Chapter II. Metric Spaces and the Topology of \( \mathbb{C} \)

Study Guide

The following is a brief list of topics covered in Chapter II of Conway's *Functions of One Complex Variable*, 2nd edition. This list is not meant to be comprehensive, but only gives a list of several important topics. You should also carefully study the proofs given in class and the homework problems.

**Section II.1. Definition and Examples of Metric Spaces.**
Metric space, metric, open ball, closed ball, taxicab metric, Euclidean metric, metric versus norm, \( \ell^2 \), \( \ell^p \) for \( 1 \leq p < \infty \), Banach space, Hilbert space, open set, properties of open sets (Theorem 1.9), closed set, properties of closed sets (Theorem 1.11), topological space, interior of a set, closure of a set, boundary of a set, relationship between open/closed/interior/boundary and \( \varepsilon \) (Theorem 1.13), dense set.

**Section II.2. Connectedness.**
Connected metric space and connected sets, separation of a set, line segment, polygon, component of a set, components partition a set (Theorem 2.7), open set in \( \mathbb{C} \) (Theorem 2.9).

**Section II.3. Sequences and Completeness.**
Convergent sequence, limit point of a set, closed sets and limit points (Propositions 3.2 and 3.4), Cauchy sequence, complete metric space, \( \mathbb{C} \) is complete (Proposition 3.6), diameter of a set, “Cantor’s Theorem.”

**Section II.4. Compactness.**
compact set, open cover, a closed and bounded set in \( \ell^2 \) that is not compact, compact sets are closed and bounded (Proposition 4.3), finite intersection property, every compact metric space is complete (Corollary 4.5), every infinite set in a compact metric space has a limit point (Corollary 4.6), sequentially compact metric space, Lebesgue’s Covering Lemma, total boundedness, equivalences to compact (Proposition 4.9), the Heine-Borel Theorem (Theorem 4.10).

**Section I.5. Continuity.**
Continuous function at a point and on a set, equivalences to continuity (Propositions 5.2 and 5.3), limit of a function between topological spaces, definition of continuity for a function between topological spaces, uniformly continuous function, Lipschitz function, distance from a point to a set and properties (Proposition 5.7), properties of sets preserved by continuous functions (Theorem 5.8), Intermediate Value Theorem (Theorem 5.11), Extreme Value Theorem (Corollary 5.12), distance from a point to a compact set (Corollary 5.14), continuity on a compact set implies uniform continuity (Theorem 5.15), distance between two sets, distance from a closed to a compact set (Theorem 5.17).

**Section II.6. Uniform Convergence.**
Uniform convergence of a sequence of functions, the uniform limit of continuous functions is continuous (Theorem 6.11), Weierstrass M-Test (Theorem 6.2).

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