Chapter 3. Differentiation

3.1 Tangents and the Derivative at a Point

Note. We now return to an idea introduced in Section 2.1: Slopes of lines tangent to curves.

Definition. Slope and Tangent Line.

The slope of the curve \( y = f(x) \) at the point \( P(x_0, f(x_0)) \) is the number

\[
m = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h},
\]

provided the limit exists. The tangent line to the curve at \( P \) is the line through \( P \) with this slope.

Figure 3.1, page 122
**Example.** Page 125 number 7.

**Definition.** Derivative at a Point.

The derivative of a function $f$ at a point $x_0$, denoted $f'(x_0)$, is

$$f'(x_0) = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}$$

provided this limit exists.

**Example.** Page 125 number 26.

**Note.** Since the derivative of a function at a point is a limit of an average rate of change (to recall a topic from Section 2.1), then we see that the derivative can be interpreted as an instantaneous rate of change of the function $f$ with respect to the variable $x$. For example, if $f(t)$ is the position of a particle at time $t$, then the instantaneous rate of change of position with respect to time (i.e. the instantaneous velocity) at time $t = t_0$ is

$$f'(t_0) = \lim_{h \to 0} \frac{f(t_0 + h) - f(t_0)}{h},$$

provided the limit exists.

**Examples.** Page 125 number 28 and Page 126 number 36 (vertical tangents).