Points missed: _____ Student's Name: ____

Total score: _____/100 points

East Tennessee State University – Department of Computer and Information Sciences
CSCI 2710 (Tarnoff) – Discrete Structures
TEST 1 for Fall Semester, 2004

Read this before starting!

- This test is closed book and closed notes
- You may ***NOT*** use a calculator
- All answers must have a box drawn around them. This is to aid the grader (who might not be me!) Failure to do so might result in no credit for answer.
- If you perform work on the back of a page in this test, indicate that you have done so in case the need arises for partial credit to be determined.
- Statement regarding academic misconduct from Section 5.7 of the East Tennessee State University Faculty Handbook, June 1, 2001:

  "Academic misconduct will be subject to disciplinary action. Any act of dishonesty in academic work constitutes academic misconduct. This includes plagiarism, the changing of falsifying of any academic documents or materials, cheating, and the giving or receiving of unauthorized aid in tests, examinations, or other assigned school work. Penalties for academic misconduct will vary with the seriousness of the offense and may include, but are not limited to: a grade of 'F' on the work in question, a grade of 'F' of the course, reprimand, probation, suspension, and expulsion. For a second academic offense the penalty is permanent expulsion."

Addition principle of cardinality:

- $|A \cup B| = |A| + |B| – |A \cap B|$
- $|A \cup B \cup C| = |A| + |B| + |C| – |A \cap B| – |A \cap C| – |B \cap C| + |A \cap B \cap C|$

Properties of characteristic functions:

- $f_{A\cap B} = f_A f_B$; that is $f_{A\cap B}(x) = f_A(x)f_B(x)$ for all x.
- $f_{A\cup B} = f_A + f_B – f_A f_B$; that is $f_{A\cup B}(x) = f_A(x) + f_B(x) – f_A(x)f_B(x)$ for all x.
- $f_{A\oplus B} = f_A + f_B – 2f_A f_B$; that is $f_{A\oplus B}(x) = f_A(x) + f_B(x) – 2f_A(x)f_B(x)$ for all x.

Properties of integers:

- If n and m are integers and n > 0, we can write m = qn + r for integers q and r with $0 \leq r < n$.
- Properties of divisibility:

If a | b and a | c, then a | (b + c)	If a | b or a | c, then a | bc
If a | b and a | c, where b > c, then a | (b - c)	If a | b and b | c, then a | c

- Every positive integer n > 1 can be written uniquely as $n = p_1^{k1}p_2^{k2}p_3^{k3}p_4^{k4} \ldots p_s^{ks}$ where $p_1 < p_2 < p_3 < p_4 < \ldots < p_s$ are distinct primes that divide n and the k's are positive integers giving the number of times each prime occurs as a factor of n.

Properties of operations for propositions

Commutative Properties

1. $p \vee q \equiv q \vee p$
2. $p \wedge q \equiv q \wedge p$

Associative Properties

3. $p \vee (q \vee r) \equiv (p \vee q) \vee r$
4. $p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$

Distributive Properties

5. $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
6. $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

Idempotent Properties

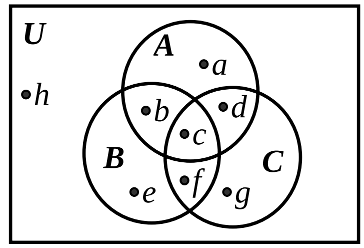
7. $p \vee p \equiv p$
8. $p \wedge p \equiv p$

Properties of Negation

9. $\sim(\sim p) \equiv p$
10. $\sim(p \vee q) \equiv (\sim p) \wedge (\sim q)$
11. $\sim(p \wedge q) \equiv (\sim p) \vee (\sim q)$

Short Short Problems (2 points each unless otherwise noted)

Problems 1 through 6 refer to the Venn Diagram shown to the right.



