Since the cylinder is not leaking, the number of gas particles present remains constant, so \( n_2 \) (number of moles after compression) = \( n_1 \) (number of moles before compression).

Here we will use Form 4 of the ideal gas law: \( PV = nRT \), and rewrite it as

\[
N = \frac{PV}{kT}
\]

SO:

\[
n_2 = n_1 \quad \text{and} \quad T_1 = 27 \degree C + 273 = 300 \text{ K}
\]

\[
\frac{P_2V_2}{RT_2} = \frac{P_1V_1}{RT_1}
\]

\[
\left( \frac{P_2V_2}{P_1V_1} \right) = \frac{T_1}{T_2}
\]

\[
T_2 = \left( \frac{0.800 \times 10^5 \text{ Pa}}{0.200 \times 10^5 \text{ Pa}} \right) \left( \frac{0.700 \text{ m}^3}{1.50 \text{ m}^3} \right) (300 \text{ K})
\]

\[
= 560 \text{ K} - 273 = 287 \degree \text{C}
\]
2.

a) \( n = 1 \text{ mol} \) and \( T = 300 \text{ K} \)

Eq. (XIII-14) gives \( \overline{KE} = \frac{3}{2} \frac{k_B}{m} T \) per particle

\[
\overline{KE} = \frac{3}{2} \left( 1.38 \times 10^{-23} \frac{J}{K} \right) (300 \text{ K})/\text{particle} \]

\[
= \frac{6.21 \times 10^{-21}}{\text{J/particle}} \]

However, there are \( n = 1 \text{ mol} \) or \( n N_A \) particles per mole, so we could also express this KE as

\[
\overline{KE} = \left( 6.21 \times 10^{-21} \frac{J}{\text{particle}} \right) n N_A \]

\[
= \left( 6.21 \times 10^{-21} \frac{J}{\text{particle}} \right) (1 \text{ mol}) (6.02 \times 10^{23} \text{ part./mol}) \]

\[
= 3740 \text{ J} \]

Total KE energy for all these particles making up this 1 mol of gas.

3.

a) \( \nu_{\text{rms}} (H_2) = ? \)

\[
\nu_{\text{rms}} = \sqrt{\frac{3k_B T}{m}} \]

\( k_B = 1.38 \times 10^{-23} \frac{J}{K} \)

\( m = 1.0 \text{ u} \)

\( \nu_{\text{rms}} = 10.3 \text{ km/s} \)
\[ m_{\text{H}_2} = 2m_H = 2 \left( 1.67 \times 10^{-27} \text{kg} \right) = 3.34 \times 10^{-27} \text{kg} \]

\[ \nu_{\text{rms}} (\text{H}_2) = \sqrt{\frac{3 \left( 1.38 \times 10^{-23} \text{J/K} \right) (240 \text{K})}{3.34 \times 10^{-27} \text{kg}}} \]

\[ (1 \text{ J} = 1 \text{ kg m}^2 \text{ s}^{-2}) \]

\[ \nu_{\text{rms}} (\text{H}_2) = \sqrt{2.97 \times 10^{6} \text{ m}^2 \text{ s}^{-2}} = 1.72 \times 10^{3} \text{ m/s} \]

\[ \nu_{\text{rms}} (\text{H}_2) = 1.72 \text{ km/s} \]

b) \[ \nu_{\text{rms}} (\text{CO}_2) = ? \]

\[ m (\text{CO}_2) = m_c + 2m_H = (12.01 \text{ amu}) (1.67 \times 10^{-27} \text{ kg/amu}) + 2 (16.00 \text{ amu}) (1.67 \times 10^{-27} \text{ kg/amu}) \]

\[ = 7.35 \times 10^{-26} \text{ kg} \]

\[ \nu_{\text{rms}} (\text{CO}_2) = \sqrt{\frac{3 \left( 1.38 \times 10^{-23} \text{ J/K} \right) (240 \text{ K})}{7.35 \times 10^{-26} \text{ kg}}} \]

\[ = 3.68 \times 10^{2} \text{ m/s} = 0.368 \text{ km/s} \]

c) \[ \nu_{\text{esc}} = 10.3 \text{ km/s} \]. If \( \nu_{\text{rms}} > \frac{\nu_{\text{esc}}}{c} \), then the gas will not be bound to the planet for an extended period of time.

