Chapter 4. Applications of Derivatives

4.1 Extreme Values of Functions

Definition. Let \( f \) be a function with domain \( D \). Then \( f(c) \) is the

(a) absolute maximum value on \( D \) if and only if \( f(x) \leq f(c) \) for all \( x \) in \( D \)

(b) absolute minimum value on \( D \) if and only if \( f(x) \geq f(c) \) for all \( x \) in \( D \).

Theorem 1. The Extreme-Value Theorem for Continuous Functions

If \( f \) is continuous at every point of a closed and bounded interval \( I = [a, b] \),
then \( f \) assumes both an absolute maximum value \( M \) and an absolute minimum value \( m \) somewhere in \( I \). That is, there are numbers \( x_1 \) and \( x_2 \) in \( I = [a, b] \) with \( f(x_1) = m, f(x_2) = M \), and \( m \leq f(x) \leq M \) for every \( x \) in \( I = [a, b] \).

Examples. Page 227 numbers 2 and 4.
Definition. Let \( c \) be an interior point of the domain of the function \( f \). Then \( f(c) \) is a

(a) local maximum value if and only if \( f(x) \leq f(c) \) for all \( x \) in some open interval containing \( c \)

(b) local minimum value if and only if \( f(x) \geq f(c) \) for all \( x \) in some open interval containing \( c \).

Theorem 2. Local Extreme Values.

If a function \( f \) has a local maximum value or a local minimum value at an interior point \( c \) of its domain, and if \( f' \) exists at \( c \), then \( f'(c) = 0 \).

Definition. A point in the domain of a function \( f \) at which \( f' = 0 \) or \( f' \) does not exist is a critical point of \( f \).
Note. How to Find the Absolute Extrema of a Continuous Function $f$ on a Closed Interval

To find extrema on a closed and bounded interval, we first find the critical points and then:

**Step 1.** Evaluate $f$ at all critical points and endpoints.

**Step 2.** Take the largest and smallest of these values.

**Examples.** Page 228 number 24, Page 229 numbers 68, 72 and 80.