1. True or False: $f \in \boldsymbol{A} \cap \boldsymbol{B}$.
2. True or False: $c \notin \overline{\boldsymbol{A}}$.
3. True or False: $b \in \boldsymbol{A} - (\boldsymbol{A} \cap \boldsymbol{B} \cap \boldsymbol{C})$.
4. True or False: $(\boldsymbol{A} \cap \boldsymbol{B} \cap \boldsymbol{C}) \subseteq \boldsymbol{C} - (\boldsymbol{B} - \boldsymbol{A})$
5. Using unions, intersections, and complements of sets ***A***, ***B***, and ***C***, write an expression to describe the set that contains *b* and *e*, but none of the other labeled points. (3 points)

6. Using unions, intersections, and complements of sets ***A***, ***B***, and ***C***, write an expression to describe the set that contains *c, f,* and *g,* but none of the other labeled points. (3 points)

7. Give the set corresponding to the sequence ***asdseeseddes***.

8. If set ***A*** has 32 elements, set ***B*** has 40 elements, and they have 5 elements in common, how many elements are a member of either ***A*** or ***B*** but not both? (3 points)

9. Identify one of the two cases where it is possible to have $|A \cup B \cup C| = |A| + |B| + |C|$.

10. True or False: Given the set A = {*ab, ca, bc*}, the string *abababc* belongs to the set ***A****.
11. True or False: The string *abb* belongs to the set corresponding to the regular expression a^*b^*c.
12. True or False: The string *c* belongs to the set corresponding to the regular expression a^*b^*c.
13. True or False: The string *aaac* belongs to the set corresponding to the regular expression $(a^* \vee c)^*$.
14. Write three elements that are members of the regular set corresponding to the regular expression $(0(01)^*) \vee (1(10)^*)$. (3 points)

15. Let *S* = {0,1}. Give the regular expression corresponding to the regular set {00, 010, 0110, 01110, 011110,…}. (3 points)

16. Write the expansion in base 7 of 218_{10}. (3 points)

    Answer: ____________________________

17. Write the expansion in base 6 of 221_{10}. (3 points)

Answer: ______________________________

18. True or False: 53436_7 is a valid base 7 number.

19. True or False: If $\boldsymbol{A}$ is a matrix, to compute $\boldsymbol{A}^p$, p is a positive integer, $p \geq 2$, A must be a square matrix.

20. True or False: Every matrix has a transpose.

21. True or False: Every matrix has a multiplicative inverse.

For problems 22, 23, and 24, use the matrices ***A***, ***B***, and ***C*** shown below.

$$\boldsymbol{A} = \begin{bmatrix} 2 & 1 & 3 \\ 3 & 1 & 1 \end{bmatrix} \qquad \boldsymbol{B} = \begin{bmatrix} 3 & 1 \\ 2 & 2 \\ 3 & 1 \end{bmatrix} \qquad \boldsymbol{C} = \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$$

22. True or False: It is possible to compute $\boldsymbol{A} + \boldsymbol{B}^T$.

23. True or False: It is possible to compute $\boldsymbol{AC}$.

24. True or False: It is possible to compute $\boldsymbol{AB} + \boldsymbol{C}$.

25. Give the negation of the statement "It will rain tomorrow or it will snow tomorrow."

For problems 26 and 27, find the truth value of each proposition if ***p* and *r* are true** and ***q* is false**.

26. $\sim p \wedge \sim q$ Answer: ______________________________

27. $\sim(p \vee q) \wedge r$ Answer: ______________________________

For problems 28 and 29, convert the sentence given to an expression in terms of p, q, r, and logical connectives if p: I studied; q: I'm having fun; and r: This test is difficult.

28. I didn't study, but I'm having fun. Answer: ______________________________

29. Either I studied or this test is easy. Answer: ______________________________

Medium-ish Problems (5 points each)

30. Convert the base 5 number 342_5 to base 10.

31. Find the greatest common divisor of 126 and 420.

32. Use any method you wish to find the least common multiple of 150 and 70.

33. Use truth tables to show that $p \Rightarrow q$ is equivalent to $\sim q \Rightarrow \sim p$. Show ***all*** intermediate steps.

34. Use truth tables to show that $\sim(p \Rightarrow q) \Rightarrow p$ is a tautology. Show ***all*** intermediate steps.

35. For the matrix $\boldsymbol{A}$ shown to the right, calculate $\boldsymbol{A}^3$. Show ***all*** work.

$$\boldsymbol{A} = \begin{bmatrix} 2 & 3 \\ 3 & 1 \end{bmatrix}$$

36. Using matrices A and B shown below, verify the theorem $(\boldsymbol{AB})^T = \boldsymbol{B}^T\boldsymbol{A}^T$. Show ***all*** work.

$$\boldsymbol{A} = \begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix} \quad \boldsymbol{B} = \begin{bmatrix} 2 & 0 \\ 3 & 1 \end{bmatrix}$$