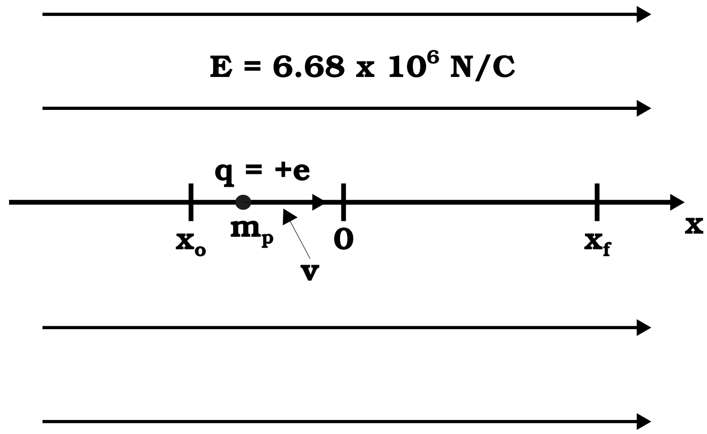
$$\begin{aligned}
|q| &= \frac{5.56 \times 10^{-4} \text{ N}}{8.99 \times 10^{9} \text{ N}\cdot\text{m}^2/\text{C}^2} \left[8.75 \times 10^{-6} \text{ C/m}^2 - 1.24 \times 10^{-6} \text{ C/m}^2\right]^{-1} \\
&= 8.24 \times 10^{-9} \text{ C} .
\end{aligned}$$

Since q has to be negative as stated in the problem, $\boxed{q = -8.24 \times 10^{-9} \text{ C.}}$

Part (b): Here we just use Newton's Second Law of Motion: $F = m\,a$, and solve for a the acceleration (remembering to change the mass from cgs units to SI units):

$$a = \frac{F_{\rm tot}}{m} = \frac{5.56 \times 10^{-4} \text{ N}}{7.65 \times 10^{-4} \text{ kg}} = \boxed{0.727 \text{ m/s}^2.}$$

2. A proton is released from rest at $x = -4.56$ m in a uniform electric field of $+6.68 \times 10^6$ N/C $\hat{x}$. (a) Calculate the change in potential energy when the proton moves along the x-axis to the $x = +7.58$ m position. (b) What will be the velocity of the proton at this position of +7.58 m? (c) What is the acceleration of this proton as it moves through the electric field? (**Show all work!**)



Solution:

Part (a): The figure to the right diagrams the problem, where our particle (a proton), has mass $m_p = 1.672 \times 10^{-27}$ kg, charge $q = +e = 1.602 \times 10^{-19}$ C, and travels along the x-axis from $x_\circ$ = -4.56 m (where it starts from rest) to $x_{\rm f}$ = 7.58 m for a total distance of $d = x_{\rm f} - x_\circ$ = 7.58 m - (-4.56 m) = 12.14 m. This proton travels in an electric field E of strength 6.68×10^6 N/C which runs parallel to the x axis. The change in potential energy can now be determined with:

$$\begin{aligned} \Delta\text{PE} &= -q\,E\,d = -(1.602 \times 10^{-19}\text{ C})\,(6.68 \times 10^6\text{ N/C})\,(12.14\text{ m}) \\ &= \boxed{-1.30 \times 10^{-11}\text{ J}\ ,} \end{aligned}$$

where the negative sign results from the fact that $q > 0$ (*i.e.*, a positive value).

