Chapter 3. Differentiation

3.4 The Derivative as a Rate of Change

Definition. Instantaneous Rate of Change

The instantaneous rate of change of $f$ with respect to $x$ at $x_0$ is the derivative

$$f'(x_0) = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h},$$

provided the limit exists.

Definition. (Instantaneous) Velocity

Velocity (instantaneous velocity) is the derivative of position with respect to time. If a body’s position at time $t$ is $s = f(t)$, then the body’s velocity at time $t$ is

$$v(t) = \frac{ds}{dt} = \lim_{\Delta t \to 0} \frac{f(t + \Delta t) - f(t)}{\Delta t}.$$

Definition. Speed

Speed is the absolute value of velocity.

$$\text{Speed} = |v(t)| = \left| \frac{ds}{dt} \right|$$
Definition. Acceleration, Jerk

*Acceleration* is the derivative of velocity with respect to time. If a body’s position at time $t$ is $s = f(t)$, then the body’s acceleration at time $t$ is

$$ a(t) = \frac{dv}{dt} = \frac{d^2 s}{dt^2}. $$

*Jerk* is the derivative of acceleration with respect to time:

$$ j(t) = \frac{da}{dt} = \frac{d^3 s}{dt^3}. $$

*Example.* Page 152 number 12.

*Note.* At the surface of the Earth, if an object is fired directly upward with an initial (upward) velocity $v_0$ from an initial height $s_0$, then the height of the object at time $t$ is

$$ s(t) = -16t^2 + v_0 t + s_0 $$

if time is measured in seconds and distances are measured in feet, or

$$ s(t) = -4.9t^2 + v_0 t + s_0 $$

if time is measured in seconds and distances are measured in meters. Notice what this implies that the accelerations are.
Example. Page 154 number 22.

Note. In economics, the term “marginal” is used when referring to derivatives. If a company produces and sells a number $x$ of objects, and the cost of producing those objects is $c(x)$ and the revenue that results from selling them is $r(x)$, then the resulting profit is $p(x) = r(x) - c(x)$. The functions $p'(x)$, $r'(x)$, and $c'(x)$ are the marginal profit, revenue, and cost functions, respectively.

Example. Page 154 number 24.