Chapter 5. Integration

5.6 Substitution and Area Between Curves

Note. We can use \( u \)-substitution in definite integrals:

\[
\int_a^b f(g(x))g'(x)dx = \int_{g(a)}^{g(b)} f(u) \, du
\]

where \( u = g(x) \), and \( du = g'(x) \, dx \).

Examples. Page 410 numbers 14a and 18.

Definition. If \( f \) and \( g \) are continuous with \( f(x) \geq g(x) \) throughout \([a, b]\), then the area of the region between the curves \( y = f(x) \) and \( y = g(x) \) from \( a \) to \( b \) is the integral of \([f - g]\) from \( a \) to \( b \):

\[
A = \int_a^b [f(x) - g(x)] \, dx.
\]
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Note. We will take a heuristic shortcut and take “$dx$” slices.

Examples. Page 412 numbers 56 and 66, page 413 number 102.

Figure 5.29, page 407