Part (b): To determine the final velocity, the simplest way is to just use the conservation of energy. Here, we do not need to include the gravitational potential energy since the mass just moves in the horizontal direction, hence $\text{PE}_{g\,\rm i} = \text{PE}_{g\,\rm f}$. Since we are starting from rest, the initial kinetic energy is equal to 0.

$$\begin{aligned} \text{KE}_{\rm i} + \text{PE}_{\rm i} &= \text{KE}_{\rm f} + \text{PE}_{\rm f} \\ \text{PE}_{\rm i} - \text{PE}_{\rm f} &= \text{KE}_{\rm f} - \text{KE}_{\rm i} \\ -\Delta\text{PE} &= \frac{1}{2}\,m\,v_{\rm f}^2 - 0 \\ v_{\rm f}^2 &= \frac{-2(\Delta\text{PE})}{m_p} \\ v_{\rm f} &= \sqrt{\frac{-2(-1.30 \times 10^{-11}\text{ J})}{1.672 \times 10^{-27}\text{ kg}}} \end{aligned}$$

$$\boxed{v_{\rm f} = 1.25 \times 10^8\text{ m/s}\ .}$$

Part (c): There are a few different ways we could solve this problem, I will show two of them here. The first. and easiest, just involves making use of Newton's Second Law of Motion ($F = m\,a$), the second involves making use of one of the equations of one dimensional motion.

Newton's Second Law solution:

$$\begin{aligned} a &= \frac{F_e}{m_p} = \frac{q\,E}{m_p} = \frac{e\,E}{m_p} \\ &= \frac{(1.602 \times 10^{-19}\ \text{C})\,(6.68 \times 10^{6}\ \text{N/C})}{1.672 \times 10^{-27}\ \text{kg}} \\ &= \boxed{6.40 \times 10^{14}\ \text{m/s}^2} \end{aligned}$$

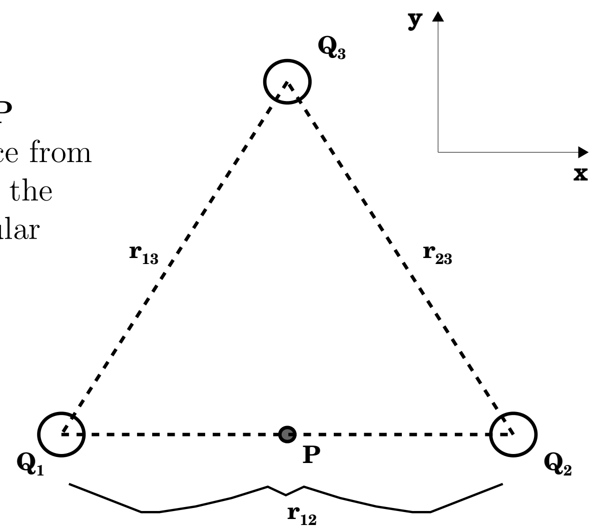
One-dimensional equation of motion solution:

$$\begin{aligned} v^2 &= v_\circ^2 + 2\,a\,(x - x_\circ) = v_\circ^2 + 2\,a\,d \\ 2\,a\,d &= v^2 - v_\circ^2 = v^2 - 0 = v^2 \\ a &= \frac{v^2}{2\,d} = \frac{(1.25 \times 10^{8}\ \text{m/s})^2}{2(12.14\ \text{m})} \\ &= \boxed{6.44 \times 10^{14}\ \text{m/s}^2} \end{aligned}$$

Both solutions are close to each other, but not exact. This results from round-off errors in the one-dimensional equation of motion solution. The Newton's Second Law solution should be considered more exact since it uses numeric values supplied to the user instead of values that were calculated for the input values.

3. In the figure below, let $Q_1 = +6.84\ \mu$C, $Q_2 =$ -16.2 μC, and $Q_3 = +9.20\ \mu$C be rigidly fixed and separated by $r_{12} = r_{13} = r_{23} = 1.34$ m. Calculate the electric potential at point **P** which is half-way between Q_1 and Q_2. (**Show all work!**)

Solution:



First we need to calculate the distances that point **P** is from each charge. We will define r_1 as the distance from Q_1 to **P**, r_2 as the distance from Q_2 to **P**, and r_3 as the distance from Q_3 to **P** — note that r_3 is perpendicular to r_{12}:

$$r_1 = \tfrac{1}{2}\, r_{12} = \tfrac{1}{2}\,(1.34\text{ m}) = 0.670\text{ m}$$
$$r_2 = \tfrac{1}{2}\, r_{12} = \tfrac{1}{2}\,(1.34\text{ m}) = 0.670\text{ m}$$
$$r_3 = \sqrt{r_{23}^2 - r_2^2} = 1.16\text{ m}$$

