Points missed: _____ Student's Name: _____

Total score: _____/100 points

East Tennessee State University – Department of Computer and Information Sciences
CSCI 2710 (Tarnoff) – Discrete Structures
TEST 1 for Spring Semester, 2005

Read this before starting!

- This test is closed book and closed notes
- You may ***NOT*** use a calculator
- All answers must have a box drawn around them. This is to aid the grader (who might not be me!) Failure to do so might result in no credit for answer.
- If you perform work on the back of a page in this test, indicate that you have done so in case the need arises for partial credit to be determined.
- ***If not otherwise identified, every integer is assumed to be represented using base 10.***
- Statement regarding academic misconduct from Section 5.7 of the East Tennessee State University Faculty Handbook, June 1, 2001:

"Academic misconduct will be subject to disciplinary action. Any act of dishonesty in academic work constitutes academic misconduct. This includes plagiarism, the changing of falsifying of any academic documents or materials, cheating, and the giving or receiving of unauthorized aid in tests, examinations, or other assigned school work. Penalties for academic misconduct will vary with the seriousness of the offense and may include, but are not limited to: a grade of 'F' on the work in question, a grade of 'F' of the course, reprimand, probation, suspension, and expulsion. For a second academic offense the penalty is permanent expulsion."

Algebraic properties of sets:

- $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
- $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
- $A \cup A = A$
- $A \cap A = A$
- $\overline{(\overline{A})} = A$
- $\overline{A} \cup A = U$
- $\overline{A} \cap A = \varnothing$
- $\overline{\varnothing} = U$
- $\overline{U} = \varnothing$
- $\overline{A \cup B} = \overline{A} \cap \overline{B}$
- $\overline{A \cap B} = \overline{A} \cup \overline{B}$
- $A \cup U = U$
- $A \cap U = A$
- $A \cup \varnothing = A$ or $A \cup \{\ \} = A$
- $A \cap \varnothing = \varnothing$ or $A \cap \{\ \} = \{\ \}$

Addition principle of cardinality:

- $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$

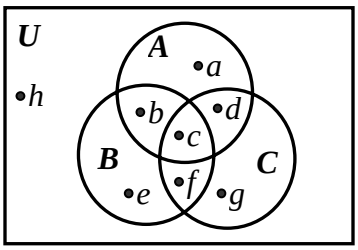
Properties of characteristic functions:

- $f_{A \cap B} = f_A f_B$; that is $f_{A \cap B}(x) = f_A(x) f_B(x)$ for all x.
- $f_{A \cup B} = f_A + f_B - f_A f_B$; that is $f_{A \cup B}(x) = f_A(x) + f_B(x) - f_A(x) f_B(x)$ for all x.
- $f_{A \oplus B} = f_A + f_B - 2 f_A f_B$; that is $f_{A \oplus B}(x) = f_A(x) + f_B(x) - 2 f_A(x) f_B(x)$ for all x.

Properties of integers:

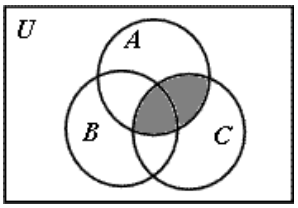
- If n and m are integers and $n > 0$, we can write $m = qn + r$ for integers q and r with $0 \leq r < n$.
- Properties of divisibility:
  If a | b and a | c, then a | (b + c) If a | b or a | c, then a | bc
  If a | b and a | c, where b > c, then a | (b - c) If a | b and b | c, then a | c
- Every positive integer $n > 1$ can be written uniquely as $n = p_1^{k1} p_2^{k2} p_3^{k3} p_4^{k4} \ldots p_s^{ks}$ where $p_1 < p_2 < p_3 < p_4 < \ldots < p_s$ are distinct primes that divide n and the k's are positive integers giving the number of times each prime occurs as a factor of n.

Problems 1 through 4 refer to the Venn Diagram shown to the right. (2 points each)

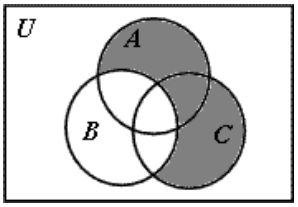


1. True or False: $f \notin \overline{\boldsymbol{A}}$
2. True or False: $d \in \boldsymbol{A} - \boldsymbol{B}$
3. True or False: $d \in \boldsymbol{A} \cup \boldsymbol{B}$
4. True or False: $b \in \boldsymbol{B} - (\boldsymbol{A} \cap \boldsymbol{B} \cap \boldsymbol{C})$

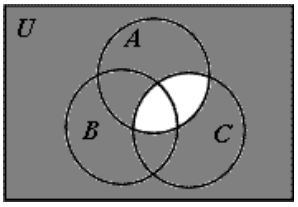
For problems 5, 6, and 7, use unions, intersections, and complements of the sets ***A***, ***B***, and ***C***, to write an expression to describe the set represented by the shaded area in the given Venn Diagram. (3 points each)

5. 

