

Basic Rules of Boolean Algebra

	OR	AND	XOR
Combined w/0	$A+0=A$	$A\cdot 0=0$	$A\oplus 0=A$
Combined w/1	$A+1=1$	$A\cdot 1=A$	$A\oplus 1=\bar{A}$
Combined w/self	$A+A=A$	$A\cdot A=A$	$A\oplus A=0$
Combined w/inverse	$A+\bar{A}=1$	$A\cdot\bar{A}=0$	$A\oplus\bar{A}=1$
Other rules	$A+A\cdot B=A$	$A+\bar{A}\cdot B=A+B$	$(A+B)\cdot(A+C)=A+B\cdot C$
DeMorgan's Th.	$\overline{A\cdot B}=\bar{A}+\bar{B}$		$\overline{A+B}=\bar{A}\cdot\bar{B}$

Prove each of the following theorems using the basic rules of boolean algebra shown above.

Please show all intermediate steps.

1. $A + \overline{A \cdot B} = 1$

$$A + \bar{A} + \bar{B}$$

First, apply DeMorgan's Theorem to distribute the bar

$$1 + \bar{B}$$

Anything OR'ed with its inverse equals 1

$$1$$

Anything OR'ed with 1 equals 1

2. $D + D \cdot \bar{B} + D \cdot \bar{C} = D$

$$D \cdot (1 + \bar{B} + \bar{C})$$

First, pull a D out of all three terms

$$D \cdot (1)$$

Anything OR'ed with 1 equals 1

$$D$$

Anything AND'ed with 1 equals itself

3. $A \cdot (C + \bar{A}) + \bar{A} + \bar{C} = 1$

$$A \cdot C + A \cdot \bar{A} + \bar{A} + \bar{C}$$

Multiply A through the sum $C + \bar{A}$

$$A \cdot C + 0 + \bar{A} + \bar{C}$$

Anything AND'ed with its inverse equals 0

$$A \cdot C + \bar{A} + \bar{C}$$

Anything AND'ed with 0 equals itself

$$A \cdot C + \overline{(A \cdot C)}$$

Apply DeMorgan's Theorem in reverse

$$1$$

Anything OR'ed with its inverse equals 1