Now we use the superposition principle (Eq. II-8) to calculate the total potential at point **P**:

$$\begin{aligned}
V &= k_e \sum_{i=1}^{3} \left(\frac{q_i}{r_i}\right) \\
&= k_e \left(\frac{Q_1}{r_1} + \frac{Q_2}{r_2} + \frac{Q_3}{r_3}\right) \\
&= \left(8.99 \times 10^9\ \text{N m}^2/\text{C}^2\right) \left[\frac{6.84 \times 10^{-6}\ \text{C}}{0.670\ \text{m}} + \frac{-16.2 \times 10^{-6}\ \text{C}}{0.670\ \text{m}} + \frac{9.20 \times 10^{-6}\ \text{C}}{1.16\ \text{m}}\right] \\
&= \left(8.99 \times 10^9\ \text{N m}^2/\text{C}^2\right) \left[-6.04 \times 10^{-6}\ \text{C/m}\right] \\
&= \boxed{-5.43 \times 10^4\ \text{V}\ .}
\end{aligned}$$

4. A fully charged capacitor stores 6.89×10^6 eV of energy while connected to a 1.50 V battery. (a) If this capacitor has circular plates of radius 1.34 cm, what is the area on one of the plates? (b) What is the capacitance of this capacitor? (c) What must be the separation between these two plates (in mm)? (d) How much charge is on these plates when fully charged? (**Show all work including units!**)

Solution:

Part (a): Since there are no other components in this circuit, the potential across the plates of the capacitor is the same as the potential delivered by the battery: $\Delta V = 1.50$ V. We now need to convert the energy of this capacitor from eVs to SI units:

$$U = 6.89 \times 10^6 \text{ eV } \cdot 1.602 \times 10^{-19} \text{ J/eV} = 1.10 \times 10^{-12} \text{ J}.$$

To calculate the area, we first must convert the radius from cgs to SI units, $r = 1.34$ cm $= 1.34 \times 10^{-2}$ m. For a circular area, we get

$$A = \pi r^2 = \pi (1.34 \times 10^{-2} \text{ m})^2 = \boxed{5.64 \times 10^{-4} \text{ m}^2 .}$$

Part (b): We determine the capacitance using the capacitor energy equation (Eq. II-28):

$$\begin{aligned} U &= \frac{1}{2} C (\Delta V)^2 \\ C &= \frac{2U}{(\Delta V)^2} = \frac{2(1.10 \times 10^{-12} \text{ J})}{(1.50 \text{ V})^2} = \boxed{9.81 \times 10^{-13} \text{ F} = 0.981 \text{ pF} .} \end{aligned}$$

Part (c): For the separation use Eq. (II-14):

$$\begin{aligned} C &= \epsilon_\circ \frac{A}{d} \\ d &= \epsilon_\circ \frac{A}{C} = \left(8.85 \times 10^{-12} \text{ C}^2/\text{N m}^2\right) \frac{5.64 \times 10^{-4} \text{ m}^2}{9.81 \times 10^{-13} \text{ F}} \\ &= 5.09 \times 10^{-3} \text{ m} = \boxed{5.09 \text{ mm} .} \end{aligned}$$

Part (d): We can now use either Eq. (II-12) or Eq. (II-28) to calculate the full charge, here we will use Eq. (II-12) first, then Eq. (II-28):

$$\begin{aligned} C &= \frac{Q}{\Delta V} \\ Q &= C\,\Delta V = (9.81 \times 10^{-13} \text{ F})(1.50 \text{ V}) = \boxed{1.47 \times 10^{-12} \text{ C} .} \\ U &= \frac{1}{2} Q\,\Delta V \\ Q &= \frac{2U}{\Delta V} = \frac{2(1.10 \times 10^{-12} \text{ J})}{1.50 \text{ V}} = \boxed{1.47 \times 10^{-12} \text{ C} .} \end{aligned}$$