For \( \text{H}_2 \) \( \nu_{\text{rms}} > \frac{1}{6} \nu_{\text{esc}} = 1.72 \text{ km/s} \)? Since \( \nu_{\text{rms}} (\text{H}_2) > \frac{1}{6} \nu_{\text{esc}} \), \( \text{H}_2 \) will escape, but just barely. Meanwhile \( \nu_{\text{rms}} (\text{CO}_2) < \frac{1}{6} \nu_{\text{esc}} \), so \( \text{CO}_2 \) will remain for an extended period of time.
(a) The initial absolute pressure in the tire is
\[ P_1 = P_0 + (P_1)_{gaje}, \text{ where } P_0 = P_{atm} = 1.00 \, \text{atm} \]
\[ = 1.00 \, \text{atm} + 1.80 \, \text{atm} = 2.80 \, \text{atm} \]
The final pressure is thus \( P_2 = 1.00 \, \text{atm} + 2.20 \, \text{atm} \)
\[ = 3.20 \, \text{atm} \]
Since volume is constant, use the Charles and Gay-Lussac Law: \( \frac{P_1}{T_1} = \frac{P_2}{T_2} \), so solving for \( T_2 \) gives
\[ T_2 = T_1 \left( \frac{P_2}{P_1} \right) = (300 \, \text{K}) \left( \frac{3.20 \, \text{atm}}{2.80 \, \text{atm}} \right) = 343 \, \text{K} \]
(b) When the quantity of gas varies, while \( T \) and \( V \) are constant, the ideal gas law gives \( \frac{V}{T} = \frac{nR}{P} = \text{constant} \), so
\[ \frac{n_2R}{P_2} = \frac{n_3R}{P_3} \text{ or } \frac{n_3}{n_2} = \frac{P_3}{P_2} \]
\[ \frac{n_3}{n_2} = \frac{2.80 \, \text{atm}}{3.20 \, \text{atm}} = 0.875 \]
At the end, we have 87.5% of the original mass of air remaining, or
\[ \frac{12.5}{100} \text{ of the original mass} \] was released.
a) Work done by changing volume is \( W = P \Delta V \)
\[
P = 0.800 \text{ atm} \times 1.013 \times 10^5 \frac{\text{Pa}}{\text{atm}} = 8.104 \times 10^4 \text{ Pa}
\]

\[
\Delta V = V_f - V_i = 2.00 \text{ L} - 9.00 \text{ L} = -7.00 \text{ L} \times 10^{-3} \text{ m}^3
\]

\[
W = -P \Delta V = - (8.104 \times 10^4 \text{ Pa}) (-7.00 \times 10^{-3} \text{ m}^3) = +567 \text{ J}
\]

b) Change in internal energy: \( \Delta U = Q + W \)

\[
Q = -400 \text{ J} \quad \text{(negative since it flows out)}
\]

\[
\Delta U = -400 \text{ J} + (567 \text{ J}) = +167 \text{ J}
\]

6a) Engine's efficiency (e)?
\[
e = \frac{W}{Q_h} = \frac{\text{work done during cycle}}{\text{heat at hottest temp}} = \frac{Q_n - Q_c}{Q_n}
\]

\[
= \frac{1700 \text{ J} - 1200 \text{ J}}{1700 \text{ J}} = \frac{500 \text{ J}}{1700 \text{ J}} = 0.29
\]

29% efficiency
6) Work done in each cycle?

\[ W = Q_n - Q_c = 1700 \text{ J} - 1200 \text{ J} = 500 \text{ J} \]

7) Power output if each cycle lasts 0.300 s?

\[ P = \frac{W}{t} = \frac{500 \text{ J}}{0.300 \text{ s}} = 1666.67 \text{ W} = 1.67 \times 10^3 \text{ W} \]

\[ T_0 = 5700 \text{ K}, \quad T_\oplus = 290 \text{ K}, \quad Q = 1000 \text{ J} \]

\[ \Delta S_\oplus = -\frac{Q}{T_0} \quad \text{(entropy is negative since Sun is losing it)} \]

\[ \Delta S_\oplus = \frac{Q}{T_\oplus} \quad \text{(positive since Earth is gaining it)} \]

\[ \Delta S_{\text{TOTAL}} = \Delta S_\oplus + \Delta S_\oplus = -\frac{Q}{T_0} + \frac{Q}{T_\oplus} = Q \left( \frac{1}{T_\oplus} - \frac{1}{T_0} \right) \]

\[ = (1000 \text{ J}) \left( \frac{1}{290 \text{ K}} - \frac{1}{5700 \text{ K}} \right) = 3.27 \text{ J/K} \]

\[ \text{Let probability} = P \]

8) a) Only 1 ace of spades per 52 cards: \[ P = \frac{1}{52} \]

b) Only 4 aces out of 52 cards: \[ P = \frac{4}{52} = \frac{1}{13} \]

c) 13 spades out of 52 cards: \[ P = \frac{13}{52} = \frac{1}{4} \]