Answer for 5: _____

6. 

Answer for 6: _____

7. 

Answer for 7: _____

8. If ***A*** = {1, 2, 3, 4, 5} and ***B*** = {4, 5, 6, 7, 8}, compute the set represented by $\boldsymbol{A} \cap \boldsymbol{B}$. (3 points)

   $\boldsymbol{A} \cap \boldsymbol{B}$=

9. If ***A*** = {1, 2, 3, 4, 5} and ***B*** = {4, 5, 6, 7, 8}, compute the set represented by $\boldsymbol{A} \oplus \boldsymbol{B}$. (3 points)

   $\boldsymbol{A} \oplus \boldsymbol{B}$=

10. If ***A*** = {1, 2, 3, 4, 5} and ***U*** = {0, 1, 2, 3, 4, 5, 6, 7, 8}, compute the set represented by $\overline{\boldsymbol{A}}$. (3 points)

    $\overline{\boldsymbol{A}}$ =

11. If set ***A*** has 32 elements, set ***B*** has 40 elements, and they have 5 elements in common, how many elements are a member of $\boldsymbol{A} \cup \boldsymbol{B}$? (3 points)

12. Give three different sequences that have {a, d, s} as the corresponding set. (3 points)

13. Write a formula for the n-th term of the sequence 1, 6, 11, 16, 21, 26, … (3 points)

For problems 14, 15, and 16, determine if the sequence defined by the formula is explicit or recursive. In addition, identify the value of the 3rd element in the sequence. (3 points each)

14. $a_n = 2 \cdot n$ ☐ explicit ☐ recursive a_3 = _____

15. $c_1 = 1$; $c_n = 2 \cdot c_{n-1}$ ☐ explicit ☐ recursive c_3 = _____

16. $d_1 = 3$; $d_2 = 5$; $d_n = d_{n-1} + d_{n-2}$ ☐ explicit ☐ recursive d_3 = _____

17. What advantage does an explicit formula have over a recursive formula? (3 points)

18. True or False: Given the set A = {*ab, bc, ba*}, the string *abbabaab* belongs to the set ***A****. (2 points)

For problems 19, 20, and 21, tell whether or not the string on the left belongs to the regular set corresponding to the regular expression on the right. (2 points each)

19. string: 01111110 regular expression: 0(1*)0 ☐ Belongs ☐ Doesn't belong

20. string: Λ (empty string) regular expression: (*ab*)**c* ☐ Belongs ☐ Doesn't belong

21. string: *cccc* regular expression: (*a** ∨ *b*)*c** ☐ Belongs ☐ Doesn't belong

22. Write three elements that are members of the regular set corresponding to the regular expression 0(11)*1. (3 points)

23. Give the regular expression corresponding to the regular set {ad, abcd, abcbcd, abcbcbcd, abcbcbcbcd, abcbcbcbcbcd,…}. (3 points)

24. True or False: 53496_9 is a valid base 9 number. (2 points)

25. Write the expansion in base 5 of 221_{10}. (4 points)

Answer for 25: _____

26. What is the result of *f*(20) if *f* is the mod-3 function, i.e., calculate 20 mod 3. (2 points)

27. Convert the base 5 number 322_5 to base 10. Just write out the formula with the correct values; do not worry about doing the final calculation. (3 points)

28. If $m = 24$ and $n = 6$, determine the values of q and r that satisfy the expression $m = q \cdot n + r$ such that $0 \leq r < n$. (2 points)

$q =$ _____ $r =$ _____

29. Write the integer 6160_{10} as a product of powers of primes. (4 points)

30. Use any method you wish to find the greatest common divisor of 84 and 770. (4 points)

31. Use any method you wish to find the least common multiple of 300 and 210. (4 points)

32. True or False: If $\boldsymbol{A}$ is a matrix, to compute $\boldsymbol{A}^2$, $\boldsymbol{A}$ must be a square matrix. (2 points)

33. True or False: If $\boldsymbol{A}$ and $\boldsymbol{B}$ are matrices, then $\boldsymbol{AB} = \boldsymbol{BA}$. (2 points)

For problems 34 through 37, use the matrices $\boldsymbol{A}$, $\boldsymbol{B}$, and $\boldsymbol{C}$ shown below.

$$\boldsymbol{A} = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \qquad \boldsymbol{B} = \begin{bmatrix} 4 & 2 \\ 3 & 1 \\ 2 & 5 \end{bmatrix} \qquad \boldsymbol{C} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

34. True or False: It is possible to compute $\boldsymbol{A}^{\boldsymbol{T}} + \boldsymbol{B}$. (2 points)

35. True or False: The result of $\boldsymbol{BC}$ is matrix with 3 rows and 1 column. (2 points)

36. True or False: It is possible to compute $\boldsymbol{CA} + \boldsymbol{B}^{\boldsymbol{T}}$. (2 points)

37. Calculate $\boldsymbol{AB}$. Show ***all*** work. (4 points)